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An Efficient Algorithm for Solving Nonlinear Systems of Partial Differential Equations with Local Fractional Operators

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Abstract. The aim of the present study is to extend the local fractional Sumudu decomposition method (LFSDM) to resolve nonlinear systems of partial differential equations with local fractional derivatives. The derivative operators are taken in the local fractional sense. The LFSDM method provides the solution in a rapid convergent series, which may lead the non-differentiable solution in a closed form, this makes them an appropriate method for similar problems. We have provided some examples to confirm their flexibility in solving these types of systems.

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1. Introduction

The perception of the notion of a non-integer order derivative (for example a real number) has ceased to be an astonishing fact. On the contrary, its contribution has allowed and still allows us to better glimpse the phenomena of nature, in other words, to model them by differential equations or systems of non-integer order, which better reflect the passage from the real to the known. In this view, the non-integer order differential systems, although more complicated, are better adapted to the mechanical modeling of certain materials that retain the memory of past deformations (return to the initial shape of the rubber after a twist), the behavior described as viscoelastic in the language of mechanics.

Many mathematicians have studied methods of solving differential equations or differential systems, especially nonlinear differential equations or systems. Among these methods, the one known as "Adomian decomposition method", is among the most famous method developed recently, where it was developed between the 1970s and 1990s by George Adomian [1, 2, 3, 4, 5].

With the advent of fractional differential equations, researchers used this method to solve the kind of this new type of equations or systems [6, 11, 19, 18, 17, 22], then they combined it with the Laplace transform method [9, 12, 13, 14] and with the Sumudu transform method [7, 15, 16], all this, to give this method more effective and faster in identifying solutions. This method and associated methods, have also been used to solve new differential equations of order slightly different from order of classical differential equations, for example, from the ordinary differential equations, we obtain new known equations under the name of: local fractional ordinary differential equations, and from partial differential equations, we obtain other new equations, known under the name of: local fractional ordinary differential equations, and from partial differential equations, we obtain another new equation, known under the name of: local fractional partial differential equations. This new concept of derivatives and integrations which is known as local fractional operators, is attributed to Xiao-Jun Yang [23, 24].

The work presented in this paper consists in extending the use of the modified method proposed by D. Ziane et al. [26] for solving nonlinear systems of partial differential equations with of local fractional derivatives. The importance of LFSDM lies in the fact that it combines two important methods to solve this type of nonlinear systems. To demonstrate the value of this method, we used it to solve two important nonlinear systems of local fractional partial differential equations.

2. Basic definitions

This work, which we will present in this article, requires basic concepts of fractional calculus, in addition, we will present the new definition of Sumudu transformation according to this concept.

2.1. Local fractional derivative.

Definition 2.2. *The local fractional derivative of $\Phi(\varkappa)$ of order σ at $\varkappa = \varkappa_0$ is defined as* [23, 24]

$$\Phi^{(\sigma)}(\varkappa) = \left.\frac{d^\sigma \Phi}{d\varkappa^\sigma}\right|_{\varkappa=\varkappa_0} = \lim_{\varkappa \to \varkappa_0} \frac{\Delta^\sigma(\Phi(\varkappa) - \Phi(\varkappa_0))}{(\varkappa - \varkappa_0)^\sigma}, \tag{2. 1}$$

where

$$\Delta^\sigma(\Phi(\varkappa) - \Phi(\varkappa_0)) \cong \Gamma(1+\sigma)\left[(\Phi(\varkappa) - \Phi(\varkappa_0))\right]. \tag{2. 2}$$

For any $\varkappa \in (\alpha, \beta)$, there exists

$$\Phi^{(\sigma)}(\varkappa) = D_\varkappa^\sigma \Phi(\varkappa),$$

denoted by

$$\Phi(\varkappa) \in D^{\sigma}_{\varkappa}(\alpha, \beta).$$

A local fractional derivative of high order is written in the form

$$\Phi^{(m\sigma)}(\varkappa) = \overbrace{D^{(\sigma)}_{\varkappa} \cdots D^{(\sigma)}_{\varkappa}\Phi(\varkappa)}^{m\,times}, \tag{2. 3}$$

and for the local fractional partial derivative of high order, we have

$$\frac{\partial^{m\sigma}\Phi(\varkappa,\tau)}{\partial\varkappa^{m\sigma}} = \overbrace{\frac{\partial^{\sigma}}{\partial\varkappa^{\sigma}} \cdots \frac{\partial^{\sigma}}{\partial\varkappa^{\sigma}}\Phi(\varkappa,\tau)}^{m\,times}. \tag{2. 4}$$

2.3. **Local fractional integral.**

Definition 2.4. *The local fractional integral of $\Phi(\varkappa)$ of order σ in the interval $[\alpha,\beta]$ is defined as* [23, 24]

$$\begin{aligned} {}_{\alpha}I^{(\sigma)}_{\beta}\Phi(\varkappa) &= \frac{1}{\Gamma(1+\sigma)}\int_{\alpha}^{\beta}\Phi(\tau)(d\tau)^{\sigma} \\ &= \frac{1}{\Gamma(1+\sigma)}\lim_{\Delta\tau\longrightarrow 0}\sum_{j=0}^{N-1} f(\tau_j)(\Delta\tau_j)^{\sigma}, \end{aligned} \tag{2. 5}$$

where $\Delta\tau_j = \tau_{j+1} - \tau_j$, $\Delta\tau = \max\{\Delta\tau_0, \Delta\tau_1, \Delta\tau_2, \cdots\}$ *and* $[\tau_j, \tau_{j+1}]$, $\tau_0 = \alpha$, $\tau_N = \beta$, *is a partition of the interval* $[\alpha,\beta]$. *For any* $r \in (\alpha,\beta)$, *there exists* ${}_{\alpha}I^{(\sigma)}_{\varkappa}\Phi(\varkappa)$, *denoted by* $\Phi(\varkappa) \in I^{(\sigma)}_{\varkappa}(\alpha,\beta)$.

