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Some Families of Convex Polytopes Labeled by 3-Total Edge Product Cordial Labeling

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Abstract. For a graph $G = (V_G, E_G)$, consider a mapping $h : E_G \rightarrow \{0, 1, 2, \ldots, k-1\}$, $2 \leq k \leq |E_G|$ which induces a mapping $h^* : V_G \rightarrow \{0, 1, 2, \ldots, k-1\}$ such that $h^*(v) = \prod_{i=1}^{n} h(e_i)(\bmod k)$, where e_i is an edge incident to v. Then h is called k-total edge product cordial (k-TEPC) labeling of G if $|s(i) - s(j)| \leq 1$ for all $i, j \in \{1, 2, \ldots, k-1\}$. Here $s(i)$ is the sum of all vertices and edges labeled by i. In this paper, we study k-TEPC labeling for some families of convex polytopes for $k = 3$.

AMS (MOS) Subject Classification Codes: 05C07
Key Words: 3-TEPC labeling, The graphs of convex polytopes.

1. INTRODUCTION AND PRELIMINARIES

Let G be an undirected, simple and finite graph with vertex-set V_G and edge-set E_G. Order of a graph G is the number of vertices and size of a graph G is the number of edges. Graph labeling is a map that assigns integers to V_G or E_G or both subject to particular condition(s). In graph G a mapping from V_G (or E_G) to positive integers is called vertex (or edge) labeling and a mapping from $V_G \cup E_G$ to positive integers is known as total labeling.

In order to understand cordial labeling h and its types, we need the following notations:

(1) $v_h(i)$ is the number of vertices labeled by i;
(2) $e_h(i)$ is the number of edges labeled by i;
(3) $v_h(i,j) = v_h(i) - v_h(j)$;
(4) $e_h(i,j) = e_h(i) - e_h(j)$ and
(5) the sum of all vertices and edges labeled by i is $s(i)$ i.e. $s(i) = v_h(i) + e_h(i)$.

Cahit introduced cordial labeling in [9].

Definition 1.1. *Let $h : V_G \longrightarrow \{0,1\}$ be a mapping that induces $h^* : E_G \longrightarrow \{0,1\}$ such that $h^*(uv) = |h(u) - h(v)|$ for each edge uv, then h is called cordial labeling if it satisfies $|v_h(1,0)| \leq 1$ and $|e_h(1,0)| \leq 1$.*

Product cordial labeling was introduced by Sundaram et al. in [15].

Definition 1.2. *Let $h : V_G \to \{0,1\}$ be a mapping that induces $h^* : E_G \to \{0,1\}$ such that $h^*(uv) = h(u)h(v)$ for each edge uv, then h is called product cordial labeling if it satisfies $|v_h(1,0)| \leq 1$ and $|e_h(1,0)| \leq 1$.*

In 2006, the total product cordial labeling was developed by Sundaram et al. in [16].

Definition 1.3. *Let $h : V_G \to \{0,1\}$ be a mapping that induces $h^* : E_G \to \{0,1\}$ such that $h^*(uv) = h(u)h(v)$ for each edge uv, then h is called total product cordial labeling if it satisfy $|s(0) - s(1)| \leq 1$.*

Definition 1.4. [13] *Let $h : V_G \to \{0,1,\ldots,k-1\}$, $2 \leq k \leq |E_G|$ be a mapping that induces $h^* : E_G \to \{0,1,\ldots,k-1\}$ such that $h^*(uv) = h(u)h(v)$(mod k) for each edge uv, then h is called k-total product cordial labeling if it satisfy $|s(a) - s(b)| \leq 1$ for all $a, b \in \{0,1,\ldots,k-1\}$.*

In 2012, Vaidya and Barasara introduced edge product cordial labeling (see [17]).

