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Magnetohydrodynamic Oscillating and Rotating Flows of Maxwell Electrically Conducting Fluids in a Porous Plane

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Abstract. Analytically, velocity field solutions for the magnetohydrodynamic (MHD) oscillating and rotating flows of Maxwell fluids in a porous plane have been set up with the help of Laplace transform technique. A fine uniform magnetic field of strength B_0 has been employed in normal direction to the angular velocity $\mathbf{\Omega}$ of fluid flow. Expressions for the Dimensionless velocity field have been given for electrically conducing, oscillating and rotating flows in a porous plane. A couple of analytical solutions have been acquired separately for two types of oscillations i.e for sine and cosine oscillations. In addition these solutions fulfil all proposed initial and boundary conditions. Graphical illustrations are also considered to find out the steady-state time for oscillating and rotating flows as well as influence of magnetic field parameter on velocity of Maxwell fluid has observed.

AMS (MOS) Subject Classification Codes: 76A05
Key Words: Maxwell fluid; Magnetohydrodynamic; Oscillatory flow; Velocity field; Porous plane.

1. Introduction

The investigation of Maxwell fluids in porous plane, wavering and rotating systems undergoes exceptional difficulties to mathematicians, numerical investigators, and civil designers [37],[38]. A portion about these reviews are outstanding and connected in paper, nourishment stuff, individual care item, material covering coatings, and suspension arrangements enterprises [1]-[21]. The non-Newtonian class of fluids have basically characterized with respect to differential, rate and integral type fluids [34, 35, 44, 45]. Among them, liquids of rate type have gotten extraordinary consideration [40]. In the present investigation our concern is Maxwell fluids, which is the subdivision of rate-type fluids [22].

It was used to concentrate different issues because of its straightforward structure. In addition, it is reasonable to acquire analytical solution from Maxwell fluid [6]. Now-a-days in technology especially for biotechnology and drug delivery applications magnetohydrodynamic pump get more attention [5], [7], [15] ,[23], [24], [30] [41], [43]. Problem become more difficult to solve, when we introduce magnet parameter i.e when we consider magnetohydrodynamic MHD with oscillatory, rotating and porous medium [2], [25]. In spite of this fact, different scientists need aid even now settling on their fascinating commitments in the field [31],[32],[39]. Salah *et al* investigate MHD and rotating flow of Maxwell fluid in a porous medium of an accelerated plate [36]. Khan *et al* calculate closed form solution of Stokes second problem for magnetohydrodynamics flow of a Burgers' fluid over a flat plate [26]. Liu *et al* study the magnetohydrodynamic flow of a fractional generalized Oldroyd-B fluid and calculate the exact solution in terms of Fox H-function [28]. Such investigations bring exceptional pertinence on Meteorology, Geophysics and Astrophysics [29].
According to best of authors information, till now no review has been accounted to investigate the unsteady magnetohydrodynamic flow of a rotating Maxwell fluid flowing in a porous medium of oscillating plane. Therefor, it is recommended to present such an endeavor. The goal of our present work is to build up analytical solutions for the velocity field contrasting with cosine and sine motions for a Maxwell fluid.

2. Statement and Formulation of the problem

Suppose that an incompressible oscillating and rotating Maxwell fluid bounded by lower plate, which is located at position $z = 0$. Under consideration fluid is in the influence of fine electrical conduction and is the upper part of the porous plate $z > 0$. The porous plate rotates around the z-axis and accepted as its axis. At first, the fluid in addition plate is at rest condition then after time $t = 0$, fluid with plate begin to oscillate in x-axis and system began to rotate with angular velocity $\mathbf{\Omega}$ in z-direction. A fine uniform transverse magnetic field of quality $\mathbf{B_o}$ is connected in z-direction. It has been considered that the outside electric field because of phenomena of polarization of charges and the induced magnetic field are irrelevant. Alluded to a pivoting frame of interest, the flow governing equations are being taken into account:

$$\operatorname{div}\mathbf{U} = 0. \tag{2. 1}$$

$$\rho\left[\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}\cdot\nabla)\,\mathbf{U} + \mathbf{2\Omega}\times\mathbf{U} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{d})\right] = -\nabla\mathbf{p} + \operatorname{div}\mathbf{S} - \sigma\mathbf{B_0^2}\mathbf{U} + \mathbf{R}, \tag{2. 2}$$

in which ρ represents density of the fluid, $\mathbf{d}$ is a radial vector in the vicinity of fluid rotation with $d^2 = x^2 + y^2$, $\mathbf{p}$ is the hydrostatic pressure and $\mathbf{R}$ is the Darcy's resistance. The extra stress tensor $\mathbf{S}$ for a Maxwell fluid satisfies

$$\mathbf{S} + \lambda\left[\frac{\partial \mathbf{S}}{\partial t} + (\mathbf{U}\cdot\nabla)\,\mathbf{S} - \mathbf{LS} - \mathbf{SL}^{\mathrm{t}}\right] = \mu\mathbf{A}_1\ , \tag{2. 3}$$

where $\mathbf{L} = \nabla\mathbf{U}$, μ indicate dynamic viscosity, λ indicate relaxation time and $\mathbf{A}_1$ represents first Rivlin-Ericken tensor given as $\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^{\mathrm{t}}$.

Taking into account the Darcy's resistance for an Oldroyd-B fluid which is true for following expression [27]

$$\left(1+\lambda\frac{\partial}{\partial \mathrm{t}}\right)\mathbf{R}= -\frac{\mu\phi}{\kappa}\left(1+\lambda_{\mathrm{r}}\frac{\partial}{\partial \mathrm{t}}\right)\mathbf{U}, \tag{2. 4}$$

Here ϕ is is the porosity, λ_{r} is retardation time and κ is the permeability of the porous medium [47]. Since we are concern with Maxwell fluid, therefore $\lambda_{\mathrm{r}} = 0$

$$\left(1+\lambda\frac{\partial}{\partial \mathrm{t}}\right)\mathbf{R}= -\frac{\mu\phi}{\kappa}\mathbf{U}, \tag{2. 5}$$

and velocity field is $\mathbf{U} = (\mathrm{u}\,(\mathrm{z},\mathrm{t})\,, \mathrm{v}\,(\mathrm{z},\mathrm{t})\,, \mathrm{w}\,(\mathrm{z},\mathrm{t}))$. From Eq.(2.1) yields $w = 0$. Therefore from Eq.(2.2), (2.3) and (2.5)

$$\rho\left(\frac{\partial u}{\partial t}-2\Omega v\right) = \frac{\partial S_{xz}}{\partial z} - \sigma {B_0}^2 u + R_x, \tag{2. 6}$$

$$\rho\left(\frac{\partial v}{\partial t}+2\Omega u\right) = \frac{\partial S_{yz}}{\partial z} - \sigma {B_0}^2 v + R_y, \tag{2. 7}$$

where

$$\left(1+\lambda\frac{\partial}{\partial t}\right)T_{xz} = \mu\frac{\partial u}{\partial z}, \tag{2. 8}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)T_{yz} = \mu\frac{\partial v}{\partial z}. \tag{2. 9}$$

R_x and R_y are Darcy's resistance in x and y-directions. From Eq.(2.6)-(2.9) we have

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial t}\right)+\left(-2\Omega+\frac{\sigma {B_0}^2}{\rho}\right)\left(1+\lambda\frac{\partial}{\partial t}\right)u+\frac{\upsilon\phi}{\kappa}u=\frac{\upsilon\partial^2 u}{\partial z^2}, \tag{2. 10}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial t}\right)+\left(2\Omega+\frac{\sigma {B_0}^2}{\rho}\right)\left(1+\lambda\frac{\partial}{\partial t}\right)v+\frac{\upsilon\phi}{\kappa}v=\frac{\upsilon\partial^2 v}{\partial z^2}. \tag{2. 11}$$