2.5. **Some important results.**

Definition 2.6. *The expressions of the functions Mittage Leffler, sine and cosine in the sense of local fractional operators (*[10, 23, 24]*, are given as follows*

$$E_{\sigma}(\varkappa^{\sigma}) = \sum_{m=0}^{+\infty}\frac{\varkappa^{m\sigma}}{\Gamma(1+m\sigma)}, \quad 0<\sigma\leqslant 1, \tag{2. 6}$$

$$E_{\sigma}(\varkappa^{\sigma})E_{\sigma}(\upsilon^{\sigma}) = E_{\sigma}(\varkappa+\upsilon)^{\sigma}, \quad 0<\sigma\leqslant 1, \tag{2. 7}$$

$$E_{\sigma}(\varkappa^{\sigma})E_{\sigma}(-\upsilon^{\sigma}) = E_{\sigma}(\varkappa-\upsilon)^{\sigma}, \quad 0<\sigma\leqslant 1, \tag{2. 8}$$

$$\sin_{\sigma}(\varkappa^{\sigma}) = \sum_{m=0}^{+\infty}(-1)^m\frac{\varkappa^{(2m+1)\sigma}}{\Gamma(1+(2m+1)\sigma)}, \quad 0<\sigma\leqslant 1, \tag{2. 9}$$

$$\cos_{\sigma}(\varkappa^{\sigma}) = \sum_{m=0}^{+\infty}(-1)^m\frac{\varkappa^{2m\sigma}}{\Gamma(1+2m\sigma)}, \quad 0<\sigma\leqslant 1, \tag{2. 10}$$

Some important properties of derivatives and integrals in the sense of local fractional operators [23, 24], are given by

$$\frac{d^{\sigma}\varkappa^{m\sigma}}{d\varkappa^{\sigma}} = \frac{\Gamma(1+m\sigma)\varkappa^{(m-1)\sigma}}{\Gamma(1+(m-1)\sigma)}. \tag{2. 11}$$

$$\frac{d^{\sigma}}{d\varkappa^{\sigma}} E_{\sigma}(\varkappa^{\sigma}) = E_{\sigma}(\varkappa^{\sigma}). \tag{2. 12}$$

$$\frac{d^{\sigma}}{d\varkappa^{\sigma}} \sin_{\sigma}(\varkappa^{\sigma}) = \cos_{\sigma}(\varkappa^{\sigma}). \tag{2. 13}$$

$$\frac{d^{\sigma}}{d\varkappa^{\sigma}} \cos_{\sigma}(\varkappa^{\sigma}) = \sinh_{\sigma}(\varkappa^{\sigma}). \tag{2. 14}$$

$${}_0I_{\varkappa}^{(\sigma)} \frac{\varkappa^{m\sigma}}{\Gamma(1+m\sigma)} = \frac{\varkappa^{(m+1)\sigma}}{\Gamma(1+(m+1)\sigma)}. \tag{2. 15}$$

2.7. **Local fractional Sumudu transform.** In this subsection, we will give the basic definition of the Sumudu transform method of local fractional derivative (${}^{lf}S_{\sigma}$), followed by some important properties [20].

We consider this new transform operator ${}^{lf}S_{\sigma} : \Phi(\varkappa) \longrightarrow F(\upsilon)$, namely,

$${}^{lf}S_{\sigma}\left\{\sum_{m=0}^{\infty} a_m \varkappa^{m\sigma}\right\} = \sum_{m=0}^{\infty} \Gamma(1+m\sigma) a_m \upsilon^{m\sigma}. \tag{2. 16}$$

As typical examples, we have

$${}^{lf}S_{\sigma}\left\{E_{\sigma}(i^{\sigma}\varkappa^{\sigma})\right\} = \sum_{m=0}^{\infty} i^{\sigma m}\upsilon^{\sigma m}. \tag{2. 17}$$

$${}^{lf}S_{\sigma}\left\{\frac{\varkappa^{\sigma}}{\Gamma(1+\sigma)}\right\} = \upsilon^{\sigma}. \tag{2. 18}$$

Definition 2.8. [20] *The formula of the Sumudu transform of local fractional derivative for the function $\Phi(\varkappa)$ of order σ,is defined as follow*

$${}^{lf}\mathrm{S}_{\sigma}\left\{\Phi(\varkappa)\right\} = F_{\sigma}(\upsilon) = \frac{1}{\Gamma(1+\sigma)}\int_0^{\infty} E_{\sigma}(-\upsilon^{-\sigma}\varkappa^{\sigma})\frac{\Phi(\varkappa)}{u^{\sigma}}(d\varkappa)^{\sigma},\ 0<\sigma\leqslant 1 \tag{2. 19}$$

The inverse formula of (2. 19), is given by

$${}^{lf}\mathrm{S}_{\sigma}^{-1}\left\{F_{\sigma}(\upsilon)\right\} = \Phi(\varkappa)\ , \quad 0<\sigma\leqslant 1\ . \tag{2. 20}$$

Theorem 2.9. *(linearity).*
If ${}^{lf}\mathrm{S}_{\sigma}\left\{\Phi(\varkappa)\right\} = F_{\sigma}(\upsilon)$ *and* ${}^{lf}\mathrm{S}_{\sigma}\left\{\varphi(\varkappa)\right\} = \Psi_{\sigma}(\upsilon)$*, then one has*

$${}^{lf}\mathrm{S}_{\sigma}\left\{\Phi(\varkappa)+\varphi(\varkappa)\right\} = F_{\sigma}(u)+\Psi_{\sigma}(u). \tag{2. 21}$$

Proof. Using formula (2. 19), we obtain

$$\begin{aligned} {}^{lf}S_\sigma\left\{\Phi(\varkappa)+\varphi(\varkappa)\right\} &= \frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma(-\upsilon^{-\sigma}\varkappa^\sigma)\frac{\Phi(\varkappa)+\varphi(\varkappa)}{u^\sigma}(d\varkappa)^\sigma \\ &= \frac{1}{\Gamma(1+\sigma)}\int_0^\infty \left[E_\sigma(-\upsilon^{-\sigma}\varkappa^\sigma)\frac{\Phi(\varkappa)}{u^\sigma}+E_\sigma(-\upsilon^{-\sigma}\varkappa^\sigma)\frac{\varphi(\varkappa)}{u^\sigma}\right](d\varkappa)^\sigma \\ &= \frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma(-\upsilon^{-\sigma}\varkappa^\sigma)\frac{\Phi(r)}{u^\sigma}(d\varkappa)^\sigma+\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma(-\upsilon^{-\sigma}\varkappa^\sigma)\frac{\varphi(\varkappa)}{u^\sigma}(d\varkappa)^\sigma \\ &= {}^{lf}S_\sigma\left\{\Phi(\varkappa)\right\}+{}^{lf}S_\sigma\left\{\varphi(\varkappa)\right\}. \end{aligned}$$

