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A Study on Cheban Abel-Grassmann's Groupoids

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Abstract. An AG-groupoid also called a Left-almost semigroup is a magma satisfying the law, $uv \cdot w = wv \cdot u \quad \forall u, v, w$. In this paper, the concept of Cheban AG-groupoid is developed and investigated as a subclass of AG-groupoid. Various non-associative examples and counterexamples are constructed by the recent computational techniques of Mace-4, GAP and Prover-9. Cheban AG-groupoids are enumerated up to order 6 and various relations of this class are established with already existing subclasses of AG-groupoids. Furthermore, ideals in Cheban AG-groupoid are introduced and investigated.

AMS (MOS) Subject Classification Codes: 20N05, 20N02, 20N99
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1. Introduction

Abel Grassmann's groupoid (shortly AG-groupoid) is generally a non-commutative and non-associative structure that holds the left invertive law. The medial law, $tu \cdot vw = tv \cdot uw$ holds in every AG-groupoid however, the paramedial law, $tu \cdot vw = wu \cdot vt$ is satisfied in AG-groupoid if it possesses the left identity. AG-groupoids have various applications in geometry, flock theory, fuzzy and finite mathematics [1, 6, 7, 12, 13]. A variety of new subclasses is being rapidly introduced and investigated [4, 19, 20, 21]. We extend the concept of left (resp. right) Cheban loops arising in [18] and introduce some new subclasses of AG-groupoids that are left (resp. right) Cheban AG-groupoid. In the Section 2 we give definitions of these AG-groupoids and provide some non-associative examples to show their existence. We use juxtaposition in our mathematical calculations to avoid frequent use of parenthesis. For instance, $tu \cdot vw$ will mean $(t \cdot u) \cdot (v \cdot w)$. Various AG-groupoids with their defining identities arising in various papers [2, 3, 14, 16, 17, 19, 22, 23, 25] are presented in the following Table 1 that we use frequently throughout this article. In Section 3 we establish relations of Cheban AG-groupoids with some other previously known subclasses of AG-groupoids. It is investigated that any Cheban AG-groupoid is Bol*, is middle and left nuclear square and is a commutative semigroup, if S is the inverse AG-groupoid. We also prove that any AG* is Cheban AG-groupoid and that a Cheban AG-groupoid is a semigroup if either it has the left identity element or it is cancellative. In Section 4 we provide enumerations of Cheban AG-groupoids up to order 6 and in Section 5 we define and investigate various ideals in a Cheban AG-groupoid. From now onward Cheban will mean a Cheban AG-groupoid.

2. MATERIALS AND METHOD

We listed various definitions of different subclasses of the AG-groupoid in Table-1 below to investigate the new class of Cheban AG-groupoid. We have used the modern techniques of GAP and Mace-4 to enumerate Cheban AG-groupoids up to order 6 and to find various examples and counterexamples.

Cheban AG-groupoid

Definition 1. *Let S be an AG-groupoid, then is called -*

(i) -left Cheban AG-groupoid if $\forall t, u, v, w \in S$,

$$t(uv \cdot w) = vt \cdot uw$$

(ii) -right Cheban AG-groupoid if $\forall t, u, v, w \in S$,

$$(t \cdot uv)w = tw \cdot vu$$

(iii) -Cheban AG-groupoid if S is both a left and a right Cheban AG-groupoid.

AG-gropoid	Defining Identitiy	AG-gropoid	Defining Identitiy
Bol*	$t(uv \cdot w) = (tu \cdot v)w$	Right alternative	$uv \cdot v = u \cdot vv$
Inverse	$uv \cdot u = u$ & $vu \cdot v = v$	Self-dual	$u \cdot vw = w \cdot vu$
Left nuclear square	$u^2v \cdot w = u^2 \cdot vw$	Right cancellative	$ux = vx \Rightarrow u = v$
Right nuclear square	$uv \cdot w^2 = u \cdot vw^2$	Left cancellative	$xu = xv \Rightarrow u = v$
Middle nuclear square	$uv^2 \cdot w = u \cdot v^2w$	AG-3-band	$u \cdot uu = uu \cdot u = u$
AG*	$uv \cdot w = v \cdot uw$	T^1	$tu = vw \Rightarrow ut = wv$

Table 1. AG-groupoids and their identities

Example 1. *The following Caley's tables respectively represent:*

(i) Right Cheban AG-groupoid of order 3.