Definition 1.5. *Let $h : E_G \to \{0,1\}$ be a mapping that induces $h^* : V_G \to \{0,1\}$ such that $h^*(u) = h(e_1)h(e_2)\ldots h(e_n)$ for edges $e_1, e_2, \ldots, e_n$ incident to u, then h is called edge product cordial labeling if it satisfies $|v_h(0,1)| \leq 1$ and $|e_h(0,1)| \leq 1$.*

Definition 1.6. *Let $h : E_G \longrightarrow \{0,1,\ldots,k-1\}$, $2 \leq k \leq |E_G|$ be a mapping that induces $h^* : V_G \to \{0,1,\ldots,k-1\}$ such that $h^*(u) = h(e_1)h(e_2)\ldots h(e_n)$(mod k) for edges $e_1, e_2, \ldots, e_n$ incident to u, then h is called k-edge product cordial labeling if it satisfies $|v_h(a,b)| \leq 1$ and $|e_h(a,b)| \leq 1$ for $0 \leq a < b \leq k-1$.*

Definition 1.7. [18] *Let $h : E_G \to \{0,1\}$ be a mapping that induces $h^* : V_G \to \{0,1\}$ such that $h^*(u) = h(e_1)h(e_2)\ldots h(e_n)$ for edges $e_1, e_2, \ldots, e_n$ incident to u, then h is called a total edge product cordial labeling if it satisfy $|s(0) - s(1))| \leq 1$.*

In 2015, Azaizeh et al. provide the k-total edge product cordial labeling in [5].

Definition 1.8. *Let $h : E_G \longrightarrow \{0, 1, \ldots, k-1\}$, $2 \leq k \leq |E_G|$ be a mapping that induces $h^* : V_G \longrightarrow \{0, 1, \ldots, k-1\}$ such that $h^*(u) = h(e_1)h(e_2)\ldots h(e_n)$(mod k) for edges $e_1, e_2, \ldots, e_n$ incident to u, then h is called k-total edge product cordial labeling if it satisfy $|s(a) - s(b)| \leq 1$ for $a, b \in \{0, 1, \ldots, k-1\}$.*

We refer to the articles [1, 3, 12] for more recent topics in labeling.
In order to study different families of convex polytopes, first we give definitions of the following two archimedean convex polytopes introduced in [11].

Definitions 1.9. (1) *A prism graph Y_m is cartesian product graph $C_m \times P_2$, where C_m is cycle graph of order m and P_2 is path graph of order 2 (see Figure 1).*

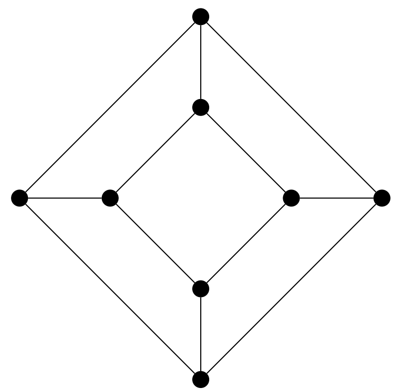

FIGURE 1. Prism graph Y_4

(2) *A m-sided anti-prism A_m is polyhedron composed of two parallel copies of some particular m-sided polygon connected by alternating band of triangle (see Figure 2).*

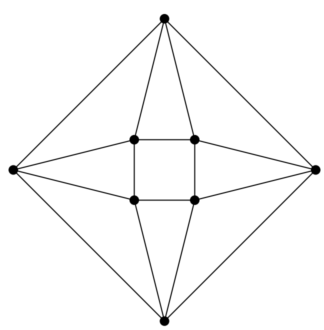

FIGURE 2. Anti-prism graph A_4

Baca introduced the following convex polytopes R_m in [6].