Joining Eq, (2.10) and (2.11)

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial G}{\partial t}+\left(2i\Omega+\frac{\sigma {B_0}^2}{\rho}\right)\left(1+\lambda\frac{\partial}{\partial t}\right)G+\frac{\upsilon\phi}{k}G=\upsilon\frac{\partial^2 G}{\partial z^2}, \tag{2. 12}$$

where $G(z,t) = u(z,t) + iv(z,t)$ is the complex velocity function for under consideration fluid, where as u represent velocity part along x-direction and v is velocity part along y-direction. Where as σ represents electrical conductivity of under considration fluid, ϕ lies between $(0, 1)$ is the porosity and $k > 0$ is the permeability of the porous medium [46].
For the present problem, suitable imposed initial and boundary conditions are

$$G(z,0) = 0, \qquad z > 0, \tag{2. 13}$$

$$\frac{\partial}{\partial t}G(z,0) = 0, \tag{2. 14}$$

$$G(0,t) = V_o\cos(\omega t) \text{ or } V_o\sin(\omega t), \qquad \text{for } t > 0,$$

$$G(z,t) \to 0 \qquad \text{as } z \to \infty, \tag{2. 15}$$

here V_o represents the amplitude of oscillations and ω is the frequency of the oscillations. To make the problem more convenient we introduce following dimensionless variables [42]

$$z^\star = \frac{z\sqrt{\omega}}{\sqrt{\nu}};\ \ t^\star = \omega t;\ \ G^\star = \frac{G}{V_0}\quad and\, \Omega^\star = \frac{\Omega}{\omega} \tag{2. 16}$$

into Eq. (2.12) and ignoring the star notation in postscript, we reached to

$$b\frac{\partial^2}{\partial t^2}G\,(z,t) + (1+cb)\,\frac{\partial}{\partial t}G(z,t) + (c+a)\,G(z,t) = \frac{\partial^2}{\partial z^2}G(z,t), \tag{2. 17}$$

$$G(z,0) = 0; \qquad z > 0, \tag{2. 18}$$

$$G(0,t) = \cos t\ \text{ or }\ \ \sin t\ \text{ for }\ t > 0\,; G(z,t) \to 0\ \text{ as }\ z \to \infty, \tag{2. 19}$$

where

$$a = \frac{\upsilon\phi}{k\omega};\ b = \lambda\,\omega;\ c = 2i\Omega + M\ \ and\ \ M = \frac{\sigma {B_0}^2}{\omega\rho}. \tag{2. 20}$$

3. Calculation of the Velocity Field

To figure out the analytical solution of the above problem (2.17)-(2.19), we might consider the Laplace transforms technique [33]. All calculations will be presented for the cosine oscillations. On the other hand we shall give final velocity expression for sine oscillations with out calculations on this paper. Applying the Laplace transform to Eqs. (2.17)-(2.19), we obtain

$$bq^2\overline{G}(z,q) + (1+cb)q\overline{G}(z,q) + (c+a)\overline{G}(z,q) = \frac{d^2}{dz^2}\overline{G}(z,q), \tag{3. 21}$$

$$\overline{G}_c(0,q) = \frac{q}{q^2+1};\ \ \overline{G}_c(z,q) \to 0\ \ as\ \ z \to \infty. \tag{3. 22}$$

The solution of Equation (3.21) fulfilling the given conditions (3.22) is

$$\overline{G_c}(z,q) = \frac{q}{1+q^2}\exp\{-z\ \sqrt{bq^2 + (1+cb)q + (c+a)}\}, \tag{3. 23}$$

and the velocity field $G_c(z,t) = \mathcal{L}^{-1}\{\overline{G}(z,q)\}$ may be written as in new form by using convolution theorem

$$G_c(z,t) = (G_1 * G_2)(t) = \int_0^t G_1(t-s)G_2(z,s)ds, \tag{3. 24}$$

where $G_1(t) = \delta(t) - sin(t)$ and $G_2(z,t)$ are the inverse Laplace transform of

$\overline{G}_1(q) = \frac{q}{1+q^2}$ and $\overline{G}_2(z,q) = \frac{1}{q}\exp\{-z\sqrt{bq^2+(1+cb)q+(c+a)}\}$.