(ii) Left Cheban AG-groupoid of order 3.

(iii) Cheban AG-groupoid of order 3.

$*$	0	1	2
0	0	0	0
1	0	0	0
2	1	1	1

(i)

$*$	0	1	2
0	0	0	0
1	0	0	0
2	0	1	1

(ii)

$*$	0	1	2
0	0	0	0
1	0	0	0
2	0	1	0

(iii)

3. RESULTS AND DISCUSSIONS

3.1. **Relations of Cheban AG-groupoid with some other known subclasses.** Here, relations of Cheban AG-groupoid are investigated with some of the existing subclasses of AG-groupoid.

Theorem 1. *A Cheban AG-groupoid is Bol*.*

Proof. Let t, u, v, w be elements of a Cheban AG-groupoid S. Then, by the left invertive law, the medial law and by the Definition 1), we have

$$\begin{aligned} t(uv \cdot w) &= t(wv \cdot u) = vt \cdot wu = vw \cdot tu = (v \cdot ut)w \\ &= (w \cdot ut)v = wv \cdot tu = (tu \cdot v)w. \end{aligned}$$

Thus, $t(uv \cdot w) = (tu \cdot v)w$. Hence S is Bol*-AG-groupoid. □

The converse statement of Theorem (1) may not be valid, as shown by the following counterexample.

Example 2. *A Bol*-AG-groupoid which is not a Cheban AG-groupoid. Here,* $0 * (1 * 2) * 3) \neq (2 * 0) * (1 * 3)$*. Thus S is not a left Cheban or a Cheban AG-groupoid.*

$*$	*0*	*1*	*2*	*3*
0	*0*	*1*	*2*	*3*
1	*1*	*0*	*3*	*2*
2	*3*	*2*	*1*	*0*
3	*2*	*3*	*0*	*1*

It has been investigated that Every Bol*-AG-groupoid is paramedial AG-groupoid [20]. Hence, using this fact and Theorem (1), we prove the following.

Corollary 1. *Cheban AG-groupoid is paramedial AG-groupoid.*

Next, we establish a relation of Cheban AG-groupoid with the left nuclear square and middle nuclear square AG-groupoids.

Theorem 2. *Any inverse Cheban AG-groupoid is a commutative semigroup.*

Proof. Let S be a be the inverse Cheban AG-groupoid, and $u, v \in S$. Then, by left invertive and medial laws and Definition (1), we have

$$\begin{aligned} uv &= (uv \cdot u)v = (vu)(uv) = u((uv)v) \\ &= u((vv)u) = vu \cdot vu = v(uv \cdot u) = vu \end{aligned}$$

Thus $uv = vu$. Therefore, S is commutative, but as each commutative AG-groupoid is also an associative, thus S is a commutative semigroup. □

Theorem 3. *For a Cheban AG-groupoid K each of the following hold:*

(i) K is middle nuclear square.
(ii) K is left nuclear square.

Proof. Let K be a Cheban AG-groupoid, and $u, v, w \in K$.

(i) Then, by left invertive, medial laws, Definition (1) and Theorem (1) we get

$$\begin{aligned} u(v^2w) &= (vu \cdot vw) = (v(wv))u = (u(wv))v = \\ &= uv \cdot vw = v(vu \cdot w) = v(wu \cdot v) = \\ &= uv \cdot wv = uw \cdot vv = (u \cdot vv)w = (uv^2)w \end{aligned}$$

Thus $u(v^2w) = (uv^2)w$. Hence, K is middle nuclear square AG-groupoid.