□

Theorem 2.10. *(local fractional Laplace-Sumudu duality).*
If $L_\sigma\left\{\Phi(\varkappa)\right\}=\Phi_s^{L,\sigma}(s)$ *and* ${}^{LF}S_\sigma\left\{\Phi(\varkappa)\right\}=F_\sigma(\upsilon)$, *then one has*

$$ {}^{lf}S_\sigma\left\{\Phi(\varkappa)\right\}=\frac{1}{u^\sigma}L_\sigma\left\{\Phi(\frac{1}{\varkappa})\right\}, \tag{2. 22}$$

$$ L_\sigma\left\{\Phi(\varkappa)\right\}=\frac{{}^{LF}S_\sigma\left\{\Phi(\frac{1}{s})\right\}}{s^\sigma}. \tag{2. 23}$$

Proof. see [20] □

Theorem 2.11. *1- (local fractional Sumudu transform of local fractional derivative).*
If ${}^{lf}\mathrm{S}_\sigma\left\{\Phi(\varkappa)\right\}=\digamma_\sigma(\upsilon)$, *then one has*

$$ {}^{lf}\mathrm{S}_\sigma\left\{\frac{d^\sigma\Phi(\varkappa)}{d\varkappa^\sigma}\right\}=\frac{\digamma_\sigma(\upsilon)-\digamma(0)}{\upsilon^\sigma}. \tag{2. 24}$$

As the direct result of (2. 24), we have the following results. If ${}^{lf}S_\sigma\left\{\Phi(r)\right\}=\digamma_\sigma(\upsilon)$, *we obtain*

$$ {}^{lf}\mathrm{S}_\sigma\left\{\frac{d^{n\sigma}\Phi(\varkappa)}{d\varkappa^{n\sigma}}\right\}=\frac{1}{\upsilon^{n\sigma}}\left[\digamma_\sigma(\upsilon)-\sum_{k=0}^{n-1}u^{k\sigma}\Phi^{(k\sigma)}(0)\right]. \tag{2. 25}$$

When $n=2$, *from (2. 25), we get*

$$ {}^{lf}\mathrm{S}_\sigma\left\{\frac{d^{2\sigma}\Phi(\varkappa)}{d\varkappa^{2\sigma}}\right\}=\frac{1}{\upsilon^{2\sigma}}\left[\digamma_\sigma(\upsilon)-\Phi(0)-u^\sigma\Phi^{(\sigma)}(0)\right]. \tag{2. 26}$$

2- (local fractional Sumudu transform of local fractional integral).
If ${}^{lf}S_\sigma\left\{\Phi(\varkappa)\right\}=\digamma_\sigma(\upsilon)$, *then we have*

$$ {}^{lf}\mathrm{S}_\sigma\left\{{}_0I_\varkappa^{(\sigma)}\Phi(\varkappa)\right\}=\upsilon^\sigma\digamma_\sigma(\upsilon). \tag{2. 27}$$

Proof. see [20] □

Theorem 2.12. *(local fractional convolution).*
If ${}^{lf}\mathrm{S}_\sigma\{\Phi(\varkappa)\} = F_\sigma(\upsilon)$ *and* ${}^{lf}\mathrm{S}_\sigma\{\varphi(\varkappa)\} = \Psi_\sigma(\upsilon)$*, then one has*

$$ {}^{lf}\mathrm{S}_\sigma\{\Phi(\varkappa)*\varphi(\varkappa)\} = \upsilon^\sigma F_\sigma(\upsilon)\Psi_\sigma(\upsilon), \tag{2. 28}$$

where

$$\Phi(\varkappa)*\varphi(\varkappa) = \frac{1}{\Gamma(1+\sigma)}\int_0^\infty \Phi(\tau)\varphi(\varkappa-\tau)(d\tau)^\sigma.$$

Proof. see [20] □

3. Analysis of the Method

Let us consider the following nonlinear system of partial differential equations with local fractional derivative

$$\begin{cases} \frac{\partial^\sigma X}{\partial\tau^\sigma} + \frac{\partial^\sigma T}{\partial\varkappa^\sigma} + N_{\sigma,1}(X,T) + R_{\sigma,1}(X,T) = \varphi(\varkappa,\tau), \\ \frac{\partial^\sigma T}{\partial\tau^\sigma} + \frac{\partial^\sigma X}{\partial\varkappa^\sigma} + N_{\sigma,2}(X,T) + R_{\sigma,2}(X,T) = \psi(\varkappa,\tau), \end{cases} \tag{3. 29}$$

where $\frac{\partial^\sigma}{\partial(\cdot)^\sigma}$ denote linear local fractional derivative operator of order σ. $R_{\sigma,1}$, $R_{\sigma,2}$ denotes linear local fractional operators, $N_{\sigma,1}$, $N_{\sigma,2}$ denotes nonlinear local fractional operators and $\varphi(\varkappa,\tau)$, $\psi(\varkappa,\tau)$ two functions denotes the non-differentiable source terms.

We apply the transformation ${}^{lf}S_\sigma$ on both sides of (3. 29), we obtain

$$\begin{cases} {}^{lf}S_\sigma\left[\frac{\partial^\sigma X}{\partial\tau^\sigma}\right] + {}^{lf}S_\sigma\left[\frac{\partial^\sigma T}{\partial\varkappa^\sigma} + N_{\sigma,1}(X,T) + R_{\sigma,1}(X,T)\right] = {}^{lf}S_\sigma[\varphi(\varkappa,\tau)] \\ {}^{lf}S_\sigma\left[\frac{\partial^\sigma T}{\partial\tau^\sigma}\right] + {}^{lf}S_\sigma\left[\frac{\partial^\sigma X}{\partial\varkappa^\sigma} + N_{\sigma,2}(X,T) + R_{\sigma,2}(X,T)\right] = {}^{lf}S_\sigma[\psi(\varkappa,\tau)] \end{cases}. \tag{3. 30}$$

