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Some Topological Indices of $L(S(CNC_k[n]))$

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Abstract. A topological index is a function which associates real number to the graphs. Graph theory is significant in the subject of structural chemistry. In this paper we calculated R_α, M_α, χ_α, ABC, GA, ABC_4 and GA_5 indices of $L(S(CNC_k[n]))$.

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1. Introduction and basic facts

Let G denote a simple graph, $V(G)$ denotes vertex set and $E(G)$ is an edge set. The degree d_a is the number incidental edges of vertex a and $S_a = \sum_{b \in N_a} d_b$ where $N_a = \{b \in V(G) | ab \in E(G)\}$. The subdivision graph $S(G)$ is constructed from G by substituting every edge with length of path 2. The line graph $L(G)$ of G have vertices that are edges of G, two vertices e and f are incident iff they have a common end vertex in G.
Topological indices are the arithmetical numbers that matches up to the configuration of any graph. They are unchanged in graph isomorphisms. The impact of topologically indices is typically connected with QSPR and QSAR (see [17]).
Thought of topological index came into view by the efforts of Wiener (see [20]) when he was finding paraffin's boiling point. At that time theory of the topological index was

established and he entitled this index as Wiener index. The Wiener index of G is

$$W(G) = \frac{1}{2} \sum_{ab \in E(G)} d(a, b)$$

where $d(a, b)$ is $a - b$ geodesic. The earliest index based on degrees of graphs build up by Randic [14] is as follows

$$R(G) = \sum_{ab \in E(G)} (d_a d_b)^{-1/2}.$$

Afterwards, this index was globalized and recognized as the generalized Randic index $R_\alpha(G)$:

$$R_\alpha(G) = \sum_{ab \in E(G)} (d_a d_b)^\alpha. \quad (1.\ 1)$$

The general Zagreb index initiated by Li and Zhao in [10]:

$$M_\alpha(G) = \sum_{a \in V(G)} (d_a)^\alpha. \quad (1.\ 2)$$

The general sum-connectivity index $\chi_\alpha(G)$ was launched in [21]:

$$\chi_\alpha(G) = \sum_{ab \in E(G)} (d_a + d_b)^\alpha. \quad (1.\ 3)$$

Estrada et al. developed atom-bond connectivity index (ABC) in [3]. The ABC index is

$$ABC(G) = \sum_{ab \in E(G)} \sqrt{\frac{d_a + d_b - 2}{d_a d_b}}. \quad (1.\ 4)$$

The geometric arithmetic index (GA) is initiated in [19]. The GA index is

$$GA(G) = \sum_{ab \in E(G)} \frac{2\sqrt{d_a d_b}}{d_a + d_b}. \quad (1.\ 5)$$

Ghorbani et al. launched 4th ABC index in [5]:

$$ABC_4(G) = \sum_{ab \in E(G)} \sqrt{\frac{S_a + S_b - 2}{S_a S_b}}. \quad (1.\ 6)$$

Graovac et al. proposed 5th GA index in [6]:

$$GA_5(G) = \sum_{ab \in E(G)} \frac{2\sqrt{S_a S_b}}{S_a + S_b}. \quad (1.\ 7)$$

For additional information on the topological indices see [2, 4, 7, 9, 13].
The following lemma is useful to prove our results and it is recognized as handshaking Lemma.

LEMMA 1.1. *Let G be a graph. Then* $\sum_{a \in V(G)} d_a = 2|E(G)|$.

2. TOPOLOGICAL INDICES OF $L(S(G))$

Ranjini et al. computed Shultz index of $S(G)$ where G is ladder, wheel, tadpole and helm graphs in [16]. They also considered the Zagreb indices of the $L(S(G))$ where G is ladder, wheel and tadpole graph in [15]. Bindusree et al. computed ABC index of the $L(S(G))$ where G is ladder, lollipop and helm graph in [1]. Su and Xu computed $\chi_\alpha(G)$ and its co-index of the $L(S(G))$ where G is ladder, wheel and tadpole graph in [18]. In [11], M. F. Nadeem et al. computed ABC_4 and GA_5 indices of $L(S(G))$ where G is ladder, wheel and tadpole graph. They also calculated R_α, M_α, χ_α, ABC, GA, ABC_4 and GA_5 indices of $L(S(G))$ where G is $2D-$lattice, nanotube and nanotorus $TUC_4C_8[p,q]$ in [12]. In this paper, we have computed R_α, M_α, χ_α, ABC, GA, ABC_4 and GA_5 indices of $L(S(CNC_k[n]))$.
The graphical structure of $CNC_k[n]$ nanocones have a cycle of k-length at its central part and n levels of hexagons positioned at the conical exterior around its central part. The graph of $CNC_k[n]$ is shown in Fig. 1. For detailed study on some topological properties of nanocones $CNC_k[n]$, we refer to the articles [8, 13].