Definition 1.10. *For $m \geq 5$, a combination of prism graph Y_m and antiprism graph A_m is known as convex polytope graph R_m. It consists of the inner cycle vertices $\{u_i, 1 \leq i \leq m\}$, the middle cycle vertices $\{v_i, 1 \leq i \leq m\}$ and the outer cycle vertices $\{w_i, 1 \leq i \leq m\}$*

(see Figure 3). Note that $V_{R_m} = \{u_i, v_i, w_i, 1 \leq i \leq m\}$ *and* $E_{R_m} = \{u_i u_{i+1}, 1 \leq i \leq m-1\} \cup \{v_i v_{i+1}, 1 \leq i \leq m-1\} \cup \{w_i w_{i+1}, 1 \leq i \leq m-1\} \cup \{u_i v_i, 1 \leq i \leq m\} \cup \{u_{i+1} v_i, 1 \leq i \leq m-1\} \cup \{v_i w_i, 1 \leq i \leq m\} \cup \{u_m u_1\} \cup \{v_m v_1\} \cup \{w_m w_1\} \cup \{u_1 v_m\}$.

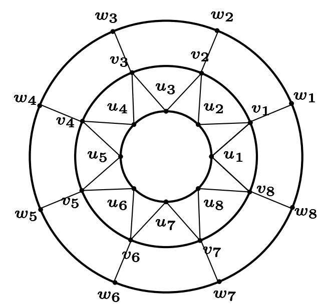

FIGURE 3. Graph of convex polytope R_8

Now we will discuss different families of convex polytopes obtained from Y_m, A_m and R_m.

Definitions 1.11. (1) *The graph of convex polytope* A_m *can be obtained from* R_m *by adding some new lines (edges). i.e.* V_{A_m}=V_{R_m} *and* $E_{A_m} = E_{R_m} \cup \{v_m w_1\} \cup \{v_i w_{i+1}, 1 \leq i \leq m-1\}$. A_m *consist of three-sided faces and a* m*-sides face (see Figure 4).*

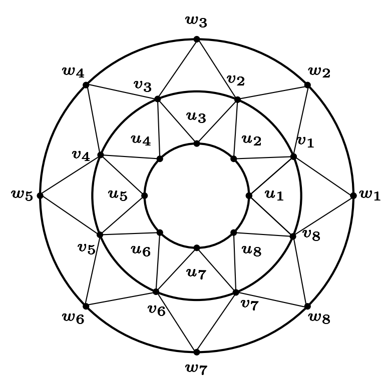

FIGURE 4. Graph of convex polytope A_8

(2) *The graph of convex polytope* S_m *is composed of two parallel copies of prism graphs connected by alternating band of triangles. It consist of three-sided faces, four-sided faces and* m*-sides face (see Figure 5). Note that* $V_G = \{u_i, v_i, w_i, z_i, 1 \leq i \leq m\}$ *and* $E_G = \{u_i u_{i+1}, 1 \leq i \leq m-1\} \cup \{v_i v_{i+1}, 1 \leq i \leq m-1\} \cup \{w_i w_{i+1}, 1 \leq i \leq m-1\} \cup \{z_i z_{i+1}, 1 \leq i \leq m-1\} \cup \{u_i v_i, 1 \leq i \leq m\} \cup \{v_i w_i, 1 \leq i \leq m\} \cup \{v_{i+1} w_i, 1 \leq i \leq m-1\} \cup \{w_i z_i, 1 \leq i \leq m\} \cup \{u_m u_1\} \cup \{v_m v_1\} \cup \{w_m w_1\} \cup \{z_m z_1\} \cup \{v_1 w_m\}$.

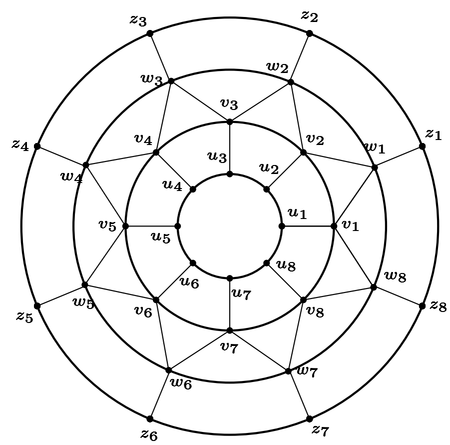

FIGURE 5. Graph of convex polytope S_8

(3) *The convex polytope Q_m can be obtained from S_m by deleting some lines (edges). i.e. V_{Q_m}=V_{S_m} and $E_{Q_m} = E_{S_m} \setminus \{w_i w_{i+1}, 1 \leq i \leq m-1\} \cup \{w_m w_1\}$. It consist of three-sided faces, four-sided faces, five-sided faces and m-sides face (see Figure 6).*