Here $\delta(.)$ is Dirac Delta function. So as to figure out the inverse Laplace transform of the $\overline{G}_2(z,q)$ in above equation by using relation $(A1)$ from Appendix and obtain

$$G_2(z,t) = \Phi(t - z\sqrt{b})\{e^{\frac{-z\sqrt{b}(l+m)}{2}} + \frac{z\sqrt{b}(l^2-m^2)}{4}$$

$$\times \int_0^t e^{\frac{-(l+m)\tau}{2}}.(\tau^2 - z^2 b)^{\frac{-1}{2}} I_1(\frac{(l-m)\sqrt{\tau^2 - z^2 b}}{2})d\tau\}, \tag{3. 25}$$

where

$$l = \frac{(\frac{1}{b}+c) - \sqrt{(\frac{1}{b}+c)^2 - \frac{4}{b}(c+a)}}{2},$$

$$m = \frac{(\frac{1}{b}+c) + \sqrt{(\frac{1}{b}+c)^2 - \frac{4}{b}(c+a)}}{2},$$

$I_1(\cdot)$ and $\Phi(\cdot)$ are the modified Bessel function of the first kind of order one and the unit step fuction respectively. Hence Eqs.(A) and (3.4) implies

$$G_c(z,q) = \int_0^t (\delta(t-s) - \sin(t-s))\, \Phi(t - z\sqrt{b})\{e^{\frac{-z\sqrt{b}(l+m)}{2}}$$

$$+\frac{z\sqrt{b}(l^2-m^2)}{4}\int_0^t e^{\frac{-(l+m)s}{2}}(s^2 - z^2 b)^{\frac{-1}{2}} I_1(\frac{(l-m)\sqrt{s^2 - z^2 b}}{2})ds\}ds. \tag{3. 26}$$

Since unit step function $\Phi(\cdot)$ is independent of variable 's' so it can be taken outside the integral, hence

$$G_c(z,t) = \Phi\left(t - z\sqrt{b}\right) e^{\frac{-z\sqrt{b}(l+m)}{2}} \int_0^t \{\delta(t-s) - sin(t-s)\}ds + \frac{z\sqrt{b}(l^2-m^2)}{4}$$

$$\times \int_0^t \{(\delta(t-s) - sin(t-s))\int_0^t \frac{e^{\frac{-(l+m)s}{2}}}{\sqrt{s^2 - z^2 b}} I_1(\frac{(l-m)}{2}\sqrt{s^2 - z^2 b})ds\}ds. \tag{3. 27}$$

Further calculations leads to

$$G_c(z,t) = \Phi(t - z\sqrt{b})\,[e^{\frac{-z\sqrt{b}(l+m)}{2}}\{-1 + \cos(t) + \int_0^t \delta(t-s)ds\} + \frac{z\sqrt{b}(l^2-m^2)}{4}$$

$$\times \int_0^t \{(\delta(t-s) - \sin(t-s))\int_0^t e^{\frac{-(l+m)s}{2}}\sqrt{s^2 - z^2 b} I_1$$

$$\times \left(\frac{(l-m)\sqrt{s^2 - z^2 b}}{2}\right) ds\}]ds. \tag{3. 28}$$

Similar calculations lead to

$$G_s(z,t) = \Phi(t - z\sqrt{b})[e^{\frac{-z\sqrt{b}(l+m)}{2}} \sin(t) + \frac{z\sqrt{b}(l^2 - m^2)}{4}$$
$$\times \int_0^t \cos(t-s) \left(\int_0^t e^{\frac{-(l+m)s}{2}} (s^2 - z^2 b)^{\frac{-1}{2}} I_1 \left(\frac{(l-m)\sqrt{s^2 - z^2 b}}{2} \right) ds \right) ds]. \quad (3.\ 29)$$

corresponding to sine oscillations of the porous plane.

