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Application of Soft Semi-Open Sets to Soft Binary Topology

Arif Mehmood Khattak
Department of Mathematics,
Department of Mathematics and Statistics, Riphah International University, Sector I-14, Islamabad, Pakistan,
Email corresponding author: mehdaniyal@gmail.com
Muhammad Rahim Khan
Department of Mathematics,
University of Science and Technology,Bannu, Pakistan,
Email: Rahimkhan4411@gmai.com
Muhammad Zamir
Department of Mathematics,
University of Science and Technology,Bannu, Pakistan,
Email: zamirburqi@ustb.edu.pk
Fawad Nadeem
Department of Mathematics,
University of Science and Technology,Bannu, Pakistan,
Email: fawadnadeem2@gmail.com
Saleem Abduallah
Department of Mathematics,
Abdul wali khan University Mardan, Pakistan,
Email: saleemabdullah81@yahoo.com
Shamoona Jabeen
School of Mathematics and system Science, Beihang University Beijing China,
Email: shamoonaafzal@yahoo.com

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Abstract. This paper introduces an application of soft semi-open sets in soft binary topology. An important outcome of this work is a formal framework for the study of information associated with ordered pairs of soft sets. Five main results concerning binary soft topological spaces are given in this paper.

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Key Words: Binary soft topology, binary soft weak open sets. binary soft weak separation axioms and binary soft T_o space.

1. INTRODUCTION

The concept of soft sets was first introduced by Molodtsov [8] in 1999 as a general mathematical for dealing with uncertain substances. In [8, 9] Molodtsov beautifully applied the soft theory in numerous ways such as smoothness of functions, game theory, operations research, Riemann integration perron integration, probability, theory of measurement, and so on. M. Shabir and M. Naz [11] discussed some new operations on soft set theory. Soft point set topology deals with a non- empty set X together with a collection τ of subset X under some set of of parameters satisfying certain conditions. Khattak et al [6] introduced the notion of soft sub-spaces and soft b-separation axioms in binary soft topological spaces. Khattak et al [7] planned the idea of binary soft most separation axioms in binary soft topological spaces. S.S.Benchalli et al [2] threw his detailed discussion on binary soft topological. A. Kalaichelvi and P.H. Malini [5] beautifully discussed application of fuzzy soft sets to investment decision and also discussed some more results related to this particular field. N. Y . Ozgulr and N. Tas, [10] studied some more applications of fuzzy soft sets to investment decision making problem. N. Tas , N.Y.Ozgur and P.Demir [13] worked over an application of soft set and fuzzy soft set theories to stock management J.C.R. Alcantud et al [1] carefully discussed valuation fuzzy soft sets : A flexible fuzzy soft sets based decision making procedure for the valuation of sets N.Cagman and S. Enginoglu [4]. In explored soft matrix theory and some very basic results to it and its decision making. Borah et al [3] discussed Soft ideal topological space and mixed fuzzy soft ideal topological space. Tahat et al [12] studied Soft topological soft groups and soft rings.
In continuation, in the present paper binary soft topological structures known as soft weak structures with respect to first coordinate as well as with respect to second coordinate are defined. Moreover some basic results related to this structures are also planted in this paper. The same structures are defined over soft points of binary soft topological structure and related results are also reflected here with respect to ordinary and soft points.

2. PRELIMINARIES

Definition 2.1. *Let X be an initial universe and let E be a set of paramaters. Let P(X) denote the power set of X and let A be a non empty subset of E . A pair (F, A) is called A soft set over X, where F is a mapping given by : A→P(X). In other some words, a soft over X is a parameterized family subsets of the universe X. for $\varepsilon \in A$, F(ε) may be considered as the set of $\varepsilon-$approximate elements of the soft set (F, A). Clearly , a soft set is not a set.Let U_1 , U_2 be two initial universe sets and E be a set of parameters Let $P(U_1)$, $P(U_2)$ denote the power set of U_1 , U_2 respectively. Also , Let A, B, C $\subseteq$ E*

Definition 2.2. [6] *A pair (F, A) is said be binary soft set over $U_{1,}U_2$ where F is defined below:*

F:A$\longrightarrow P(U_1) \times P(U_2), F(e)$ = (X, Y) for each e$\in A$ such that X$\subseteq U_1$ Y$\subseteq U_2$

Definition 2.3. [6] *A binary soft set (F, A) over U_1, U_2 is called a binary absolute soft set denoted by $\overset{\approx}{A}$ if F(e)=(U_1, U_2) for each e$\in A$*

Definition 2.4. [6] *The extended union of two binary soft sets of (F, A) and (G, B) over common U_1, U_2 is the binary soft set (H, C) where $c = A \cup B$ and for all $e \in C$*

$$h(e) = \begin{cases} (X_1, Y_1)\ if\ e \in A - B \\ (X_2, Y_2)\ if\ e \in B - A \\ (X_1 \cup X_2, Y_1 \cup Y_2)\ if\ e \in A \cap B \end{cases} \tag{2. 1}$$

Such that F(e)= (X_1, Y_1) for each $e \in A$ and G(e)= (X_2, Y_2) for each $e \in B$ We denote it (F, A)$\overset{\approx}{\cup}$(G, B)=(H,C)

Definition 2.5. [6]. *The restricted intersection of two binary soft sets of (F, A) and (G, B) over common U_1, U_2 is the binary soft set (H, C) where $c = A \cap B$ and for all $e \in C$ and H(e)= ($X_1 \cap X_2$, $Y_1 \cap Y_2$) for each $e \in C$ such that F(e)= (X_1, Y_1) for each $e \in A$ and G(e)= (X_2, Y_2) for each $e \in B$ We denote it as (F, A)$\overset{\approx}{\cap}$(G, B)=(H,C)*

Definition 2.6. [6] *Let (F, A) and (G, B) be two binary soft set over a common U_1, U_2 . (F, A) is called a binary soft subset of (G, B) if*
(i) $A \subseteq B$
(ii)$X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ such that F(e)= (X_1, Y_1), G(e)= (X_2, Y_2) or each $e \in A$. We denote it is (F, A)$\overset{\approx}{\subseteq}$(G, B)

Definition 2.7. [6] *A binary soft set (F, A) over U_1, U_2 is called a binary null soft set, denoted by if F(e)= (φ, φ) for each $e \in A$*