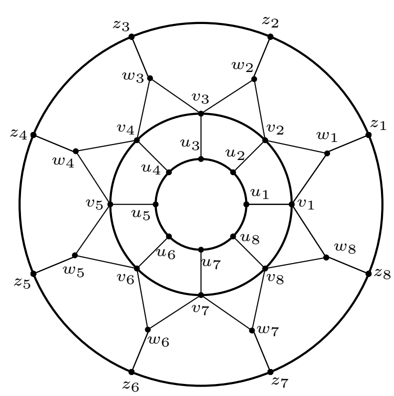

FIGURE 6. Graph of convex polytope Q_8

(4) *The graph of convex polytope T_m can be obtained from Q_m by adding some new lines (edges). i.e. V_{T_m}=V_{Q_m} and $E_{T_m} = E_{Q_m} \cup \{u_{i+1} v_i, 1 \leq i \leq m\} \cup \{u_1 v_m\}$. It consist of three-sided faces, five-sided faces and m-sides face (see Figure 7).*

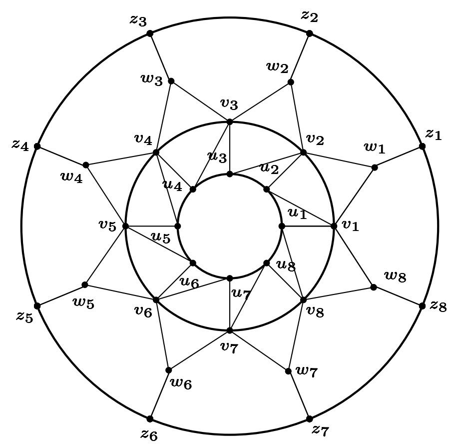

FIGURE 7. Graph of convex polytope T_8

Convex polytopes are geometrical objects. In recent years, different families of convex polytopes were studied in the context of graph labeling and metric dimension. For more details, we refer to the articles [4, 6, 7, 8, 10].

Azaizeh et al. discussed 3-TEPC labeling of path, circle and star graphs in [5]. Madiha et al. in [14] have discussed 3-TEPC labeling of Dutch Windmill graph and m isomorphic copies of n-cycle graphs. Yasir et al. in [2] has discussed 3-TEPC labeling of gear, web and helm graphs. In this paper, we study 3-TEPC labeling for convex polytopes graphs A_m, S_m and T_m.

2. MAIN RESULTS

In this section, we will discuss 3-total edge product cordial (3-TEPC) labeling of convex polytopes.

Theorem 2.1. *Let G be a graph of convex polytope (double antiprism) A_m then G admits 3-TEPC labeling.*

Proof. In order to show that A_m is 3-TEPC, we consider three cases as follows:

Case 1: Let $m \equiv 0 \pmod 3$ which implies $m = 3l$, for some integer $l \geq 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_iu_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 3l-1; \end{cases} \text{ and } h(u_{3l}u_1) = 2.$$

$$h(v_iv_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 3l-1; \end{cases} \text{ and } h(v_{3l}v_1) = 2.$$

$$h(w_iw_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l-1; \\ 2, & \text{if } l \leq i \leq 3l-1; \end{cases} \text{ and } h(w_{3l}w_1) = 2.$$

$$h(v_{i+1}w_i) = \begin{cases} 0, & \text{if } 1 \le i \le l-1; \\ 1, & \text{if } l \le i \le 3l-1; \end{cases} \text{ and } h(v_1 w_{3l}) = 1.$$

$$h(u_{i+1}v_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l-1; \end{cases} \text{ and } h(u_1 v_{3l}) = 1.$$

$$h(u_i v_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l-1; \\ 1, & \text{if } i = 3l. \end{cases}, \; h(v_i w_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 1, & \text{if } l+1 \le i \le 3l. \end{cases}$$

In this case, we have $s(0) = s(1) = s(2) = 10l$. Therefore $|s(x) - s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is 3-TEPC labeling.