(ii) Again by the use of left invertive and medial laws, Theorem (3, Part i) and Definition (1), we have

$$\begin{aligned} (u^2v)w &= (wv \cdot u^2) = (w \cdot uu)v = (uw \cdot uv) = \\ &= uu \cdot wv = u(wu \cdot v) = u(vu \cdot w) = uu \cdot vw = (u^2 \cdot vw \end{aligned}$$

Thus $(u^2v)w = u^2(vw)$. Hence, K is is left nuclear square AG-groupoid and the theorem is proved.

□

Now, a counterexample is provided to show that each Cheban AG-groupoid is neither a right nuclear square nor a right alternative. However, we prove in the following that a Cheban AG-groupoid is right nuclear square if it is the right alternative AG-groupoid.

Example 3. *A Cheban AG-groupoid of size 4 that is neither a right alternative nor a right nuclear square.*

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	3	2	1	0
3	0	0	2	2

The next theorem has extended the idea of Theorem (3) to establish a relation for Cheban AG-groupoid with nuclear square if it is either a right alternative AG-groupoid, or a self-dual AG-groupoid or a T^1-AG-groupoid.

Theorem 4. *A Cheban AG-groupoid S is nuclear square, if any of the following is true:*

(i) S is right alternative AG-groupoid.
(ii) S is self-dual AG-groupoid.
(iii) S is T^1-AG-groupid.

Proof. Let S is Cheban AG-groupoid, and $u, v, w \in S$. Then

(i) If S is a right alternative, then by left invertive and medial laws and Definition (1), we have

$$\begin{aligned} u(vw^2) &= u(vw \cdot w) = u(w^2v) = wu \cdot wv = \\ &= ww \cdot uv = (uv \cdot w)w = uv \cdot ww = (uv)w^2. \end{aligned}$$

Thus $u(uw^2) = (uv)w^2$. Hence, S is right nuclear square AG-groupoid, and thus by Theorem (3) the result follows.

(ii) If S is self-dual AG-groupoid, then again by Corollary (1), and medial law, we have

$$uv(w^2) = wv \cdot wu = ww \cdot vu = u \cdot vw^2.$$

that is $uv \cdot w^2 = u \cdot vw^2$. Hence, S is right nuclear square, and whence is nuclear square by Theorem (3).

(iii) If S is T^1-AG-groupoid, then by the left invertive and medial laws and Definition (1), we have

$$\begin{aligned} uv \cdot w^2 &= uw \cdot vw \Rightarrow w^2 \cdot uv = vw \cdot uw = \\ &= vu \cdot ww = (v \cdot ww)u \Rightarrow (uv)w^2 = u(vw^2). \end{aligned}$$

that is $uv \cdot w^2 = u \cdot vw^2$. Hence, S is right nuclear square, and whence is nuclear square by Theorem (3).

□

Theorem 5. *Any AG*-groupoid is a Cheban AG-groupoid.*

Proof. Assume that K is an AG*-groupoid, and let $u, v, w, x \in K$. We prove via in turn that K is left and a right Cheban AG-groupoid.

(i) For left Cheban AG-groupoid let $u, v, w, x \in K$. Then, using Part (viii) of Table 1, we have

$$u(vw \cdot x) = u(w \cdot vx) = wu \cdot vx \Rightarrow u(vw \cdot x) = wu \cdot vx.$$

This shows that K is left Cheban AG-groupoid.

(ii) For right Cheban AG-groupoid let $u, v, w, x \in K$. We use again the left invertive, medial laws and Part (viii) of Table 1, we have

$$\begin{aligned} u(vw \cdot x) &= (vu \cdot w)x = xw \cdot vu \\ &\Rightarrow (v \cdot xw)u = (xv \cdot w)u = uw \cdot xv = ux \cdot wv \\ &\Rightarrow (u \cdot vw)x = ux \cdot wv. \end{aligned}$$

This shows that K is right Cheban AG-groupoid. Hence the result is established.

□

A Cheban AG-groupoid becomes a commutative semigroup under the following variety of conditions.

Theorem 6. *For each of the following assertions, a Cheban AG-groupoid S is a commutative semigroup.*

(i) S has the left identity element.
(ii) S is AG-3-band.
(iii) S is left cancellative.
(iv) S is left cancellative.