Definition 2.8. [6] *The difference of two binary soft sets (F, A) and (G, A) over the common U_1, U_2 is the binary soft set (H, A) where H(e) $(X_1 - X_2, Y_1 - Y_2)$ for each $e \in A$ such that (F, A)= (X_1, Y_1) and (G, A)= (X_2, Y_2)*

Definition 2.9. [7]. *Let $\tau_\triangle$ be the collection of soft sets over U_1, U_2 then $\tau_\triangle$ is said to be a binary soft topology on U_1, U_2 if*
(i) $\overset{\approx}{\varphi}, \overset{\approx}{X} \in \tau_\triangle$.
(ii) the union of any member of any binary soft sets in $\tau_\triangle$ belongs to $\tau_\triangle$.
(iii) the intersection of any member of any binary soft sets in $\tau_\triangle$ belongs to $\tau_\triangle$.
Then (U_1, U_2, $\tau_\triangle$, E) is called a binary soft topological space over U_1, U_2.

Definition 2.10. *Let (F, A) be any binary soft subset of a binary soft topological space (X, Y, τ, E) then (F, A) will be termed soft semi open (written S.S.O.) if and only if there exists soft open set (O, E) such that e (O, E) $\overline{\subseteq}$ $(F, E) \subseteq Cl(O, E)$.*

Definition 2.11. *Let (F, A) be any binary soft subset of a binary soft topological space (X, Y, τ, E) then (F, A) will be termed soft semi open (written S.S.O.) if its relative complement is soft semi-open e.g, there exists a soft closed set (F, E) such that (F, E) $\overline{\subseteq}$ $(G, E) \subseteq (F, E)$. The set of all soft binary semi-open soft sets is denoted by BSOSS (X, Y, τ, E) and the set of all binary s-closed sets is denoted by BSOSS (X, Y, τ, E)*

3. BINARY SOFT SEMI-SEPARATION AXIOMS

In this section, binary soft semi-separation axioms are discussed with respect to ordinary and soft points in binary soft topological spaces.

Definition 3.1. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, M, A)$ *is called a binary soft* S_0 *space if for any two binary points* $(x_1, y_1), (x_2, y_2)\widetilde{\varepsilon}(\widetilde{X}, \widetilde{Y})$ *such that* $x_1 < x_2, y_1 < y_2$ *there exists binary soft s - open sets* (F_1, A) and (F_2, A) *which behaves as*
$(x_1, y_1)\widetilde{\widetilde{\in}}(F_1, A), (x_2, y_2)\widetilde{\widetilde{\notin}}(F_1, A)$ *or* $(x_2, y_2)\widetilde{\widetilde{\in}}(F_2, A)$ *and* $(x_1, y_1)\widetilde{\widetilde{\notin}}(F_2, A)$*.*

Definition 3.2. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, M, A)$ *is called a binary soft* S_1 *space if for any two binary points* $(x_1, y_1), (x_2, y_2)\widetilde{\varepsilon}(\widetilde{X}, \widetilde{Y})$ *such that* $x_1 < x_2, y_1 < y_2$ *If there exists binary soft s - open sets* $(F_1, A) and (F_2, A)$ *which behaves as* $(x_1, y_1)\widetilde{\widetilde{\in}}(F_1, A)$ *and* $(x_2, y_2)\widetilde{\widetilde{\notin}}(F_1, A)$
or $(x_2, y_2)\widetilde{\widetilde{\in}}(F_2, A)$ and $(x_1, y_1)\widetilde{\widetilde{\notin}}(F_2, A)$*.*

Definition 3.3. *Two binary soft s-open sets* $(F, A), (G, A)$ *and* $(H, A), (I, A)$ *are said to be is joint if* $((F, A) \sqcap (H, A), (G, A) \sqcap (I, A)) = (\Phi, \Phi)$ *and* $(G, A) \sqcap (I, A) = (\Phi, \Phi)$*.*

Definition 3.4. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, M, A)$ *is called a binary soft* S_2 *space if for any two binary points* $(x_1, y_1), (x_2, y_2)\widetilde{\varepsilon}(\widetilde{X}, \widetilde{Y})$ *such that* $x_1 < x_2, y_1 < y_2$ *If there exists binary soft s-open sets* (F_1, A) and (F_2, A) *which behaves as* $(x_1, y_1)\widetilde{\widetilde{\in}}(F_1, A)$ *and* $(x_2, y_2)\widetilde{\widetilde{\in}}(F_2, A)$ *and moreover* (F_1, A) and (F_2, A) *are disjoint that is*
$(F_1, A) \sqcap (F_2, A) = (\Phi, \Phi)$

Definition 3.5. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, M, A)$ *is called a binary soft* S_2 *space if for any two binary points* $(x_1, y_1), (x_2, y_2)\widetilde{\varepsilon}(\widetilde{X}, \widetilde{Y})$ *such that* $x_1 < x_2, y_1 < y_2$ *If there exists binary soft s-open sets* (F_1, A) and (F_2, A) *which behaves as* $(x_1, y_1)\widetilde{\widetilde{\in}}(F_1, A)$ *and* $(x_2, y_2)\widetilde{\widetilde{\in}}(F_2, A)$ *and moreover* (F_1, A) and (F_2, A) *are disjoint that is*
$(F_1, A) \sqcap (F_2, A) = (\Phi, \Phi)$

Definition 3.6. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ *is called a binary soft s-*T_0 *with respect to the first coordinate if for every pair of binary points* $(x_1, \alpha), (y_1, \alpha)$ *there exist* $((F, A), (G, A))\widetilde{\varepsilon}\tau \times \sigma$ *with* $x_1\widetilde{\varepsilon}(F, A),\ y_1\widetilde{\notin}(F, A),\ \alpha\widetilde{\varepsilon}(G, A)$*. Where s-open (F, A) in* τ and $s - open$ (G, A) in σ*.*

Definition 3.7. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ *is called a binary soft s-*T_0 *with respect to the second coordinate if for every pair of binary points* $(\beta, x_2), (\beta, y_2)$ *there exist* $((F, A), (G, A))\widetilde{\varepsilon}\tau \times \sigma$ *with* $\beta\widetilde{\varepsilon}(F, A),\ x_2\widetilde{\varepsilon}(G, A),\ y_2\widetilde{\notin}(G, A)$*. Where s-open (F, A) in* τ and $s - open(G, A)$ in σ*.*