Depending on the properties of this transform, we have

$$\begin{cases} {}^{lf}S_\sigma[X] = X(\varkappa,0) + \upsilon^\sigma\left({}^{lf}S_\sigma[\varphi(\varkappa,\tau)]\right) - \upsilon^\sigma\left({}^{lf}S_\sigma\left[\frac{\partial^\sigma T}{\partial\varkappa^\sigma} + N_{\sigma,1}(X,T) + R_{\sigma,1}(X,T)\right]\right) \\ {}^{lf}S_\sigma[T] = T(\varkappa,0) + \upsilon^\sigma\left({}^{lf}S_\sigma[\psi(\varkappa,\tau)]\right) - \upsilon^\sigma\left({}^{lf}S_\sigma\left[\frac{\partial^\sigma X}{\partial\varkappa^\sigma} + N_{\sigma,2}(X,T) + R_{\sigma,2}(X,T)\right]\right) \end{cases}. \tag{3. 31}$$

Taking the inverse transformation on both sides of (3. 31) gives

$$\begin{cases} X = X(\varkappa,0) + {}^{lf}S_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}S_\sigma[\varphi(\varkappa,\tau)]\right)\right) - {}^{lf}S_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}S_\sigma\left[\frac{\partial^\sigma T}{\partial\varkappa^\sigma} + N_{\sigma,1}(X,T) + R_{\sigma,1}(X,T)\right]\right)\right) \\ T = T(\varkappa,0) + {}^{lf}S_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}S_\sigma[\psi(\varkappa,\tau)]\right)\right) - {}^{lf}S_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}S_\sigma\left[\frac{\partial^\sigma X}{\partial\varkappa^\sigma} + N_{\sigma,2}(X,T) + R_{\sigma,2}(X,T)\right]\right)\right) \end{cases}. \tag{3. 32}$$

According to the Adomian method [1], we can introduce the two unknown functions X and T into two infinite series, as follows

$$\begin{aligned} X(\varkappa,\tau) &= \textstyle\sum_{n=0}^\infty X_n(\varkappa,\tau), \\ T(\varkappa,\tau) &= \textstyle\sum_{n=0}^\infty T_n(\varkappa,\tau). \end{aligned} \tag{3. 33}$$

and the nonlinear terms can be decomposed as

$$\begin{array}{c} N_{\sigma,1}(X,T)=\sum_{n=0}^{\infty}A_n, \\ N_{\sigma,2}(X,T)=\sum_{n=0}^{\infty}B_n, \end{array} \quad (3.\ 34)$$

where A_n and B_n are Adomian polynomials [25].

Substituting (3. 33) and (3. 34) in (3. 32), give us the following result

$$\left\{ \begin{array}{c} \sum_{n=0}^{\infty}X_n(\varkappa,\tau)=X(\varkappa,0)+\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\varphi(\varkappa,\tau)\right]\right)\right) \\ -\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\frac{\partial^{\sigma}}{\partial\varkappa^{\sigma}}\left(\sum_{n=0}^{\infty}T_n\right)+\sum_{n=0}^{\infty}A_n+R_{1,\sigma}\left(\sum_{n=0}^{\infty}X_n,\sum_{n=0}^{\infty}T_n\right)\right]\right)\right), \\ \sum_{n=0}^{\infty}T_n(\varkappa,\tau)=T(\varkappa,0)+\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\psi(\varkappa,\tau)\right]\right)\right) \\ -\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\frac{\partial^{\sigma}}{\partial\varkappa^{\sigma}}\left(\sum_{n=0}^{\infty}X_n\right)+\sum_{n=0}^{\infty}B_n+R_{2,\sigma}\left(\sum_{n=0}^{\infty}X_n,\sum_{n=0}^{\infty}T_n\right)\right]\right)\right). \end{array}\right. \quad (3.\ 35)$$

On comparing both sides of (3. 35), we have

$$\left[\begin{array}{c} X_0(\varkappa,\tau)=X(\varkappa,0)+\ ^{lf}S_{\sigma}^{-1}\left(u^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\varphi(\varkappa,\tau)\right]\right)\right), \\ X_1(\varkappa,\tau)=-\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\frac{\partial^{\sigma}T_0}{\partial\varkappa^{\sigma}}+A_0+R_{1,\sigma}\left(X_0,T_0\right)\right]\right)\right), \\ X_2(\varkappa,\tau)=-\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\frac{\partial^{\sigma}T_1}{\partial\varkappa^{\sigma}}+A_1+R_{1,\sigma}\left(X_1,T_1\right)\right]\right)\right), \\ X_3(\varkappa,\tau)=-\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\frac{\partial^{\sigma}T_2}{\partial\varkappa^{\sigma}}+A_2+R_{1,\sigma}\left(X_2,T_2\right)\right]\right)\right), \\ \vdots \end{array}\right] \quad (3.\ 36)$$

$$\left[\begin{array}{c} T_0(\varkappa,\tau)=T(\varkappa,0)+\ ^{lf}S_{\sigma}^{-1}\left(u^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\psi(\varkappa,\tau)\right]\right)\right), \\ T_1(\varkappa,\tau)=-\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\frac{\partial^{\sigma}X_0}{\partial\varkappa^{\sigma}}+B_0+R_{2,\sigma}\left(X_0,T_0\right)\right]\right)\right), \\ T_2(\varkappa,\tau)=-\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\frac{\partial^{\sigma}X_1}{\partial\varkappa^{\sigma}}+B_1+R_{2,\sigma}\left(X_1,T_1\right)\right]\right)\right), \\ T_3(\varkappa,\tau)=-\ ^{lf}S_{\sigma}^{-1}\left(\upsilon^{\sigma}\left(\ ^{lf}S_{\sigma}\left[\frac{\partial^{\sigma}X_2}{\partial\varkappa^{\sigma}}+B_2+R_{2,\sigma}\left(X_2,T_2\right)\right]\right)\right), \\ \vdots \end{array}\right] \quad (3.\ 37)$$

In the latter case, the solution of a system (3. 29) can be obtained by calculating the following two limits

$$\left\{ \begin{array}{c} X(\varkappa,\tau)=\lim_{N\to\infty}\sum_{n=0}^{N}X_n(\varkappa,\tau) \\ T(\varkappa,\tau)=\lim_{N\to\infty}\sum_{n=0}^{N}T_n(\varkappa,\tau) \end{array}\right. . \quad (3.\ 38)$$

4. Applications

In this section, we will apply the proposed method to solve two nonlinear system of partial differential equations with local fractional derivative.