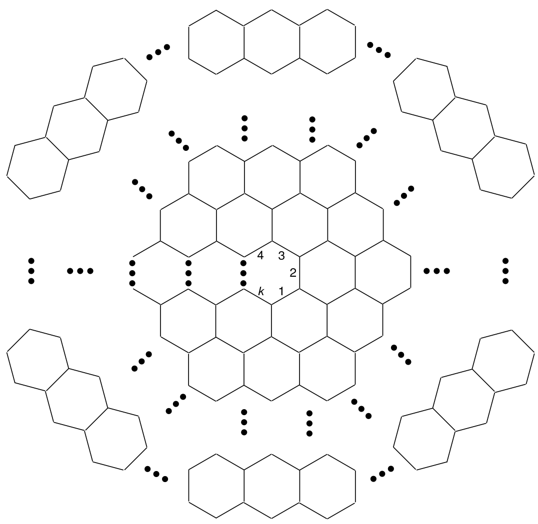

Figure 1: $CNC_k[n]$

THEOREM 2.1. *Let G be* $L(S(CNC_k[n]))$. *Then*

$$M_\alpha(G) = k\,(n+1)\,2^{\alpha+1} + k\left(n^2+n\right)3^{\alpha+1}.$$

Proof. The graph G is shown in Fig. 2. In G we have $k(3n^2+5n+2)$ vertices. The vertices of degree 2 are $2k(n+1)$ and remaining vertices are of degree 3. Hence we get $M_\alpha(G)$ by using Formula (1. 2). □

THEOREM 2.2. *Let G be* $L(S(CNC_k[n]))$. *Then:*

(1) $R_\alpha(G) = k\,(n+2)\,4^\alpha + 2\cdot 6^\alpha kn + \left(\frac{9}{2}kn^2+\frac{7}{2}kn\right)9^\alpha$;
(2) $\chi_\alpha(G) = k\,(n+2)\,4^\alpha + 2\cdot 5^\alpha kn + \left(\frac{9}{2}kn^2+\frac{7}{2}kn\right)6^\alpha$;

(3) $ABC(G) = 3\,kn^2 + \left(\frac{3}{\sqrt{2}} + \frac{7}{3}\right) nk + \sqrt{2}k;$

(4) $GA(G) = \frac{9}{2}kn^2 + \left(\frac{9}{2} + \frac{4\sqrt{6}}{5}\right) nk + 2k.$

Proof. It is easily seen from Theorem 2.1 and Lemma 1.1 that $|E(G)| = k(\frac{9}{2}n^2+\frac{13}{2}n+2)$. Hence we obtain the edge division, on the basis of the vertex degrees as shown in Table 1. Formulas (1. 1), (1. 3), (1. 4) and (1. 5) are applied to the information in Table 1 and obtain the required indices. □

(d_a, d_b) where $ab \in E(G)$	$(2,2)$	$(2,3)$	$(3,3)$
Number of edges	$k\,(n+2)$	$2kn$	$k\left(\frac{9}{2}n^2 + \frac{7}{2}n\right)$

Table 1: The edge division of the graph G w.r.t degree

THEOREM 2.3. *Let G be $L(S(CNC_k[n]))$. Then:*

(1) $ABC_4(G) = 2kn^2+\left(\frac{2\sqrt{2}}{5} + \frac{\sqrt{110}}{10} + \frac{\sqrt{14}}{8} + \frac{\sqrt{30}}{6} + \frac{2}{9}\right) kn+\left(\frac{\sqrt{6}}{4} + \frac{\sqrt{35}}{5} - \frac{2\sqrt{2}}{5}\right) k;$

(2) $GA_5(G) = \frac{9}{2}kn^2 + \left(\frac{5}{2} + \frac{8\sqrt{10}}{13} + \frac{24\sqrt{2}}{17}\right) kn + \frac{8\sqrt{5}}{9}k.$

Proof. The edge division, on the basis of the degree sum of neighbor vertices of each edge is shown in Table 2. Formula (1. 6) and (1. 7) are applied to the Table 2 and obtain the required indices.

□

(S_a, S_b) where $ab \in E(G)$	$(4,4)$	$(4,5)$	$(5,5)$	$(5,8)$	$(8,8)$	$(8,9)$	$(9,9)$
Number of edges	k	$2k$	$k(n-1)$	$2kn$	kn	$2kn$	$k\ \ \frac{9}{2}n^2 + \frac{1}{2}n$

Table 2: The edge division of the graph G w.r.t degree sum

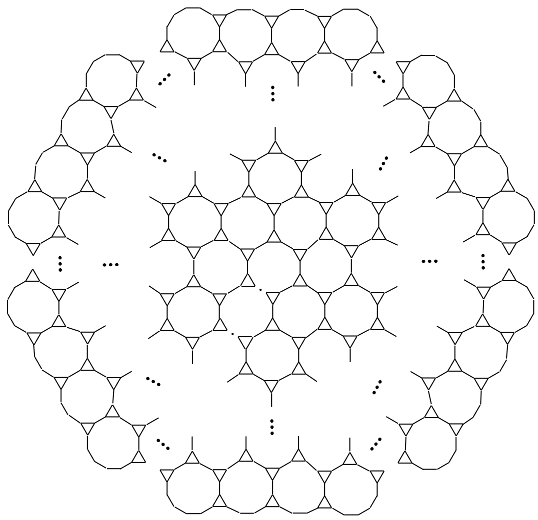

Figure 2: $L(S(CNC_k[n]))$

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