Definition 3.8. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, M, A)$ *is called a binary soft s -* T_0 *space if for any two binary soft points* $(e_{\mathbb{G}1}, e_{\mathbb{H}1}), (e_{\mathbb{G}2}, e_{\mathbb{H}2})\widetilde{\varepsilon}(\widetilde{X_A}, \widetilde{Y_A})$ *such that* $e_{\mathbb{G}1} < e_{\mathbb{G}2}, e_{\mathbb{H}1} < e_{\mathbb{H}1}$ *there exists binary soft s - open sets* $(F_1, A) and (F_2, A)$ *which behaves as* $(e_{\mathbb{G}1}, e_{\mathbb{H}1})$
$\widetilde{\widetilde{\in}}(F_1, A), (e_{\mathbb{G}2}, e_{\mathbb{H}2})\widetilde{\widetilde{\notin}}(F_1, A)$ *or* $(e_{\mathbb{G}2}, e_{\mathbb{H}2})\widetilde{\widetilde{\in}}(F_2, A)$ *and* $(e_{\mathbb{G}1}, e_{\mathbb{H}1})\widetilde{\widetilde{\notin}}(F_2, A)$*.*

Definition 3.9. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, M, A)$ *is called a binary soft s -* T_1 *space if for any two binary soft points* $(e_{\mathbb{G}1}, e_{\mathbb{H}1}), (e_{\mathbb{G}2}, e_{\mathbb{H}2})\widetilde{\varepsilon}(\widetilde{X_A}, \widetilde{Y_A})$ *such that*

$e_{\mathbb{G}1} < e_{\mathbb{G}2}, e_{\mathbb{H}1} < e_{\mathbb{H}1}$ *there exists binary soft s - open sets* (F_1, A) and (F_2, A) *which behaves as* $(e_{\mathbb{G}1}, e_{\mathbb{H}1})$

$$\widetilde{\widetilde{\in}}(F_1, A) \; and \; (e_{\mathbb{G}2}, e_{\mathbb{H}2})\widetilde{\widetilde{\notin}}(F_1, A) \text{ and } (e_{\mathbb{G}2}, e_{\mathbb{H}2})\widetilde{\widetilde{\in}}(F_2, A) \text{ and } (e_{\mathbb{G}1}, e_{\mathbb{H}1})\widetilde{\widetilde{\notin}}(F_2, A).$$

Definition 3.10. *A binary soft topological space* $(\widetilde{X}, \widetilde{Y}, M, A)$ *is called a binary soft s -* T_2 *space if for any two binary soft points* $(e_{\mathbb{G}1}, e_{\mathbb{H}1}), (e_{\mathbb{G}2}, e_{\mathbb{H}2})\widetilde{\varepsilon}(\widetilde{X_A}, \widetilde{Y_A})$ *such that* $e_{G_1} < e_{G_2}$, $e_{H_1} < e_{H_2}$ *if there exists binary soft s-open sets* (F_1, A) *and* (F_2, A) *which behaves as (* e_{G_1} e_{H_1}*)* $\widetilde{\widetilde{\epsilon}}$ (F_1, A) *and (* e_{G_2} e_{H_2}*)* $\widetilde{\widetilde{\epsilon}}$ (F_2, A) *and* (F_1, A) *and* (F_2, A) *are disjoint*

Example 3.11. *Let* $U_1 = \{x_1, x_2, x_3\}$ $U_2 = \{y_1, y_2\}$ $E = \{e_1, e_2\}$ *and*
$\tau_\Delta = \{\tilde{X}, \tilde{\varphi}, \{(e_1, (\{x_2\}, \{y_2\})), (e_2, (\{x_1\}, \{y_1\}))\},$
$\{(e_1, (\{x_1\}\{y_1\})), (e_2, (\{x_2\}, \{y_2\}))\}, \{(e_1, (\{x_1\}, \{y_1\}))\},$
$\{(e_1, (\{\tilde{U_1}\}, \{\tilde{U_2}\}, (e_2, (\{x_1\}, \{y_1\}))\}\}$, *where* $(F_1, E) = \{(e_1, (\{x_2\}, \{y_2\}), (e_2, (\{x_1\}, \{y_1\}))\}$, $(F_2, E) = \{(e_1, (\{x_1\}, \{y_1\}), (e_2, (\{x_2\}, \{y_1\}))\}$, $(F_3, E) = \{(e_1, (\{x_1\}, \{y_1\}))\}$, $(F_4, E) = \{(e_1, (\{\tilde{U_1}\}, \{\tilde{U_2}\})), (e_2, (\{x_1\}, \{y_1\}))\}$ *clearly* $(U_1, U_2, \tau_\Delta, E)$ *is a binary soft topological space of* $\tilde{X}$ *over* $(U_1 \times U_2)$ *note that* $\tau_{\Delta_1} = \{\tilde{X}, \tilde{\varphi}, \{(e_1, (\{x_1\}, \{y_1\}))\},$ $(e_1, (\{x_2\}, \{y_2\}))\}, T_{\Delta_2} = \{\tilde{X}, \tilde{\varphi}, \{(e_2, (\{x_1\}, \{y_1\}))\}, (e_2, (\{x_2\}, \{y_2\}))\}$ *are a binary soft topological space on* $\tilde{X}$ *over* $(U_1 \times U_2)$*. There are two pairs of distinct binary soft points,namely,* $F_{e_1} = \{(e_1, (\{x_2\}, \{y_2\})\}, G_{e_1} = \{(e_1, (\{x_1\}, \{y_1\})\}$ *and* $F_{e_2} = \{(e_2, (\{x_1\}, \{y_1\})\}, G_{e_2} = \{(e_1, (\{x_1\}, \{y_1\})\}$*. Then for binary soft pair* $F_{e_1} \neq G_{e_1}$ *of points there are binary soft s-open set* (F_1, E)*. and* (F_2, E) *such that* $F_{e_1} \tilde{\notin} (F_2, E), G_{e_2} \tilde{\in} (F_2, E)$ *and* $G_{e_2} \tilde{\notin} (F_1, E), F_{e_2} \tilde{\in} (F_1, E)$*. This show that* $(U_1, U_2, \tau_\Delta, E)$ *is a binary soft space* $s - T_{\Delta_1}$ *space and hence a binary soft* $s - T_{\Delta_0}$ *space. Note that* $(U_1, U_2, \tau_\Delta, E)$ *is a binary soft* $s - T_{\Delta_2}$ *space.*