Example 4.1. *We will apply the proposed method to solve the following coupled Burguer system within local fractional derivative.*

$$\begin{cases} \frac{\partial^{\sigma} X}{\partial \tau^{\sigma}} - \frac{\partial^{2\sigma} X}{\partial \varkappa^{2\sigma}} - 2XX_{\varkappa}^{(\sigma)} + (XT)_{\varkappa}^{(\sigma)} = 0 \\ \frac{\partial^{\sigma} T}{\partial \tau^{\sigma}} - \frac{\partial^{2\sigma} T}{\partial \varkappa^{2\sigma}} - 2TT_{\varkappa}^{(\sigma)} + (XT)_{\varkappa}^{(\sigma)} = 0 \end{cases}, \ 0 < \sigma \leqslant 1, \tag{4. 39}$$

with initial conditions

$$X(\varkappa, 0) = \sin_{\sigma}(\varkappa^{\sigma}),\ T(\varkappa, 0) = \sin_{\sigma}(\varkappa^{\sigma}). \tag{4. 40}$$

Depending to the formula (3. 32), we obtain

$$\begin{cases} X(\varkappa,\tau) = \sin_{\sigma}(\varkappa^{\sigma}) - {}^{lf}\mathrm{S}_{\sigma}^{-1}\left(\upsilon^{\sigma}\left({}^{lf}\mathrm{S}_{\sigma}\left[-\frac{\partial^{2\sigma} T}{\partial \varkappa^{2\sigma}} - 2XX_{\varkappa}^{(\sigma)} + (XT)_{\varkappa}^{(\sigma)}\right]\right)\right) \\ T(\varkappa,\tau) = \sin_{\sigma}(\varkappa^{\sigma}) - {}^{lf}\mathrm{S}_{\sigma}^{-1}\left(\upsilon^{\sigma}\left({}^{lf}\mathrm{S}_{\sigma}\left[-\frac{\partial^{2\sigma} X}{\partial \varkappa^{2\sigma}} - 2TT_{\varkappa}^{(\sigma)} + (XT)_{\varkappa}^{(\sigma)}\right]\right)\right) \end{cases}. \tag{4. 41}$$

Referring to the method adopted in this research [1], *each function of the solution* (X, T) *can be decomposed by an infinite series defined by*

$$\begin{aligned} X(\varkappa,\tau) &= \textstyle\sum_{n=0}^{\infty} X_n(\varkappa,\tau), \\ T(\varkappa,\tau) &= \textstyle\sum_{n=0}^{\infty} T_n(\varkappa,\tau), \end{aligned} \tag{4. 42}$$

and the nonlinear terms can be decomposed as

$$XX_{\varkappa}^{(\sigma)} = \sum_{n=0}^{\infty} A_n(X), \tag{4. 43}$$

$$(XT)_{\varkappa}^{(\sigma)} = \sum_{n=0}^{\infty} B_n(X,T), \tag{4. 44}$$

and

$$TT_{\varkappa}^{(\sigma)} = \sum_{n=0}^{\infty} C_n(X). \tag{4. 45}$$

Substituting (4. 42), (4. 43), (4. 44) and (4. 45) in (4. 41), we get

$$\begin{cases} \sum_{n=0}^{\infty} X_n(\varkappa,\tau) = \sin_{\sigma}(\varkappa^{\sigma}) - {}^{lf}\mathrm{S}_{\sigma}^{-1}\left(\upsilon^{\sigma}\left({}^{lf}\mathrm{S}_{\sigma}\left[\begin{array}{c} -\frac{\partial^{2\sigma}}{\partial \varkappa^{2\sigma}}\left(\sum_{n=0}^{\infty} X_n(\varkappa,\tau)\right) \\ -2\sum_{n=0}^{\infty} A_n(X) + \sum_{n=0}^{\infty} B_n(X,T) \end{array}\right]\right)\right) \\ \sum_{n=0}^{\infty} T_n(\varkappa,\tau) = \sin_{\sigma}(\varkappa^{\sigma}) - {}^{lf}\mathrm{S}_{\sigma}^{-1}\left(\upsilon^{\sigma}\left({}^{lf}\mathrm{S}_{\sigma}\left[\begin{array}{c} -\frac{\partial^{2\sigma}}{\partial \varkappa^{2\sigma}}\left(\sum_{n=0}^{\infty} T_n(\varkappa,\tau)\right) \\ -2\sum_{n=0}^{\infty} C_n(X) + \sum_{n=0}^{\infty} B_n(X,T) \end{array}\right]\right)\right) \end{cases}. \tag{4. 46}$$

On comparing both sides of (4. 46), we have

$$\begin{array}{c} X_0(\varkappa,\tau)=\sin_\sigma(\varkappa^\sigma),\\ X_1(\varkappa,\tau)=-\,^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left(\,^{lf}\mathrm{S}_\sigma\left[-\frac{\partial^{2\sigma}T_0}{\partial\varkappa^{2\sigma}}-2A_0(X)+B_0(X,T)\right]\right)\right),\\ X_2(\varkappa,\tau)=-\,^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left(\,^{lf}\mathrm{S}_\sigma\left[-\frac{\partial^{2\sigma}T_1}{\partial\varkappa^{2\sigma}}-2A_1(X)+B_1(X,T)\right]\right)\right),\\ X_3(\varkappa,\tau)=-\,^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left(\,^{lf}\mathrm{S}_\sigma\left[-\frac{\partial^{2\sigma}T_2}{\partial\varkappa^{2\sigma}}-2A_2(X)+B_2(X,T)\right]\right)\right),\\ \vdots \end{array} \tag{4. 47}$$

$$\begin{array}{c} T_0(\varkappa,\tau)=\sin_\sigma(\varkappa^\sigma),\\ T_1(\varkappa,\tau)=-\,^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left(\,^{lf}\mathrm{S}_\sigma\left[-\frac{\partial^{2\sigma}X_0}{\partial\varkappa^{2\sigma}}-2C_0(T)+B_0(X,T)\right]\right)\right),\\ T_2(\varkappa,\tau)=-\,^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left(\,^{lf}\mathrm{S}_\sigma\left[-\frac{\partial^{2\sigma}X_0}{\partial\varkappa^{2\sigma}}-2C_1(T)+B_1(X,T)\right]\right)\right),\\ T_3(\varkappa,\tau)=-\,^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left(\,^{lf}\mathrm{S}_\sigma\left[-\frac{\partial^{2\sigma}X_0}{\partial\varkappa^{2\sigma}}-2C_2(T)+B_2(X,T)\right]\right)\right),\\ \vdots \end{array} \tag{4. 48}$$