Proof. Let S be a Cheban AG-groupoid. Then

(i) Assume S has the left identity e, and let $u, v \in S$. Then by left invertive, medial laws and Definition (1), we have

$$\begin{aligned} uv &= eu \cdot ev = (e \cdot ve)u = (u \cdot ve)e = \\ &= ue \cdot ev = e(eu \cdot v) = e(vu \cdot e) = \\ &= ue \cdot ve = uv \cdot ee = uv \cdot e = ev \cdot u = vu. \end{aligned}$$

Thus the commutativity of S is established, that leads to associativity in AG. Therefore, S is a commutative semigroup.

(ii) Assume S is AG-3-band, then Using the left invertive, medial laws, the assumption, Definition (1) and Theorem (1), we get

$$\begin{aligned}
uv &= (uv\cdot uv)uv = (v(uu\cdot v))uv = (v(vu\cdot u))uv = \\
&= ((vv\cdot u)u)uv = (uv\cdot u)(vv\cdot u) = ((uv)(vv))(uv) = \\
&= ((u\cdot vv)v)(uu) = ((v(vv))u)(uu) = (vu\cdot vv)(uu) = \\
&= (uu\cdot vv)(vu) = (uv\cdot uv)(vu) = ((u\cdot vu)v)(vu) = \\
&= ((v(vu))u)(vu) = (vu\cdot uv)(vu) = (u(uv\cdot v))(vu) = \\
&= (u(vv\cdot u))(uu) = (vu\cdot vu)vu = vu
\end{aligned}$$

Thus $uv = vu$. That is S commutative, and thus is an associative. Equivalently, S is commutative semigroup.

(iii) Assume S is left cancellative-AG-groupoid. Now Definition (1) the assumption, left invertive and medial laws are used to get

$$\begin{aligned}
x(xu\cdot v) &= x(vu\cdot x) = ux\cdot vx = uv\cdot xx = (u\cdot xx)v = \\
&= (v\cdot xx)u = vu\cdot xx = vx\cdot ux = x(uv\cdot x) = \\
&= x(xv\cdot u) \Rightarrow x(xu\cdot v) = x(xv\cdot u) = \\
&\Rightarrow xu\cdot v = xv\cdot u
\end{aligned}$$

Equivalently $uv = vu$. Thus, S commutative, and hence associative. Equivalently, S is commutative semigroup.

(iv) Assume S is right cancellative-AG-groupoid. Again, using Definition (1) the assumption, left invertive, medial laws and Theorem (1), we have

$$\begin{aligned}
(uv\cdot x)x &= (xv\cdot u)x = x(vu\cdot x) = ux\cdot vx = uv\cdot xx = \\
&= (u\cdot xx)v = (v\cdot xx)u = vu\cdot xx = vx\cdot ux = \\
&= (v\cdot xu)x = x(xu\cdot v) = xv\cdot ux = v((ux)x) = (vu\cdot x)x \\
&\Rightarrow (uv\cdot x)x = ((vu)x)x \\
&\Rightarrow uv\cdot x = vu\cdot x
\end{aligned}$$

Therefore $uv = vu$. Equivalently, S is commutative, and thus associative.

□

The following counterexample has depicted that each Cheban AG-groupoid may neither be a right nuclear square nor a T^1-AG-groupoid.

Example 4. *Let $Q = \{1, 2, 3, 4\}$, then, $(Q, *)$ in the table below is Cheban AG-groupoid that is neither right nuclear square nor T^1-AG-groupoid, as* $(1*1)*(4*4) \neq 1*(1*(4*4))$ *and* $1*1 = 1*2$ *do not imply* $1*1 = 2*1$.

$*$	1	2	3	4
1	2	2	3	2
2	3	3	3	3
3	3	3	3	3
4	2	2	3	2

Based on our data, the following reality may be true; however, we failed to prove or to provide a counterexample to disprove it. Thus, we have proposed it as a conjecture to the reader.