Definition 3.12. *A binary soft topological space (*$\tilde{X}, \tilde{Y}, \tau \times \sigma, A$*) is called binary soft* $s{-}T_0$ *with respective to first coordinate if for every pair of binary points* (e_{G1}, α),(e_{H1}, α) $\exists$ $((F, A), (G, A))\tilde{\varepsilon}\tau \times \sigma$ *with*$(e_{G1}\tilde{\varepsilon}(F, A)$, $(e_{H1} \tilde{\notin} (F, A)$, $\alpha\tilde{\varepsilon}(G, A)$*. Where s-open* (F, A) *in* τ *and* $s - open$ *in* (G, A) *in* σ

Definition 3.13. *A binary soft topological space (*$\tilde{X}, \tilde{Y}, \tau \times \sigma, A$*) is called a binary soft* $s-$ T_0 *with respective to the second coordinate if for every pair of binary points* $(\beta, e_{\mathbb{G}2}), (\beta, e_{\mathbb{H}2})$ *there exists* $((F, A), (G, A))\widetilde{\varepsilon}\tau \times \sigma with \beta\widetilde{\varepsilon}(F, A), e_{\mathbb{G}2}\widetilde{\in}(G, A), e_{\mathbb{H}2} to \widetilde{\notin}(G, A)$*. Where s - open (F, A) in* τ and $s - open(G, A)$ in σ*.*

4. BINARY SOFT STRUCTURES WITH RESPECT TO ORDINARY POINT

Theorem 4.1. *If the binary soft topological space* $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ *is a binary soft s-*T_0*, then* $(\widetilde{X}, \rho, A) and (\widetilde{Y}, \sigma, A)$ *are soft s-*T_0*.*

Proof. *We suppose $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is a binary soft s - T_0. Suppose $x_1, x_2 \widetilde{\varepsilon} \widetilde{X} and y_1, y_2 \widetilde{\varepsilon} \widetilde{Y}$ with such that $x_1 < x_2, y_1 < y_2$. Since $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is a binary soft s-T_0, accordingly there binary soft s-open set ((F, A), (G, A)) such that $(x_1, y_1)\widetilde{\varepsilon}((F, A), (G, A))$; $(x_2, y_2)\widetilde{\varepsilon}(F^c, A), (G^c, A)$ or $(x_1, y_1)\widetilde{\varepsilon}(F^c, A), (G^c, A)$; $(x_2, y_2)\widetilde{\varepsilon}((F, A), (G, A))$. This implies that either $x_1\widetilde{\varepsilon}(F, A); x_2\widetilde{\varepsilon}(F^c, A)$ $y_1\widetilde{\varepsilon}(G, A); y_2\widetilde{\varepsilon}(G^c, A)$; or $x_1\widetilde{\varepsilon}(F^c, A)$; $y_1\widetilde{\varepsilon}(G^c, A); y_2\widetilde{\varepsilon}(G, A)$. This implies either $x_1\widetilde{\varepsilon}(F, A); x_2\widetilde{\varepsilon}(F^c, A)$ or $x_1\widetilde{\varepsilon}(F^c, A); x_1\widetilde{\varepsilon}(F, A)$ and either*
$y_1\widetilde{\varepsilon}(G, A)$;

$y_2\widetilde{\varepsilon}(G^c, A) or y_1\widetilde{\varepsilon}(G^c, A); y_2\widetilde{\varepsilon}(G, A)$. Since $((F, A), (G, A))\widetilde{\varepsilon}\rho \times \sigma$, We have s - open $(F, A)\widetilde{\varepsilon}\rho and s - open(F, E)(E)^{\sim}\sigma$. This proves that $(\widetilde{X}, \rho, A) and (\widetilde{Y}, \sigma, A)$ are soft s - T_0.

Theorem 4.2. *A binary soft topological space $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is binary soft s - T_0 space with respect to first and second coordinates, then $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft s - T_0 space.*

Proof. *Let $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is binary soft s - T_0 space with respect to first and second coordinates. Let $(x_1, y_1), (x_2, y_2)\widetilde{\varepsilon}X \times Y with x_1 < x_2, y_1 < y_2$. Take $\alpha\widetilde{\varepsilon}Y and \beta\widetilde{\varepsilon}X$. Then $(x_1, \alpha), (x_2, \alpha)\widetilde{\varepsilon}X \times Y$. since $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is a binary soft s - T_0 space with respect to first and second coordinates, by using definition, there exists s - open sets (F, A) such that (G, A) ((F, A), (G, A)) $\widetilde{\varepsilon}\tau \times \sigma with x_1\widetilde{\varepsilon}(F, A), x_2 to \widetilde{\varepsilon}(F, A), \alpha\widetilde{\varepsilon}(G, A)$. Since $(\beta, y_1), (\beta, y_2)\widetilde{\varepsilon}X \times Y$, by using arguments and using definition there exist ((H, A), (K, A))$\widetilde{\varepsilon}\tau \times \sigma$*

$y_1\widetilde{\varepsilon}$(K, A), $y_1 \tilde{\notin} (K, A)$, $\widetilde{\beta}\varepsilon$(H, A). therefore, $(x_1, y_2)\widetilde{\varepsilon}$((F, A), (K, A))
$(x_2, y_2)\widetilde{\varepsilon}((F^c, A)$, (k^c, A)). Hence $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is called a binary soft s- T_o

Theorem 4.3. *A binary soft topological space ($\tilde{X}$,τ,A) and ($\tilde{y}$,α ,A) are soft s- T_1 spaces if and only if the binary soft topological space ($\tilde{X}$, $\tilde{Y}$,$\tau, \times, \alpha, A)$ is a soft binary s-T_1*

Proof. *Suppose ($\tilde{X}, \tau, A)$ and ($\tilde{X}, \alpha, A)$ are soft s-T_1 space .*

Let $(x_1, y_1$), (x_2, y_2) $\tilde{\varepsilon}X \times Y$ with $x_1 < x_2$ $y_1 < y_2$ since ($\tilde{X}$,τ,A) is a soft s-T_1 there exists soft -open set such that (F, A) , $(G, A$) $\tilde{\varepsilon}\tau$ $x_1\varepsilon$ (F, A) and x_2 $\varepsilon(G, A$) such that