□

Case 2: Let $m \equiv 1 \pmod 3$ which implies $m = 3l + 1$, for some integer $l \ge 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_i u_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l; \end{cases} \text{ and } h(u_{3l+1} u_1) = 2.$$

$$h(v_i v_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l; \end{cases} \text{ and } h(v_{3l+1} v_1) = 2.$$

$$h(w_i w_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l; \end{cases} \text{ and } h(w_{3l+1} w_1) = 2.$$

$$h(v_{i+1}w_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 1, & \text{if } l+1 \le i \le 3l; \end{cases} \text{ and } h(v_1 w_{3l+1}) = 1.$$

$$h(u_{i+1}v_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l; \end{cases} \text{ and } h(u_1 v_{3l+1}) = 1.$$

$$h(u_i v_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l; \\ 1, & \text{if } i = 3l+1. \end{cases}, \; h(v_i w_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 1, & \text{if } l+1 \le i \le 3l+1. \end{cases}$$

In this case, we have $s(0) = s(1) = 10l + 3$, $s(2) = 10l + 4$. Therefore $|s(x) - s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is 3-TEPC labeling.

Case 3: Let $m \equiv 2 \pmod 3$ which implies $m = 3l + 2$, for some integer $l \ge 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_i u_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l+1; \\ 2, & \text{if } l+2 \le i \le 3l+1; \end{cases} \text{ and } h(u_{3l+2} u_1) = 2.$$

$$h(v_i v_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l+1; \end{cases} \text{ and } h(v_{3l+2} v_1) = 2.$$

$$h(w_iw_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l+1; \end{cases} \text{ and } h(w_{3l+2}w_1) = 2.$$

$$h(v_{i+1}w_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 1, & \text{if } l+1 \le i \le 3l+1; \end{cases} \text{ and } h(v_1w_{3l+2}) = 1.$$

$$h(u_{i+1}v_i) = \begin{cases} 0, & \text{if } 1 \le i \le l+1; \\ 2, & \text{if } l+2 \le i \le 3l+1; \end{cases} \text{ and } h(u_1v_{3l+2}) = 1.$$

$$h(u_iv_i) = \begin{cases} 0, & \text{if } 1 \le i \le l+1; \\ 2, & \text{if } l+2 \le i \le 3l+1; \\ 1, & \text{if } i = 3l+2. \end{cases}, h(v_iw_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 1, & \text{if } l+1 \le i \le 3l+2. \end{cases}$$

In this case, we have $s(0) = s(1) = 10l + 7$, $s(2) = 10l + 6$. Therefore $|s(x) - s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is 3-TEPC labeling.

Example 2.2. *The* 3*-TEPC labeling of* A_8 *is shown in Figure 8.*

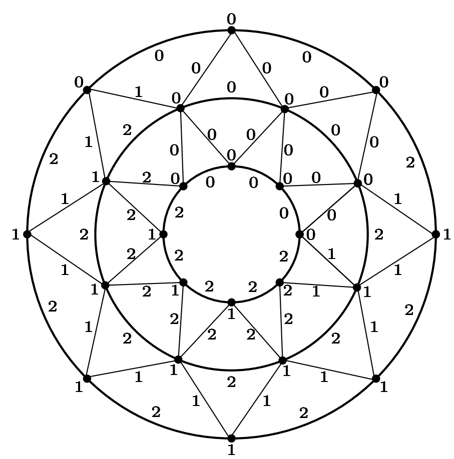

FIGURE 8. 3-TEPC labeling of A_8

Theorem 2.3. *Let* G *be a graph of convex polytope* S_m *then* G *admits* 3*-TEPC labeling.*

Proof. In order to show that S_m is 3-TEPC, we consider three cases as follows:

Case 1: Let $m \equiv 0 \pmod 3$ which implies $m = 3l$, for some integer $l \ge 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_iu_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l-1; \end{cases} \text{ and } h(u_{3l}u_1) = 2.$$

$$h(v_iv_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l-1; \end{cases} \text{ and } h(v_{3l}v_1) = 2.$$

$$h(w_iw_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l-1; \\ 2, & \text{if } l \le i \le 3l-1; \end{cases} \text{ and } h(w_{3l}w_1) = 2.$$

$$h(z_iz_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l-1; \\ 1, & \text{if } l \le i \le 3l-1; \end{cases} \text{ and } h(z_{3l}z_1) = 2.$$

$$h(v_{i+1}w_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 1, & \text{if } l+1 \le i \le 3l-1; \end{cases} \text{ and } h(v_1w_{3l}) = 1.$$

$$h(u_iv_i) = \begin{cases} 0, & \text{if } 1 \le i \le l+1; \\ 2, & \text{if } l+2 \le i \le 3l. \end{cases}, h(v_iw_i) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 1, & \text{if } l+1 \le i \le 3l. \end{cases}$$

$$h(w_iz_i) = \begin{cases} 0, & \text{if } 1 \le i \le l-1; \\ 1, & \text{if } l \le i \le 3l. \end{cases}$$

In this case, we have $s(0) = s(1) = s(2) = 12l$. Therefore $|s(x) - s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is 3-TEPC labeling.

□

Case 2: Let $m \equiv 1 \pmod 3$ which implies $m = 3l + 1$, for some integer $l \ge 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_iu_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l; \end{cases} \text{ and } h(u_{3l+1}u_1) = 2.$$

$$h(v_iv_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 3l; \end{cases} \text{ and } h(v_{3l+1}v_1) = 2.$$

$$h(w_iw_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l; \\ 2, & \text{if } l+1 \le i \le 2l; \\ 1, & \text{if } 2l+1 \le i \le 3l; \end{cases} \text{ and } h(w_{3l+1}w_1) = 1.$$

$$h(z_iz_{i+1}) = \begin{cases} 0, & \text{if } 1 \le i \le l-1; \\ 2, & \text{if } l \le i \le 2l-1; \\ 1, & \text{if } 2l \le i \le 3l; \end{cases} \text{ and } h(z_{3l+1}z_1) = 1.$$

$$h(v_{i+1}w_i) = \begin{cases} 0, & \text{if } 1 \le i \le l+1; \\ 1, & \text{if } l+2 \le i \le 3l; \end{cases} \text{ and } h(v_1w_{3l+1}) = 1.$$

$$h(u_iv_i) = \begin{cases} 0, & \text{if } 1 \le i \le l+1; \\ 2, & \text{if } l+2 \le i \le 3l+1. \end{cases}, h(v_iw_i) = \begin{cases} 0, & \text{if } 1 \le i \le l+1; \\ 1, & \text{if } l+2 \le i \le 3l+1. \end{cases}$$

$$h(w_iz_i) = \begin{cases} 0, & \text{if } 1 \le i \le l-1; \\ 1, & \text{if } l \le i \le 3l+1. \end{cases}$$

In this case, we have $s(0) = s(1) = s(2) = 12l+4$. Therefore $|s(x)-s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is 3-TEPC labeling.

Case 3: Let $m \equiv 2 \pmod 3$ which implies $m = 3l + 2$, for some integer $l \geq 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_i u_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(u_{3l+2} u_1) = 2.$$

$$h(v_i v_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(v_{3l+2} v_1) = 2.$$

$$h(w_i w_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 2l; \\ 1, & \text{if } 2l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(w_{3l+2} w_1) = 1.$$

$$h(z_i z_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 2, & \text{if } l+2 \leq i \leq 2l; \\ 1, & \text{if } 2l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(z_{3l+2} z_1) = 1.$$

$$h(v_{i+1} w_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 1, & \text{if } l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(v_1 w_{3l+2}) = 1.$$

$$h(u_i v_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 2, & \text{if } l+2 \leq i \leq 3l+2. \end{cases}, h(v_i w_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 1, & \text{if } l+2 \leq i \leq 3l+2. \end{cases}$$

$$h(w_i z_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 1, & \text{if } l+1 \leq i \leq 3l+2. \end{cases}$$

In this case, we have $s(0) = s(1) = s(2) = 12l+8$. Therefore $|s(x)-s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is 3-TEPC labeling.