Conjecture 1. *Every unipotent Cheban AG-groupoid is right alternative.*

4. ENUMERATION OF CHEBAN AG-GROUPOIDS

Classification and enumeration of various mathematical entries is considered as a sound area of research in finite and pure mathematics. Classification of the finite simple groups is considered as an achievement of the current century. The enumeration of various structures is obtained indifferent ways like; combinatorial or in algebraic consideration. Using the combinatorial consideration and the bespoke exhaustive generation software some non-associative algebraic structures, like, quasigroups and loops are enumerated up to size 11 [15]. FINDER (Finite domain enumeration) [24] is used for enumeration of IP loops up to the size 13 [5]. The developed constraint solving techniques for semigroups and the monoids, the third author of this article, in collaboration with A. Distler (the author of [8, 9, 10]) has used GAP to enumerate AG-groupoids. It is worth mentioning that the data presented in [11] is verified by one of the referees of the said article with Mace-4 and Isofilter as acknowledged in the mentioned article. All this validate the enumeration and classification results for the Cheban AG-groupoids, a subclass of AG-groupoids, as the same techniques and relevant data of [11] with modified coding in GAP (GAP, 2012) has been used for the purpose. Further, all the tables of size up to 3 have been verified manually for our Cheban AG-groupoids. Enumeration of non-associative Cheban AG-groupoids of order 3 to 6 is given in Table 2 below:

AG-groupoid/Order	3	4	5	6
Total	8	269	31467	40097003
Left Cheban AG-groupoid	2	50	2983	1356457
Right Cheban AG-groupoid	6	204	24482	35538962
Cheban AG-groupoid	2	49	2913	1334621

Table 2. Enumeration of non-associative Cheban AG-groupoids up to order 6.

5. IDEALS IN CHEBAN AG-GROUPOID

Here, we have characterized Cheban AG-groupoid S by the properties of their ideals. A subgroupoid K of S is a right (resp. left) ideal of S if

$$KS \subseteq K(resp. \quad SK \subseteq K)$$

Theorem 7. *For any left ideal L of a Cheban AG-groupoid S and $\forall x \in S$, the following holds:*

(i) xL^2 is right ideal of S.
(ii) L^2x is left ideal of S.

Proof. Let S be Cheban AG-groupoid and L be a left ideal of S, then $m \in L, s \in S$, then

(i) By Theorem (1) the property of Cheban AG-groupoid and by the left invertive law, we have

$$\begin{aligned}
(x\,(LL)\,S) = \underset{s\in S, m\in L}{U}(x\cdot mm)\,s &= \underset{s\in S, m\in L}{U} xs\cdot mm\\
\underset{s\in S, m\in L}{U} s\,(mx\cdot m) &= \underset{s\in S, m\in L}{U}(sm\cdot x)\,m\\
\underset{s\in S, m\in L}{U}(xm\cdot s)\,m &= \underset{s\in S, m\in L}{U}(ms\cdot xm)\\
\underset{s\in S, m\in L}{U}(mx\cdot sm) &= \underset{s\in S, m\in L}{U} x\,(sm\cdot m)\\
\underset{s\in S, m\in L}{U} x\,(sm\cdot m) &\subseteq \underset{s\in S, m\in L}{U} x\,(mm) \subseteq xL^2
\end{aligned}$$

Hence xL^2 is right ideal of S.

(ii) Again by the properties of Cheban AG-groupoid, left ideal and left invertive law, we have

$$\begin{aligned}
S\,((LL)\,x) = \underset{s\in S, m\in L}{U} s\,(mm\cdot x) &= \underset{s\in S, m\in L}{U} s\,(xm\cdot m)\\
\underset{s\in S, m\in L}{U} ms\cdot xm &= \underset{s\in S, m\in L}{U} mx\cdot sm\\
\underset{s\in S, m\in L}{U}(m\cdot ms)\,x &= \underset{s\in S, m\in L}{U}(x\cdot ms)\,m\\
\underset{s\in S, m\in L}{U} xm\cdot sm &= \underset{s\in S, m\in L}{U} xm\cdot m\\
\underset{s\in S, m\in L}{U} mm\cdot x &\subseteq L^2x
\end{aligned}$$

So L^2x is left ideal for S.