and so on.
The first few components of $A_n(X)$, $B_n(X,t)$ and $C(T)$ polynomials [25] *are given by*

$$\begin{array}{c} A_0(X)=X_0X_{0,\varkappa}^{(\sigma)},\\ A_1(X)=X_0X_{1,\varkappa}^{(\sigma)}+X_1X_{0,\varkappa}^{(\sigma)},\\ A_2(X)=X_0X_{2,\varkappa}^{(\sigma)}+X_2X_{0,\varkappa}^{(\sigma)}+X_1X_{1,\varkappa}^{(\sigma)},\\ \vdots \end{array} \tag{4. 49}$$

$$\begin{array}{c} B_0(X,T)=(X_0T_0)_{\varkappa}^{(\sigma)},\\ B_1(X,T)=(X_0T_1+X_1T_0)_{\varkappa}^{(\sigma)},\\ B_2(X,T)=(X_1T_1+X_0T_2+X_2T_0)_{\varkappa}^{(\sigma)},\\ \vdots \end{array} \tag{4. 50}$$

and

$$\begin{array}{c} C_0(T)=T_0T_{0,\varkappa}^{(\sigma)},\\ C_1(T)=T_0T_{1,\varkappa}^{(\sigma)}+T_1T_{0,\varkappa}^{(\sigma)},\\ C_2(T)=T_0T_{2,\varkappa}^{(\sigma)}+T_2T_{0,\varkappa}^{(\sigma)}+T_1T_{1,\varkappa}^{(\sigma)},\\ \vdots \end{array} \tag{4. 51}$$

According to the equations (4. 47)-(4. 48) and formulas (4. 49)-(4. 51), the first terms of the solution (X,T) obtained using the proposed method are as follows

$$\begin{array}{c} X_0(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma), \\ X_1(\varkappa,\tau) = -\sin_\sigma(\varkappa^\sigma)\frac{\tau^\sigma}{\Gamma(1+\sigma)}, \\ X_2(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)\frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}, \\ X_3(\varkappa,\tau) = -\sin_\sigma(\varkappa^\sigma)\frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}, \\ \vdots \end{array} \tag{4. 52}$$

and

$$\begin{array}{c} T_0(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma), \\ T_1(\varkappa,\tau) = -\sin_\sigma(\varkappa^\sigma)\frac{\tau^\sigma}{\Gamma(1+\sigma)}, \\ T_2(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)\frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}, \\ T_3(\varkappa,\tau) = -\sin_\sigma(\varkappa^\sigma)\frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}, \\ \vdots \end{array} \tag{4. 53}$$

Thus, the approximate solution (X,T), *is given by*

$$\begin{cases} X(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)\left(1 - \frac{\tau^\sigma}{\Gamma(1+\sigma)} + \frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)} - \frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)} + \cdots\right), \\ T(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)\left(1 - \frac{\tau^\sigma}{\Gamma(1+\sigma)} + \frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)} - \frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)} + \cdots\right), \end{cases} \tag{4. 54}$$

and in a closed form, we obtain the non-differentiable solution (X,T) *defined by*

$$\begin{cases} X(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)E_\sigma(-\tau^\sigma), \\ T(\varkappa,\tau) = \sin_\sigma(\varkappa^\sigma)E_\sigma(-\tau^\sigma). \end{cases} \tag{4. 55}$$

Substituting $\sigma = 1$ *into (4. 55), we obtain*

$$\begin{cases} X(\varkappa,\tau) = \sin(\varkappa)e^{-\tau}, \\ T(\varkappa,\tau) = \sin(\varkappa)e^{-\tau}. \end{cases} \tag{4. 56}$$

Note that, our solution (4. 55) satisfies the initial conditions (4. 40), and in the case $\sigma = 1$, *we obtain the same solution obtained in* [21] *by the homotopy perturbation method.*

Example 4.2. *In this second example, we will solve the following nonlinear system of three partial differential equations with local fractional derivative*

$$\begin{cases} X_\tau^{(\sigma)} + T_\varkappa^{(\sigma)} Z_\upsilon^{(\sigma)} - T_\upsilon^{(\sigma)} Z_\varkappa^{(\sigma)} = -X \\ T_\tau^{(\sigma)} + X_\varkappa^{(\sigma)} Z_\upsilon^{(\sigma)} + X_\upsilon^{(\sigma)} Z_\varkappa^{(\sigma)} = T \\ Z_\tau^{(\sigma)} + X_\varkappa^{(\sigma)} T_\upsilon^{(\sigma)} + X_\upsilon^{(\sigma)} T_\varkappa^{(\sigma)} = Z \end{cases}, \; 0 < \sigma \leqslant 1, \tag{4. 57}$$

subject to the initial conditions

$$X(\varkappa,\upsilon,0)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma),\ T(\varkappa,\upsilon,0)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma),\ Z(\varkappa,\upsilon,0)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma). \tag{4. 58}$$

The application of the transformation ${}^{lf}\mathrm{S}_\sigma$ *to each of the system equations (4. 57), gives the following result*

$$\begin{cases} {}^{lf}\mathrm{S}_\sigma\left[X(\varkappa,\upsilon,\tau)\right]=X(\varkappa,\upsilon,0)-\upsilon^\sigma\left({}^{lf}\mathrm{S}_\sigma\left[T_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)}-T_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)}+X\right]\right)\\ {}^{lf}\mathrm{S}_\sigma\left[T(\varkappa,\upsilon,\tau)\right]=T(\varkappa,\upsilon,0)-\upsilon^\sigma\left({}^{lf}\mathrm{S}_\sigma\left[X_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)}+X_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)}-T\right]\right)\\ {}^{lf}\mathrm{S}_\sigma\left[Z(\varkappa,\upsilon,\tau)\right]=Z(\varkappa,\upsilon,0)-\upsilon^\sigma\left({}^{lf}\mathrm{S}_\sigma\left[X_\varkappa^{(\sigma)}T_\upsilon^{(\sigma)}+X_\upsilon^{(\sigma)}T_\varkappa^{(\sigma)}-Z\right]\right) \end{cases}.$$