4. Graphical results and conclusions

In this work, analytical technique used to acquire the velocity expressions for magnetohydrodynamic oscillating and rotating flow of a Maxwell fluid through a porous medium plate. Integral transform technique (laplace) has been taking into account for this purpose. A magnetic field of uniform strength has been acted parallel to z-direction which is taken as axis of rotation and analytical velocity expressions have been established for oscillating as well as rotating flows. Velocity expressions obtained satisfy all initial and boundary conditions as well as the governing equation.

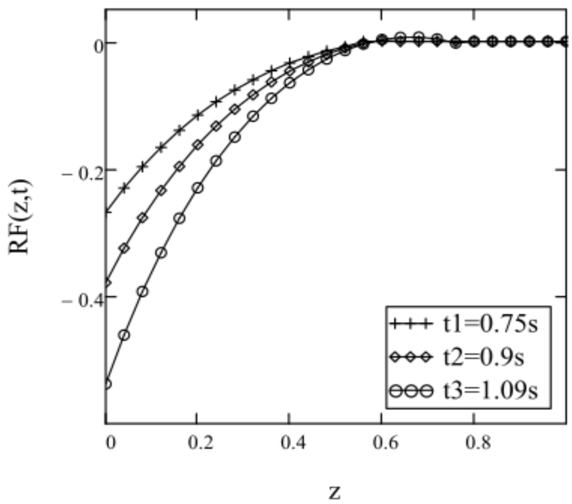

FIGURE 1. Time needed to approach the steady-state for a cosine vibrating plate, for $G_c(z,t)$ set up by Eq. (3.28) for $\sigma = 26,\ \lambda = 2,\ \rho = 610,\ \omega = 1,\ B_0 = 10, \upsilon = 3,\ \phi = 0.5,\ \kappa = 0.2\ , \Omega = 0.5$ and varying values of t.

In consequence, the main results for real parts of velocity having cosine and sine oscillations are being discussed,

• The steady-state time for cosine is t=1.02s, as it results from Fig.1, i.e the variation of time is not important after t=1.02 s.

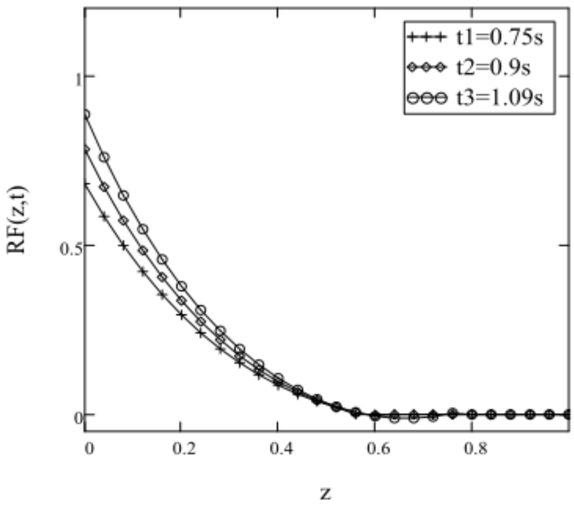

FIGURE 2. Time needed to approach the steady-state for a sine vibrating plate, for $G_s(z,t)$ set up by Eq. (3.29) for $\sigma = 26,\ \lambda = 2,\ \rho = 610,\ \omega = 1,\ B_0 = 10, \upsilon = 3,\ \phi = 0.5,\ \kappa = 0.2\ , \Omega = 0.5$ and varying values of t.