$x_1 \tilde{\notin}$ $(G, A$) and $x_2 \tilde{\notin}$ (F, A) also ,since ($\tilde{y}, \alpha, A)$ is soft s-T_1 space there exists soft -open set such that $(H, A), (I, A)\tilde{\varepsilon}\alpha, y_1\varepsilon(H, A)$ and $y_2\varepsilon(I, A)$ such that

y_1 $\tilde{\notin}$ (I, A) and y_2 $\tilde{\notin}$ (H, A) .thus $(x_1, y_1$)$\varepsilon((F, A), (H, A))$ and $(x_2, y_2)\varepsilon$ $((G, A), (I, A))$ with $(x_1, y_1$)ε $((G^c, A), (I^c, A))$ and $(x_1, y_1$)$\varepsilon((F^c, A), (H^c, A))$. this implies that

($\tilde{X}$, $\tilde{Y}$,$\tau, \times, \alpha, A)$ is a soft binary s-T_1 conversly assume that ($\tilde{X}$, $\tilde{Y}$,$\tau, \times, \alpha, A)$ is a soft binary s-T_1.let $x_1, x_2\varepsilon X$ and y_1, y_2 εY such that $x_1 < x_2$ $y_1 < y_2$.

there fore $(x_1, y_1$), (x_2, y_2) $\tilde{\varepsilon}X \times Y$.since ($\tilde{X}$, $\tilde{Y}$,$\tau, \times, \alpha, A)$ is a soft s-T_1 there exists soft -open set (F, A) , $(G, A$) s-open sets $(H, A), (I, A)\varepsilon(\tau, \times, \alpha)$.

$(x_1, y_1$)$\varepsilon((G, A), (H, A))$ and $(x_1, y_1$)$\varepsilon((F, A), (H, A))$ such that $(x_1, y_1$)ε $((H^c, A), (I^c, A))$ and $(x_2, y_2)\varepsilon$ $((F^c, A), (G^c, A))$there fore $x_1\varepsilon(F, A), x_2\varepsilon(H, A)$

and $x_1\varepsilon$ *(*H^C, A*)*(I^c, A) *and* $x_{2,}\varepsilon(F^c, A)$ *and* $y_1\varepsilon(G^c, A)$ *and* $y_1\varepsilon(I, A)$ *and* $y_1\varepsilon(I^c, A)$ *and* $y_2\varepsilon(G^c, A)$ *since (F, A),(G, A)*$\varepsilon\tau\times, \alpha$ *we have* $(F, A), (H, A)\varepsilon\tau$ *and* $(G, A), (I, A)\varepsilon\tau$ *,this prove that*

($\tilde{X}, \tau, A$*) and (*$\tilde{X}, \alpha, A$*) are soft s-*T_1 *space*

Theorem 4.4. *A binary soft topological space (*$\tilde{X}, \tilde{y}, M, A$*) is a binary soft* $s - T_1$ *space if and only if the binary soft point* $\wp(X) \times\wp(Y)$ *is binary soft s-closed.*

Proof. *suppose that (*$\tilde{X}$*, ỹ,M,,A) is a binary soft* $s - T_1$ *space . Let* (x, y) $\tilde{\varepsilon}X\times$ Y *. let ({x},{y})* $\wp(X) \times\wp(Y)$*. we shall show that* $(\{x\}, \{y\})$ *is binary soft s-closed. it is sufficient to show that* $(X$ $\{x\}y$ $\{y\}$ *hence a=x,b=y. that is* (a, b) *and* (x, y) *are distinct binary soft point of* $X \times Y$*. Since (*$\tilde{X}$*, ỹ,M,,A) is a binary soft s-*T_1 *space, there exists soft -open set* $((F, A), (G, A))$ *and (H, A) , (I, A) such that (a,b)*ε $((F, A), (G, A))$ *and (x,y)*ε *((*H, A*) , (*I, A *)) such that (a,b)*$\varepsilon((H^c, A), (I^c, A))$ *and (x,y)*$\varepsilon((F^c, A), (G^c, A))$ *there fore* $((F, A), (G, A) \subseteq$ $(\{x\}^c, \{y\}^c)$ *). Hence* $(\{x\}^c, \{y\}^c)$ *is a soft neighborhood of (a,b) this implies that* $(\{x\}, \{y\})$ *is binary soft b-closed. Conversely, suppose that* $(\{x\}, \{y\})$ *is binary soft s-closed for every* (x, y) $\varepsilon X \times Y$*. suppose (*x_1, y_1 *) ,(*$x_{2,}$ y_2*)* $\tilde{\varepsilon}X\times$ Y *with* $x_1 < x_2$*,* $y_1 < y_2$*. Therefore (*$x_{2,}$ y_2*)*ε $(\{x_1\}^c, \{y_1\}^c)$ *and* ε $(\{x_1\}^c, \{y_1\}^c)$ *is binary soft s-open also (*x_1, y_1 *)*ε $(\{x_2\}^c, \{y_2\}^c)$ *and* $(\{x_1\}, \{y_1\})$ *is a binary soft s-open set .also (*x_1, y_1 *)*ε $(\{x_2\}^c, \{y_2\}^c)$ *and* ε $(\{x_2\}^c, \{y_2\}^c)$*is binary soft s-open set. This shows that (*$\tilde{X}, \tilde{y}, M, A$*) is a binary soft* $s - T_1$ *space.*

Theorem 4.5. *A binary soft topological space* $\left(\tilde{X}, \tau, A\right)$ *and* $(\tilde{Y}, \sigma, A)$ *are soft-* T_2 *spaces if and only if the binary soft topological space* $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ *is soft binary* $s - T_{2.}$