Taking ${}^{lf}S_\sigma^{-1}$ *on both sides of (4. 57), taking into account the initial condition (4. 58), gives the following formulas*

$$\begin{cases} X(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma)-\ {}^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathrm{S}_\sigma\left[T_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)}-T_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)}+X\right]\right)\right)\\ T(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)-\ {}^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathrm{S}_\sigma\left[X_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)}+X_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)}-T\right]\right)\right)\\ Z(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)-\ {}^{lf}\mathrm{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathrm{S}_\sigma\left[X_\varkappa^{(\sigma)}T_\upsilon^{(\sigma)}+X_\upsilon^{(\sigma)}T_\varkappa^{(\sigma)}-Z\right]\right)\right) \end{cases}. \tag{4. 59}$$

Since the Adomian decomposition method [1] *depends on the decomposed each function of the solution* (X,T,Z) *by an infinite series as follows*

$$\begin{aligned} X(\varkappa,\upsilon,\tau)&=\textstyle\sum_{n=0}^{\infty}X_n(\varkappa,\upsilon,\tau),\\ T(\varkappa,\upsilon,\tau)&=\textstyle\sum_{n=0}^{\infty}T_n(\varkappa,\upsilon,\tau),\\ Z(\varkappa,\upsilon,\tau)&=\textstyle\sum_{n=0}^{\infty}Z_n(\varkappa,\upsilon,\tau), \end{aligned} \tag{4. 60}$$

and the nonlinear terms can be decomposed as

$$T_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)}=\sum_{n=0}^{\infty}A_n(T,Z),\quad T_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)}=\sum_{n=0}^{\infty}A'_n(T,Z), \tag{4. 61}$$

$$X_\varkappa^{(\sigma)}Z_\upsilon^{(\sigma)}=\sum_{n=0}^{\infty}B_n(X,Z),\quad X_\upsilon^{(\sigma)}Z_\varkappa^{(\sigma)}=\sum_{n=0}^{\infty}B'_n(X,Z), \tag{4. 62}$$

and

$$X_\varkappa^{(\sigma)}T_\upsilon^{(\sigma)}=\sum_{n=0}^{\infty}C_n(X,T),\quad X_\upsilon^{(\sigma)}T_\varkappa^{(\sigma)}=\sum_{n=0}^{\infty}C'_n(X,T). \tag{4. 63}$$

Substituting (4. 60), (4. 61), (4. 62) and (4. 69) in (4. 59), we get

$$\begin{cases} \sum_{n=0}^{\infty} X_n(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma+\upsilon^\sigma) \\ -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[\sum_{n=0}^{\infty} A_n(T,Z)-\sum_{n=0}^{\infty} A'_n(T,Z)+\sum_{n=0}^{\infty} X_n(\varkappa,\upsilon,\tau)\right]\right)\right), \\ \sum_{n=0}^{\infty} T_n(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma-\upsilon^\sigma) \\ -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[\sum_{n=0}^{\infty} B_n(X,Z)+\sum_{n=0}^{\infty} B'_n(X,Z)-\sum_{n=0}^{\infty} T_n(\varkappa,\upsilon,\tau)\right]\right)\right), \\ \sum_{n=0}^{\infty} Z_n(\varkappa,\upsilon,\tau) = E_\sigma(-\varkappa^\sigma+\upsilon^\sigma) \\ -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[\sum_{n=0}^{\infty} C_n(X,T)+\sum_{n=0}^{\infty} C'_n(X,T)-\sum_{n=0}^{\infty} Z_n(\varkappa,\upsilon,\tau)\right]\right)\right). \end{cases} \tag{4. 64}$$

On comparing both sides of (4. 64), we have

$$\begin{gathered} X_0(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma+\upsilon^\sigma), \\ X_1(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[A_0(T,Z)-A'_0(T,Z)+X_0(\varkappa,\upsilon,\tau)\right]\right)\right), \\ X_2(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[A_1(T,Z)-A'_0(T,Z)+X_1(\varkappa,\upsilon,\tau)\right]\right)\right), \\ X_3(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[A_2(T,Z)-A'_0(T,Z)+X_2(\varkappa,\upsilon,\tau)\right]\right)\right), \\ \vdots \end{gathered} \tag{4. 65}$$

$$\begin{gathered} T_0(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma-\upsilon^\sigma), \\ T_1(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[B_0(X,Z)+B'_0(X,Z)-T_0(\varkappa,\upsilon,\tau)\right]\right)\right), \\ T_2(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[B_1(X,Z)+B'_1(X,Z)-T_1(\varkappa,\upsilon,\tau)\right]\right)\right), \\ T_3(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[B_2(X,Z)+B'_2(X,Z)-T_2(\varkappa,\upsilon,\tau)\right]\right)\right), \\ \vdots \end{gathered} \tag{4. 66}$$

and

$$\begin{gathered} Z_0(\varkappa,\upsilon,\tau) = E_\sigma(-\varkappa^\sigma+\upsilon^\sigma), \\ Z_1(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[C_0(X,T)+C'_0(X,T)-Z_0(\varkappa,\upsilon,\tau)\right]\right)\right), \\ Z_2(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[C_1(X,T)+C'_1(X,T)-Z_1(\varkappa,\upsilon,\tau)\right]\right)\right), \\ Z_3(\varkappa,\upsilon,\tau) = -\,{}^{lf}\mathsf{S}_\sigma^{-1}\left(\upsilon^\sigma\left({}^{lf}\mathsf{S}_\sigma\left[C_2(X,T)+C'_2(X,T)-Z_2(\varkappa,\upsilon,\tau)\right]\right)\right), \\ \vdots \end{gathered} \tag{4. 67}$$

and so on.