□

Theorem 8. *For a Cheban AG-groupoid S, the set $u\cdot Su$ is right ideal of S if $u\in S$*

Proof. Let u be an element of a Cheban AG-groupoid, then by the definition of Cheban AG-groupoid, medial and left invertive laws, we have

$$\begin{aligned}
(u\cdot Su)\,S = \underset{x,y\in S}{U}(u\cdot xu)\,y &= \underset{x,y\in S}{U}(uy\cdot ux)\\
\underset{x,y\in S}{U}(uu\cdot yx) &= \underset{x,y\in S}{U} u\,(yu\cdot x)\\
\underset{x,y\in S}{U} u\,(xu\cdot y) &= \underset{x,y\in S}{U}(uu\cdot xy)\\
\underset{x,y\in S}{U}(ux\cdot uy) &= \underset{x,y\in S}{U}(u\cdot yu)\,x\\
\underset{x,y\in S}{U}(x\cdot yu)\,u &= \underset{x,y\in S}{U}(xu\cdot uy)\\
\underset{x,y\in S}{U} u\,(ux\cdot y) &= \underset{x,y\in S}{U} u\,(yx\cdot u)\\
\Rightarrow (u\cdot Su)\,S &\subseteq u\cdot Su
\end{aligned}$$

Thus $(u\cdot Su)S\subseteq u\cdot Su$ is right ideal of S. □

Theorem 9. *For any element u of a Cheban AG-groupoid S, the set $uS\cdot u$ is ideal of S.*

Proof. For any $u \in S$, by Theorem (1), properties of Cheban AG-groupoid, the medial and the left invertive laws, we have

$$\begin{aligned}
(uS \cdot u)\, S = \underset{s,s' \in S}{U} (us \cdot u)\, s' &= \underset{s,s' \in S}{U} u\,(su \cdot s') \\
\underset{s,s' \in S}{U} (uu \cdot ss') &= \underset{s,s' \in S}{U} (u \cdot ss')\, u \\
(uS \cdot u)\, S &\subseteq uS \cdot u
\end{aligned}$$

So $(uS \cdot u)$ is right ideal of S. Again by the properties of Cheban AG-groupoid and medial law, we have

$$\begin{aligned}
S\,(uS \cdot u) = \underset{s,s' \in S}{U} s\,(us' \cdot u) &= \underset{s,s' \in S}{U} (ss' \cdot uu) \\
\underset{s,s' \in S}{U} (s'u \cdot su) &= \underset{s,s' \in S}{U} u\,(ss' \cdot u) \\
\underset{s,s' \in S}{U} u\,(us' \cdot s) &= \underset{s,s' \in S}{U} (s'u \cdot us) \\
\underset{s,s' \in S}{U} (s' \cdot su)\, u &= \underset{s,s' \in S}{U} (u \cdot su)\, s' \\
\underset{s,s' \in S}{U} (us' \cdot us) &= \underset{s,s' \in S}{U} (uu \cdot s's) \\
&= \underset{s,s' \in S}{U} (u \cdot ss')\, u \\
S\,(uS \cdot u) &\subseteq uS \cdot u
\end{aligned}$$

So $(uS \cdot u)$ is left ideal of S. Equivalently, $uS \cdot u$ is an ideal. □

Theorem 10. *For any right ideal R of a Cheban AG-groupoid S and $u \in S$, the following are true.*

(i) $u \cdot Ru$ is a right ideal of S.
(ii) $uR \cdot u$ is a left ideal of S.