Proof. *We Suppose* $(\tilde{X}, \tau, A)$ *and* $(\tilde{Y}, \sigma, A)$ *are soft* $s{-}T_2$ *spaces. Let* $(x_1, y_1), (x_2, y_2)\tilde{\varepsilon}X\times$ Y *with* $x_1 < x_2, y_1 < y_2$*. Since* $\left(\tilde{X}, \tau, A\right)$ *is soft* $s - T_2$ *space, there exist soft s-open sets such that* $(F, A), (G, A)$ $\tilde{\varepsilon}\tau, x_1\varepsilon(F, A)$ *and* $x_2\varepsilon(G, A)$ *such that* x_1 $\tilde{\notin}(G, A)$ *and* $x_2\varepsilon(F, A)$*.Also, since* $(\tilde{Y}, \sigma, A)$ *is soft* $s{-}T_2$ *space, there exist distoint soft s-open sets such that* $(H, A), (I, A)$ $\tilde{\varepsilon}\sigma, y_1\varepsilon(H, A)$ *and* $y_2\varepsilon(I, A)$ *such that* $y_1\tilde{\notin}(I, A)$ *and* $y_2\tilde{\notin}(H, A)$*. Thus* $(x_1, y_1)\varepsilon((F, A), (H, A))$ *and* $(x_2, y_2)\varepsilon((G, A), (I, A))$ *with* $(x_1, y_1)\varepsilon((G^c, A), (I^c, A))$ *and* $(x_1, y_1)\varepsilon((F^c, A), (H^c, A))$*. Since* (F, A) *and* (G, A) *are disjoint,* $(F, A) \sqcap (H, A) = (\phi, \phi)$*. This implies that* $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ *is soft binary* $s - T_2$*. Conversely assume that* $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ *is soft binary* $s - T_2$*. Let* $x_1, x_2\varepsilon X$ *and* $y_1, y_2\varepsilon Y$ *such that* $x_1 < x_2$*,* $y_1 < y_2$*. Therefore* $(x_1, y_1), (x_2, y_2)\tilde{\varepsilon}X \times Y$*. Since* $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ *is soft binary* $s - T_2$*, there exist binary soft-open sets* $(H, A), (I, A)\varepsilon(\tau \times \sigma), (x_1, y_1)\tilde{\varepsilon}((F, A), (G, A))$ *and* $(x_2, y_2)\tilde{\varepsilon}((H, A), (I, A))$ *such that* $(x_1, y_1)\varepsilon((H^c, A), (I^c, A))$ *and* $(x_2, y_2)\varepsilon((F^c, A), (G^c, A))$*. therefore,* $x_1\varepsilon(F, A)$*,* $x_2\varepsilon(H, A)$ *and* $x_1\varepsilon(H^c, A)$ *and* $x_2\varepsilon(F^c, A)$ *and* $y_1\varepsilon(G^c, A)$ *and* $y_1\varepsilon(I, A)$ *and* $y_1\varepsilon(I^c, A)$ *and* $y_2\varepsilon(G^c, A)$*. Since* $(F, A), (G, A)\tilde{\varepsilon}\tau$ *and* $(G, A), (I, A)\varepsilon\sigma$*. This proves that* $\left(\tilde{X}, \tau, A\right)$ *and* $(\tilde{Y}, \sigma, A)$ *are soft* $s - T_2$ *spaces.*

5. BINARY SOFT STRUCTURES WITH RESPECT TO SOFT POINTS

Theorem 5.1. *If the binary soft topological space* $(\tilde{X}, \tilde{Y}, \rho \times \sigma, A)$ *is a binary soft* $s - T_0$ *, then* $(\tilde{X}, \rho, A)$ *and* $(\tilde{Y}, \sigma, A)$ *are soft* $s - T_0$*.*

Proof. *We suppose $(\tilde{X}_A, \tilde{Y}_A, \rho \times \sigma, A)$ is a binary soft $s - T_0$. Suppose e_{G_1},$e_{G_2}\tilde{\epsilon}\tilde{X}_A$ and $e_{H_1}, e_{H_2}\bar{\epsilon}Y_A$ with such that $e_{G_1} < e_{G_2}, e_{H_1} < e_{H_2}$. Since $(\tilde{X}_A, \tilde{\tilde{Y}}_A, \rho \times \sigma, A)$ is a binary soft $s - T_0$, accordingly there binary soft s – open set $((F, A), (G, A))$ such that $(e_{G_1}, e_{H_1})\tilde{\epsilon}((F, A), (G, A)); (e_{G_2}, e_{H_2})\tilde{\varepsilon}(F^C, A), (G^C, A)$ or $(e_{G_1}, e_{H_1})\tilde{\epsilon}((F^C, A), (G^C, A))$;$(e_{G_2}, e_{H_2})\tilde{\epsilon}((F, A), (G, A))$. This implies that either $_{G_1}\tilde{\epsilon}(F, A); e_{|G_2}\tilde{\epsilon}(F^C, A); e_{H_1}\tilde{\epsilon}(G, A); e_{H_2}\tilde{\epsilon}(G^C, A)$; or $e_{G_1}\tilde{\epsilon}(F^C, A); e_{H_1}\tilde{\epsilon}(G^C, A); e_{H_2}\tilde{\epsilon}(G, A)$. This implies either $e_{G_1}\tilde{\epsilon}(F, A), e_{G_2}\tilde{\epsilon}(F^C, A)$ or $e_{G_1}\tilde{\epsilon}(F^C, A); x_1\tilde{\epsilon}(F, A)$ and either $e_{H_1}\tilde{\epsilon}(G, A); e_{H_2}\tilde{\epsilon}(G^C, A)$ or $e_{H_1}\tilde{\epsilon}(G^C, A); e_{H_2}\tilde{\epsilon}(G, A)$. Since $((F, A), (G, A))\tilde{\epsilon}\rho\times\sigma$, We have s – open $(F, A)\tilde{\epsilon}\rho$ and s – open$(F, A)\tilde{\epsilon}\rho$ and s– open $(F, A)\tilde{\epsilon}\sigma$, this proves that $(\tilde{X}, \rho, A)$ and $(\tilde{Y}, \sigma, A)$ are soft $s - T_{0.}$*

Theorem 5.2. *A binary soft topological space $\left(\tilde{X}, \tilde{Y}, \tau \times \sigma, A\right)$ is binary soft $s - T_0$ space with respect to first and second coordinates, then $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft $s - T_{0.}$spacs.*