The first few components of $A_n(T,Z)$, $B_n(X,Z)$ *and* $C_n(X,T)$ *polynomials* [25] *are given by*

$$\begin{gathered} A_0(T,Z) = T_{0\varkappa}^{(\sigma)} Z_{0\upsilon}^{(\sigma)}, \\ A_1(T,Z) = T_{1\varkappa}^{(\sigma)} Z_{0\upsilon}^{(\sigma)} + T_{0\varkappa}^{(\sigma)} Z_{1\upsilon}^{(\sigma)}, \\ A_2(T,Z) = T_{0\varkappa}^{(\sigma)} Z_{2\upsilon}^{(\sigma)} + T_{2\varkappa}^{(\sigma)} Z_{0\upsilon}^{(\sigma)} + T_{1\varkappa}^{(\sigma)} Z_{1\upsilon}^{(\sigma)}, \\ \vdots \end{gathered} \tag{4. 68}$$

$$\begin{array}{r}B_0(X,Z)=X_{0\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)},\\ B_1(X,Z)=X_{1\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)}+X_{0\varkappa}^{(\sigma)}Z_{1\upsilon}^{(\sigma)},\\ B_2(X,Z)=X_{0\varkappa}^{(\sigma)}Z_{2\upsilon}^{(\sigma)}+X_{2\varkappa}^{(\sigma)}Z_{0\upsilon}^{(\sigma)}+X_{1\varkappa}^{(\sigma)}Z_{1\upsilon}^{(\sigma)},\\ \vdots\end{array} \tag{4. 69}$$

and

$$\begin{array}{r}C_0(X,T)=X_{0\varkappa}^{(\sigma)}T_{0\upsilon}^{(\sigma)},\\ C_1(X,T)=X_{1\varkappa}^{(\sigma)}T_{0\upsilon}^{(\sigma)}+X_{0\varkappa}^{(\sigma)}T_{1\upsilon}^{(\sigma)},\\ C_2(X,T)=X_{0\varkappa}^{(\sigma)}T_{2\upsilon}^{(\sigma)}+X_{2\varkappa}^{(\sigma)}T_{0\upsilon}^{(\sigma)}+X_{1\varkappa}^{(\sigma)}T_{1\upsilon}^{(\sigma)},\\ \vdots\end{array} \tag{4. 70}$$

For other polynomials A'_n, B'_n and C'_n, it can be calculated in the same manner.

From the equations (4. 65)-(4. 67) and formulas of the polynomial terms, the first terms of the solution of (4. 57) are given by

$$\begin{array}{r}X_0(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma),\\ X_1(\varkappa,\upsilon,\tau)=-E_\sigma(\varkappa^\sigma+\upsilon^\sigma)\frac{\tau^\sigma}{\Gamma(1+\sigma)},\\ X_2(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma)\frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)},\\ X_3(\varkappa,\upsilon,\tau)=-E_\sigma(\varkappa^\sigma+\upsilon^\sigma)\frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)},\\ \vdots\end{array} \tag{4. 71}$$

$$\begin{array}{r}T_0(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma),\\ T_1(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)\frac{\tau^\sigma}{\Gamma(1+\sigma)},\\ T_2(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)\frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)},\\ T_3(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)\frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)},\\ \vdots\end{array} \tag{4. 72}$$

and

$$\begin{array}{r}Z_0(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma),\\ Z_1(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)\frac{\tau^\sigma}{\Gamma(1+\sigma)},\\ Z_2(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)\frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)},\\ Z_3(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)\frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)},\\ \vdots\end{array} \tag{4. 73}$$

So, the approximate solution is given

$$\left\{\begin{array}{r}X(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma+\upsilon^\sigma)(1-\frac{\tau^\sigma}{\Gamma(1+\sigma)}+\frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}-\frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}+\cdots),\\ T(\varkappa,\upsilon,\tau)=E_\sigma(\varkappa^\sigma-\upsilon^\sigma)(1+\frac{\tau^\sigma}{\Gamma(1+\sigma)}+\frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}+\frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}+\cdots),\\ Z(\varkappa,\upsilon,\tau)=E_\sigma(-\varkappa^\sigma+\upsilon^\sigma)(1+\frac{\tau^\sigma}{\Gamma(1+\sigma)}+\frac{\tau^{2\sigma}}{\Gamma(1+2\sigma)}+\frac{\tau^{3\sigma}}{\Gamma(1+3\sigma)}+\cdots),\end{array}\right. \tag{4. 74}$$

and in the closed form, the non-differentiable solution (X, T, Z), *takes the following form*

$$\begin{cases} X(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma + \upsilon^\sigma)E_\sigma\left(-\tau^\sigma\right), \\ T(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma - \upsilon^\sigma)E_\sigma\left(\tau^\sigma\right), \\ Z(\varkappa,\upsilon,\tau) = E_\sigma(-\varkappa^\sigma + \upsilon^\sigma)E_\sigma\left(\tau^\sigma\right). \end{cases} \quad (4.\ 75)$$

Depending on the results presented in [10], *we can write the solution (4. 75) as follows*

$$\begin{cases} X(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma + \upsilon^\sigma - \tau^\sigma), \\ T(\varkappa,\upsilon,\tau) = E_\sigma(\varkappa^\sigma - \upsilon^\sigma + \tau^\sigma), \\ Z(\varkappa,\upsilon,\tau) = E_\sigma(-\varkappa^\sigma + \upsilon^\sigma + \tau^\sigma). \end{cases} \quad (4.\ 76)$$

Substituting $\sigma = 1$ *into (4. 76), we obtain*

$$\begin{cases} X(\varkappa,\upsilon,\tau) = e^{\varkappa+\upsilon-\tau}, \\ T(\varkappa,\upsilon,\tau) = e^{\varkappa-\upsilon+\tau}, \\ Z(\varkappa,\upsilon,\tau) = e^{-\varkappa+\upsilon+\tau}. \end{cases} \quad (4.\ 77)$$

Note that, our solution (4. 76) satisfies the initial conditions (4. 58), and in the case $\sigma = 1$, *we obtain the same solution obtained in* [8] *by projected differential transform method and Elzaki transform.*

5. Conclusion

The extension of the modified method (LFSDM) for solving nonlinear systems of partial differential equations with local fractional derivatives, leads to establishing an efficient algorithm. The advantages of the LFSDM are that it converges rapidly to the exact solution if it exists, as shown by the results obtained through the two examples suggested in this paper. From the results obtained, it can be concluded that this algorithm is powerful and effective in applying to this type of nonlinear systems of local fractional partial differential equations, and thus can be applied to other nonlinear systems with local fractional derivative.

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