Example 2.4. *The* 3*-TEPC labeling of* S_8 *is shown in Figure 9.*

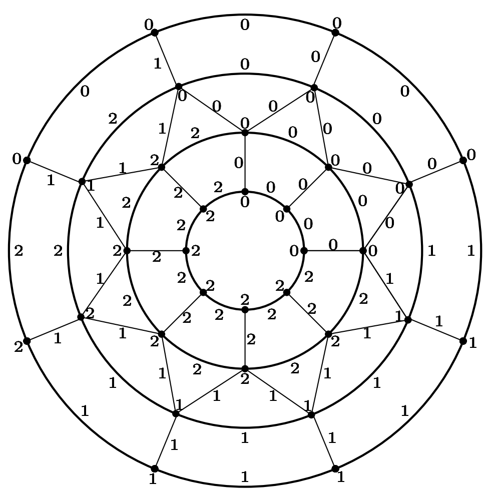

FIGURE 9. 3-TEPC labeling of S_8

Theorem 2.5. *Let G be a graph of convex polytope T_m then G admits 3-TEPC labeling.*

Proof. In order to show that T_m is 3-TEPC, we consider three cases as follows:

Case 1: Let $m \equiv 0 \pmod 3$ which implies $m = 3l$, for some integer $l \geq 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_iu_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 2, & \text{if } l+2 \leq i \leq 3l-1; \end{cases} \text{ and } h(u_{3l}u_1) = 2.$$

$$h(v_iv_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 3l-1; \end{cases} \text{ and } h(v_{3l}v_1) = 2.$$

$$h(z_iz_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l-1; \\ 2, & \text{if } l \leq i \leq l+2; \\ 1, & \text{if } l+3 \leq i \leq 3l-1; \end{cases} \text{ and } h(z_{3l}z_1) = 1.$$

$$h(u_{i+1}v_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 1, & \text{if } l+1 \leq i \leq 3l-1; \end{cases} \text{ and } h(u_1v_{3l}) = 1.$$

$$h(v_{i+1}w_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l-1; \\ 2, & \text{if } l \leq i \leq 2l; \\ 1, & \text{if } 2l+1 \leq i \leq 3l-1; \end{cases} \text{ and } h(v_1w_{3l}) = 1.$$

$$h(u_iv_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 2, & \text{if } l+2 \leq i \leq 3l. \end{cases}, \ h(w_iz_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l-1; \\ 1, & \text{if } l \leq i \leq 3l. \end{cases}$$

$$h(v_iw_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l-1; \\ 2, & \text{if } l \leq i \leq 2l; \\ 1, & \text{if } 2l+1 \leq i \leq 3l. \end{cases}$$

In this case, we have $s(0) = s(1) = s(2) = 12l$. Therefore $|s(x) - s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is 3-TEPC labeling.

□

Case 2: Let $m \equiv 1 \pmod 3$ which implies $m = 3l + 1$, for some integer $l \geq 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_iu_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 2, & \text{if } l+2 \leq i \leq 3l; \end{cases} \text{ and } h(u_{3l+1}u_1) = 2.$$

$$h(v_iv_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 3l; \end{cases} \text{ and } h(v_{3l+1}v_1) = 2.$$

$$h(z_iz_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l-1; \\ 2, & \text{if } l \leq i \leq 2l; \\ 1, & \text{if } 2l+1 \leq i \leq 3l; \end{cases} \text{ and } h(z_{3l+1}z_1) = 1.$$