Proof. *Let $\left(\tilde{X}, \tilde{Y}, \tau \times \sigma, A\right)$ is binary soft $s - T_0$ space with respect to first and second coordinates. Let (e_{G_1}, e_{H_1}), $(e_{G_2}, e_{H_2})\tilde{\varepsilon}X \times Y$ with $e_{G_1} < e_{G_2}$, $e_{H_1} < e_{H_2}$. Takes $\alpha\tilde{\varepsilon}Y$ and $\beta\tilde{\varepsilon}X$. Then (e_{G_1}, α), $(e_{G_2}, \alpha)\tilde{\varepsilon}X \times Y$ Since $\left(\tilde{X}, \tilde{Y}, \tau \times \sigma, A\right)$ is binary soft $s - T_0$ space with respect to first coordinate,by using the definition, there exists s-open sets ($(F, A)(G, A))\tilde{\varepsilon}\tau \times \sigma$ with $e_{G_1}\tilde{\varepsilon}(F, A)$ $e_{G_2}\tilde{\neg}\varepsilon(F, A), \alpha\tilde{\varepsilon}(G, A)$. Since $(\beta, e_{H_1}), (\beta, e_{H_2})\tilde{\varepsilon}X \times Y$, by using the arguments and using the definition, $\exists$ s-open sets $((H, A), (K, A))\tilde{\varepsilon}\tau \times \sigma$ with $e_{H_1}\tilde{\varepsilon}(K, A), e_{H_1}\tilde{\neg}\varepsilon(K, A), \widetilde{\beta\varepsilon}(H, A)$. Therefore, $(e_{G_1}, e_{H_1})\tilde{\varepsilon}((F, A), (K, A))$ and $(e_{G_2}, e_{H_2})\tilde{\varepsilon}((F^c, A), (K^c, A))$. Hence $\left(\tilde{X}, \tilde{Y}, \tau \times \sigma, A\right)$ is called a binary soft $s - T_0$.*

Theorem 5.3. *A binary soft topological space $\left(\tilde{X}, \tau, A\right)$ and $(\tilde{Y}, \sigma, A)$ are soft- T_1 spaces if and only if the binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary s-T_1 space*

Proof. Suppose $\left(\tilde{X}, \tau, A\right)$ and $(\tilde{Y}, \sigma, A)$ are soft- T_1 spaces . Let (e_{G_1},e_{H_1}), (e_{G_2}, e_{H_2})$\widetilde{\varepsilon}X \times Y$ with $e_{G_1} < e_{G_2}$, $e_{H_1} <$ e_{G_2} since $\left(\tilde{X}, \tau, A\right)$ is soft s-T_1 space, there exists s-open sets such that (F, A) , (G,A) $\widetilde{\varepsilon}\tau$, $e_{G_1}\varepsilon(F, A)$ and $e_{G_2}\varepsilon(F, A)$ such that e_{G_1} $\widetilde{\notin}$ (G, A) and e_{G_2} $\widetilde{\notin}$ (G, A) Also , since $(\tilde{Y}, \sigma, A)$ is soft s-T_1 space , there exists s-open sets such that (H, A) , (I,A) $\widetilde{\varepsilon}\sigma$, $e_{H_1}\varepsilon(H, A)$ and $e_{H_2}\varepsilon(I, A)$ such that e_{H_1} $\widetilde{\notin}$ (I, A) and e_{H_2} $\widetilde{\notin}$ (H, A). Thus (e_{G_1},e_{H_1})$\varepsilon(F, A)$,(H,A) , and (e_{G_2}, e_{H_2})$\varepsilon(G, A)$,(I,A) with (e_{G_2}, e_{H_2})$\varepsilon(G^c, A)$,(I^c,A) and (e_{G_2}, e_{H_2})$\varepsilon(F^C, A)$,(H^C,A). This implies $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ that is soft binary $s - T_1$. conversely assume that is soft binary s-T_1. Let e_{G_1},$e_{G_2}\varepsilon X$ and e_{H_1},$e_{H_2}\varepsilon Y$ such that $e_{G_1} > e_{G_2}$, $e_{H_1} > e_{G_2}$. Therefore (e_{G_1},e_{H_1}), (e_{G_2}, e_{H_2})$\widetilde{\varepsilon}X \times Y$ is soft s-T_1 there exists s-open sets (F,A)(G, A) and s-open sets (H, A)(I,A)($\tau \times \sigma$), (e_{G_1},e_{H_1})$\widetilde{\varepsilon}(F, A)$,(G,A) and ($e_{G_2}$, e_{H_2})$\widetilde{\varepsilon}(H, A)$,(I,A) such that ($e_{G_1}$, e_{H_1})$\varepsilon\widetilde{(H^c, A)}$, ($I^c$,A) and ($e_{G_2}$, e_{H_2})$\widetilde{\varepsilon}((F^C, A)$,($G^C$,A)). therefore $e_{G_1}\varepsilon(F, A)$, $e_{G_2}\varepsilon(H, A)$ and $e_{G_1}\varepsilon(F^c, A)$

and e$_{G_2}\varepsilon(F^c, A)$ and Y$_1\varepsilon(G^c, A)$ and e$_{H_1}\varepsilon(I, A)$ and e$_{H_1}\varepsilon(I^c, A)$ and e$_{H_2}\varepsilon(G^c, A)$ since (F, A)(G,A)($\widetilde{\varepsilon}\tau \times \sigma$), we have (F, A)(G,A)$\varepsilon\tau$ and (G,A)(I,A)$\varepsilon\sigma$. This proves that ($\widetilde{X}$, τ, A) and ($\widetilde{X}$, σ, A) are soft s-T$_1$ space

Theorem 5.4. *A binary soft topological space ($\tilde{X}$, $\tilde{Y}$,M ,A) is binary soft s-T_1 space if and only if every binary soft point $\wp(X) \times \wp(y)$ is binary soft s-closed.*