Proof. For any right ideal R of a Cheban AG-groupoid S and $u, s \in S, r \in R$, by the properties of a Cheban AG-groupoid, the medial and the left invertive laws, we have

(i)

$$\begin{aligned}
(u\,(Ru))\, S = \underset{r \in R, s \in S}{U} (u\,(ru))\, s &= \underset{r \in R, s \in S}{U} (us)\,(ur) \\
\underset{r \in R, s \in S}{U} s\,((uu)\, r) &= \underset{r \in R, s \in S}{U} s\,((ru)\, u) \\
\underset{r \in R, s \in S}{U} ((us)\,(ru)) &= \underset{r \in R, s \in S}{U} (u\,(ur))\, s \\
\underset{r \in R, s \in S}{U} ((su)\,(ru)) &= \underset{r \in R, s \in S}{U} u\,((rs)\, u) \\
(RS \cdot u)\, S &\subseteq u \cdot Ru.
\end{aligned}$$

Thus $u \cdot Ru$ is a right ideal of S.

(ii) Again by the properties of Cheban AG-groupoid and the medial law, we have

$$\begin{aligned}
S(uR\cdot u) &= \underset{r\in R, s\in S}{U} s(ur\cdot u)\\
\underset{r\in R, s\in S}{U}(su\cdot r)u &= \underset{k\in S, l\in L}{U}(ur\cdot su)\\
\underset{r\in R, s\in S}{U} r(su\cdot u) &= \underset{r\in R, s\in S}{U}(rs\cdot u)u\\
\underset{r\in R, s\in S}{U}(us\cdot r)u &= \underset{r\in R, s\in S}{U}(ur\cdot us)\\
\underset{r\in R, s\in S}{U}(uu\cdot rs) &= \underset{r\in R, s\in S}{U} u(ru\cdot s)\\
\underset{r\in R, s\in S}{U} u(su\cdot r) &= \underset{r\in R, s\in S}{U}(uu\cdot sr)\\
\underset{r\in R, s\in S}{U}(u\cdot rs)u &\subseteq (uR)u
\end{aligned}$$

Thus $uR\cdot u$ is a left ideal of S.

□

Theorem 11. *For a Cheban AG-groupoid S and $u,v\in S, uv\cdot S\ U\ vu\cdot S$ is ideal of S.*

Proof. Let $u,v,x,y\in S$, then using the properties of a Cheban AG-groupoid, the medial and the left invertive laws, we have

$$\begin{aligned}
S(uv\cdot S\ U\ vu\cdot S) &= S(uv\cdot S)\ U\ S(vu\cdot S)\\
\underset{x,y\in S}{U}[x(uv\cdot y)\ U\ x(uv\cdot y)] &= \underset{x,y\in S}{U}[(vx\cdot uy)\ U\ (ux\cdot vy)]\\
\underset{x,y\in S}{U}[(vu\cdot xy)\ U\ (uv\cdot xy)] &= \underset{x,y\in S}{U}[(v\cdot yx)u\ U\ (u\cdot yx)v]\\
\underset{x,y\in S}{U}[(u\cdot yx)v\ U\ (v\cdot yx)u] &= \underset{x,y\in S}{U}[(uv\cdot xy)\ U\ (vu\cdot xy)]\\
S(uv\cdot S\ U\ vu\cdot S) &\subseteq (uv\cdot S\ U\ vu\cdot S)
\end{aligned}$$

It means $S(uv\cdot S\ U\ vu\cdot S)\subseteq(uv\cdot S\ U\ vu\cdot S)$. Thus $(uv\cdot S\ U\ vu\cdot S)$ is a right ideal of S. Now, by the property of Cheban AG-groupoid and the medial law and Theorem (1) above, we have

$$\begin{aligned}
(uv\cdot S\ U\ vu\cdot S)S &= (uv\cdot S)S\ U\ (vu\cdot S)S\\
\underset{x,y\in S}{U}[(uv\cdot y)x\ U\ (uv\cdot y)x] &= \underset{x,y\in S}{U}[(xy\cdot uv)\ U\ (xy\cdot vu)]\\
\underset{x,y\in S}{U}[y(ux\cdot v)\ U\ y(vx\cdot u)] &= \underset{x,y\in S}{U}[(yu\cdot x)v\ U\ (yv\cdot x)u]\\
\underset{x,y\in S}{U}[(vx\cdot yu)\ U\ (ux\cdot yv)] &= \underset{x,y\in S}{U}[x(yv\cdot y)\ U\ x(yu\cdot v)]\\
\underset{x,y\in S}{U}[(xy\cdot v)u\ U\ (xy\cdot u)v] &= \underset{x,y\in S}{U}[(uv\cdot xy)\ U\ (vu\cdot xy)]\\
&\subseteq (uv\cdot S\ U\ vu\cdot S)
\end{aligned}$$