$$h(u_{i+1}v_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 1, & \text{if } l+1 \leq i \leq 3l; \end{cases} \text{ and } h(u_1v_{3l+1}) = 1.$$

$$h(v_{i+1}w_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 2l; \\ 1, & \text{if } 2l+1 \leq i \leq 3l; \end{cases} \text{ and } h(v_1w_{3l+1}) = 1.$$

$$h(u_iv_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 2, & \text{if } l+2 \leq i \leq 3l+1. \end{cases}, h(w_iz_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 1, & \text{if } l+1 \leq i \leq 3l+1. \end{cases}$$

$$h(v_iw_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 2l; \\ 1, & \text{if } 2l+1 \leq i \leq 3l+1. \end{cases}$$

In this case, we have $s(0) = s(1) = s(2) = 12l+4$. Therefore $|s(x)-s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is 3-TEPC labeling.

Case 3: Let $m \equiv 2 \pmod 3$ which implies $m = 3l+2$, for some integer $l \geq 1$. We define the edge labeling $h : E_G \to \{0, 1, 2\}$ as

$$h(u_iu_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 2, & \text{if } l+2 \leq i \leq 3l+1; \end{cases} \text{ and } h(u_{3l+2}u_1) = 2.$$

$$h(v_iv_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(v_{3l+2}v_1) = 2.$$

$$h(z_iz_{i+1}) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 2, & \text{if } l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(z_{3l+2}z_1) = 2.$$

$$h(u_{i+1}v_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 1, & \text{if } l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(u_1v_{3l+2}) = 1.$$

$$h(v_{i+1}w_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 1, & \text{if } l+1 \leq i \leq 3l+1; \end{cases} \text{ and } h(v_1w_{3l+2}) = 2.$$

$$h(u_iv_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 2, & \text{if } l+2 \leq i \leq 3l+2. \end{cases}, h(w_iz_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l; \\ 1, & \text{if } l+1 \leq i \leq 3l+2. \end{cases}$$

$$h(v_iw_i) = \begin{cases} 0, & \text{if } 1 \leq i \leq l+1; \\ 1, & \text{if } l+2 \leq i \leq 3l+1; \\ 2, & \text{if } i = 3l+2. \end{cases}$$

In this case, we have $s(0) = s(1) = s(2) = 12l+8$. Therefore $|s(x)-s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is 3-TEPC labeling.

Example 2.6. *The* 3*-TEPC labeling of* T_8 *is shown in Figure 10.*

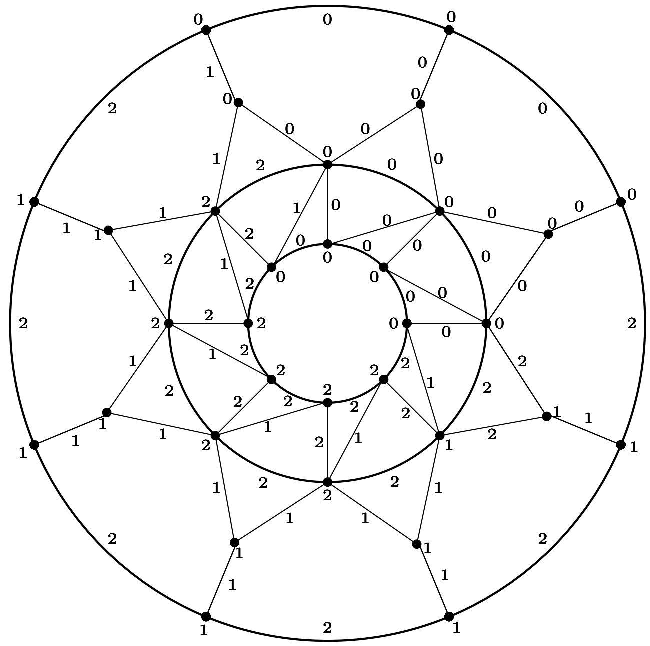

FIGURE 10. 3-TEPC labeling of T_8

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