Proof. Suppose that ($\tilde{X}$,$\tilde{Y}$,$\tilde{M}$,$\tilde{A}$) is binary soft s-T$_1$ space. Let(x,y) εX×Y. Let ({x},{e$_H$} $\tilde{\varepsilon}\wp(X)$ we shall show that ($\{e_G\}, \{e_H\}$)is binary soft s-closed. It is sufficient to show that $(X\setminus\setminus\{e_G\}Y\setminus\setminus\{e_H\}$ is binary soft s-open. Let $(a, b)\ \varepsilon(X\setminus\{e_G\}, Y\setminus$ {e$_H$}). This implies that $a\tilde{\varepsilon}X\setminus\{e_G\}$and b$\tilde{\varepsilon}Y\setminus\{e_H\}$. Hence $a \neq e_G$ and $b \neq e_H$. That is (a, b) and $e_G\ e_H$ are distinct binary soft points of X×Y .Since ($\tilde{X}$, $\tilde{Y}$,M,A)is binary soft s-T$_1$ space, there exists binary soft s-open sets ((F,A),(G,A))and (H,A),(I,A) such that (a,b)ε((F,A),(G,A)) and (x,y) $\varepsilon((H, A),(I, A))$ such that (a,b)ε((H$^C A), (I^C, A))$ and (e$_{G,}e_H)\varepsilon$((F^{C,}A),(G^C,A). Therefore, (F,A),(G,A) $\subseteq$ $(\{c\}^C, (\{e_H\}$ Hence($\{e_G\ \}, \{e_H\}^c$is a soft neighborhood of (a,b). This implies that is ($\{e_G\ \}, \{e_H\}$) binary soft s-closed. Conversely, suppose that ($\{e_G\ \}, \{e_H\}$) is binary soft s-closed for every.(e_G), (e_H). Suppose (e_{G1} ,e_{H1}), (e_{G2},e_{H2}).$\varepsilon X \times Y$ with $e_{G1}\ < e_{G2,}\ e_{H1}\ < e_{H2,}$. is a binary soft s-open. Also (e_{G2},e_{H2})ε.($\{e_G\ \}, \{e_H\}^c$ and ($\{e_G\ \}, \{e_H\}^c$ is binary soft s-open set. Also (e_{G1},e_{H1})ε.($\{e_{G2}\ \}, \{e_{H2}\}^c$ and ($\{e_{G1}$ $\}, \{e_{H1}\}^c$ is binary soft s-open set.This shows that is binary soft s_T$_1$ space binary soft s-open set . also .(e_{G1},e_{H1})ε.($\{e_{G2}\ \}, \{e_{H2}\}^{c)}$ and ($\{e_{G2}\ \}^c, \{e_{H2}\}^c$) is binary soft s-open set. This shows $(\tilde{X}, \tilde{Y}, M, A)$ is a binary soft $s - T_1$ space

Theorem 5.5. *A binary soft topological space* $\left(\tilde{X}, \tau, A\right)$ *and* $(\tilde{Y}, \sigma, A)$ *are* $soft - T_2$ *spaces if and only if the binary soft topological space* $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ *is soft binary* $s - T_2$ *space*

Proof. Suppose ($\widetilde{X}$, τ, A) and ($\widetilde{X}$, σ, A) are soft s-T$_2$ spaces. Let (e$_{G_1}$, e$_{H_1}$), (e$_{G_2}$, e$_{H_2}$)$\widetilde{\varepsilon}X \times Y$ with $e_{G1}\ < e_{G2,}$,$e_{H1}\ < e_{H2,}$. since ($\widetilde{X}$, τ, A) is soft s-T$_2$ spaces, there exists soft s-open sets such that (F,A), (G,A)$\widetilde{\varepsilon}\tau$, e$_{G_1}\varepsilon(F, A)$ and e$_{G_2}\varepsilon(G, A)$ such that e$_{G_1}$ $\widetilde{\notin}$ (G, A) and e$_{G_2}$ $\widetilde{\notin}$ (F, A). Also since ($\widetilde{X}$, σ, A) is soft s-T$_2$ spaces, there exists disjoint soft s-open sets such that $(H, A),(I, A)\ \widetilde{\varepsilon}\sigma$, e$_{H_1}\varepsilon(H, A)$ and e$_{H_2}\varepsilon(I, A)$ such that e$_{H_1}$ $\widetilde{\notin}$ (I, A) and e$_{H_2}$ $\widetilde{\notin}$ (H, A) thus (e_{G_1}, e_{H_1}), ε $((F, A)(G, A))$ and (e_{G_2}, e_{H_2}) ε $((G, A)(I, A))$. with (e_{G_1}, e_{H_1}) ε $((G^c, A)(I^c, A))$. and (e_{G_1}, e_{H_1}), ε $((F^c, A)(H^c, A))$. (F, A) and (C, A) are disjoint $(F, A)\Pi(H, A) = (\varphi, \varphi)$ Also since (H,A)$\Pi(I, A) = (\varphi, \varphi)$. thus ((F,A)$\Pi(H, A),(G, A)\Pi(I, A) = (\varphi, \varphi)$ this implies that we have this implies that $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is soft binary s-T$_2$. conversely assume that $(\widetilde{X}, \widetilde{Y}, \tau \times \sigma, A)$ is soft binary s-T$_2$. Let e$_{G_1}$,e$_{G_2}\varepsilon X$ and e$_{H_!}$,e$_{H_2}\varepsilon Y$ such that e$_{G_1}$ $>$ e$_{G_2}$, e$_{H_1} >$ e$_{G_2}$. Therefore (e$_{G_1}$,e$_{H_1}$), (e$_{G_2}$, e$_{H_2}$)$\widetilde{\varepsilon}X \times Y$ is soft s-T$_2$ there exists s-open sets (F,A)(G, A) and there exists binary soft s-open sets (H, A)(I,A)($\tau \times \sigma$), (e$_{G_1}$,e$_{H_1}$)$\widetilde{\varepsilon}(F, A)$,(G,A) and (e$_{G_2}$, e$_{H_2}$)$\widetilde{\varepsilon}(H, A)$,(I,A) such that (e$_{G_1}$, e$_{H_1}$)$\varepsilon(H^c, \widetilde{A})$, (I^c,A) and (e$_{G_2}$, e$_{H_2}$)$\widetilde{\varepsilon}((F^C, A)$,(G^C ,A)). therefore e$_{G_1}\varepsilon(F, A)$, e$_{G_2}\varepsilon(H, A)$ and e$_{G_1}\varepsilon(H^c, A)$ and e$_{G_2}\varepsilon(F^c, A)$ and and e$_{H_1}\varepsilon(I, A)$ and e$_{H_1}\varepsilon(I^c, A)$ and e$_{H_2}\varepsilon(G^c, A)$ since (F, A)(G,A)($\widetilde{\varepsilon}\tau \times \sigma$), we have (F, A)(G,A)$\varepsilon\tau$ and (G,A)(I,A)$\varepsilon\sigma$. This proves that ($\widetilde{X}$, τ, A) and ($\widetilde{X}$, σ, A) are soft s-T$_2$ space

6. CONCLUSION

The soft binary $S - T_0$, $s - T_1$ structure with respect to first and second coordinates are introduced in this paper.

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