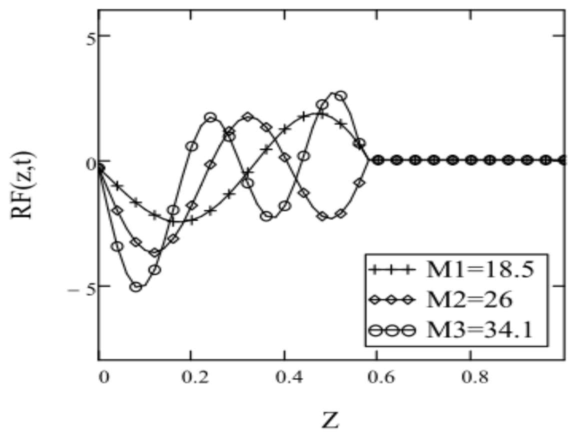

FIGURE 3. The variations of velocity profile $G_c(z,t)$ set up by Eq. (3.28) for different values of M when $\sigma = 26,\ \lambda = 2,\ \rho = 610,\ \omega = 1,\ B_0 = 10, \upsilon = 3,\ \phi = 0.5,\ \kappa = 0.2$ and $\Omega = 2$.

• The steady-state time for sine is t=1.02s, as it results from Fig.2, i.e the variation of time is not important after $t = 1.02$ s.

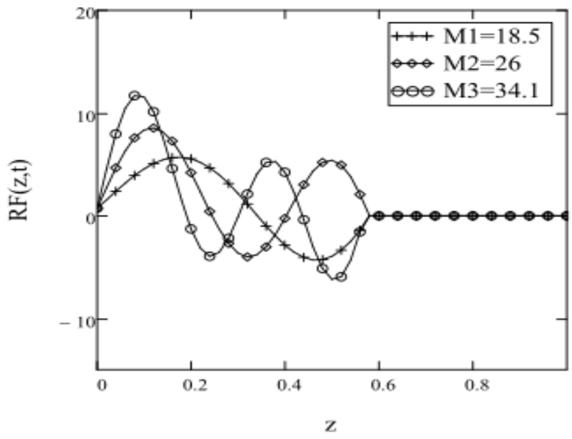

FIGURE 4. The variations of velocity profile $G_s(z,t)$ set up by Eq. (3.29) for different values of M when $\sigma = 26,\ \lambda = 2,\ \rho = 610,\ \omega = 1,\ B_0 = 10, \upsilon = 3,\ \phi = 0.5,\ \kappa = 0.2$ and $\Omega = 2$.

From Fig.1 and Fig.2 it is very clear that direction of velocity is opposite to each other for cosine and sine oscillations.

• Consequence of magnetic parameter M on real part cosine oscillations of velocity from Fig.3 elaborate that frequency of oscillations increases with increase in magnetic parameter M. Also as we keep increasing M velocity field shows some interesting results of damping velocity.

• Consequence of magnetic parameter M on real part sine oscillations of velocity from Fig.4 elaborate that frequency of oscillations increases with increase in magnetic parameter M. Also as we keep increasing M velocity field shows some interesting results of damping velocity.

• Figure-3 and 4 elaborate that frequency of oscillations increases with increase in magnetic parameter M.

From Fig.3 and Fig.4 it is clear that the influence of magnetohydrodynamic parameter M on the real part of velocity for cosine oscillation is in contradiction to that of the sine oscillation.

5. APPENDIX

$$\mathcal{L}^{-1}\left[\frac{1}{q}\mathrm{e}^{(-\alpha\sqrt{(q+\beta)(q+\gamma)})}\right] = \Phi(t-\alpha)\left[\begin{array}{l}\mathrm{e}^{(\frac{-\alpha(\beta+\gamma)}{2})}+\dfrac{\alpha(\beta^2-\gamma^2)}{4}\times \\ \displaystyle\int_0^t \mathrm{e}^{(\frac{-(\beta+\gamma)\tau}{2})}(\tau^2-\alpha^2)^{\frac{-1}{2}} I_1(\frac{(\beta-\gamma)\sqrt{(\tau^2-\alpha^2)}}{2})d\tau\end{array}\right]. \quad (A1)$$

6. DECLARATIONS

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