Thus, $(uv\cdot S\ U\ vu\cdot S)S\subseteq(uv\cdot S\ U\ vu\cdot S)$. So $(uv\cdot S\ U\ vu\cdot S)$ is a left ideal of S. Hence $(uv\cdot S\ U\ vu\cdot S)$ is an ideal of S. □

Theorem 12. *Let L be left ideal of a Cheban AG-groupoid S. Then for any $u,v\in S,(uL)v$ is an ideal of S.*

Proof. Let L be a left ideal of S, then for $u, v \in S$, $l \in L$, by the property of Cheban AG-groupoid,the medial and the left invertive laws and definition of the left ideal, we have,

$$\begin{aligned}
(uL \cdot v)\, S &= \underset{k\in S, l\in L}{U} (ul \cdot v)\, k \\
\underset{k\in S, l\in L}{U} (kv \cdot ul) &= \underset{k\in S, l\in L}{U} (k \cdot lu)\, v \\
\underset{k\in S, l\in L}{U} (v \cdot lu)\, k &= \underset{k\in S, l\in L}{U} (vk \cdot ul) \\
\underset{k\in S, l\in L}{U} (vu \cdot kl) &= \underset{k\in S, l\in L}{U} u\, (kv \cdot l) \\
\underset{k\in S, l\in L}{U} u\, (lv \cdot k) &= \underset{k\in S, l\in L}{U} (vu \cdot lk) \\
\underset{k\in S, l\in L}{U} (v \cdot kl)\, u &= \underset{k\in S, l\in L}{U} (u \cdot kl)\, v \\
&\subseteq (uL)\, v
\end{aligned}$$

That is, $(uL \cdot v)\, S \subseteq (uL)\, v$. Thus $(uL)\, v$ is a right ideal of S. Now, by the property of a Cheban AG-groupoid, medial law and by Theorem (1), we have

$$\begin{aligned}
S\, (uL \cdot v) &= \underset{k\in S, l\in L}{U} k\, (ul \cdot v) \\
\underset{k\in S, l\in L}{U} (lk \cdot uv) &= \underset{k\in S, l\in L}{U} (lu \cdot kv) \\
\underset{k\in S, l\in L}{U} (l \cdot vk)\, u &= \underset{k\in S, l\in L}{U} (u \cdot vk)\, l \\
\underset{k\in S, l\in L}{U} (ul \cdot kv) &= \underset{k\in S, l\in L}{U} (uk \cdot lv) \\
\underset{k\in S, l\in L}{U} (ul \cdot kv) &= \underset{k\in S, l\in L}{U} l\, (ku \cdot v) \\
\underset{k\in S, l\in L}{U} l\, (vu \cdot k) &= \underset{k\in S, l\in L}{U} (ul \cdot vk) \\
\underset{k\in S, l\in L}{U} (uv \cdot lk) &= \underset{k\in S, l\in L}{U} (u \cdot kl)\, v \\
&\subseteq (uL)\, v
\end{aligned}$$

Thus $S(uL \cdot v) \subseteq uL \cdot v$. Hence $uL \cdot v$ is left ideal. Equivalently, $uL \cdot v$ is an ideal of S. $\square$

6. CONCLUSION

A new class of AG-groupoid is investigated which is called Cheban AG-groupoid. Various examples and counterexamples are constructed using the modern computational methods of Prover-9 and Mace-4 to justify the results. The Cheban AG-groupoids is enumerated up to order 6. Moreover, various relations of these types are established with other algebraic structures and with other subclasses of AG-groupoids and ideals in these AG-groupoids are discussed and investigated.

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