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Extension of Mangat Randomized Response Technique Using Alternative Beta Priors

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Abstract. In this study, an extension of Mangat Randomized Response Technique using alternative beta priors has been considered and new Bayes estimators of population proportion of respondents possessing stigmatized attribute were developed when data were gathered through administration of survey questionnaire on induced abortion on 300 matured women in the metropolis. Dominance picture of the proposed Bayes estimators has been portrayed for a wide range of values of population proportion assuming alternative Beta distributions as Prior information. It is observed that the proposed Bayes estimators performed better than the Bayes estimator proposed by Hussain et al [15] when a simple Beta prior was used for small, medium as well as large sample sizes respectively. This is evident as our proposed Bayes estimators have least mean squared errors (MSEs) as π approaches one.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Proposed Bayes estimators (PBEs), Alternative beta priors (ABPs), Stigmatized attribute, Mean Square Error (MSE), Absolute Bias.

1. Introduction

Untruthful responses might be obtained from respondents in a direct interrogation approach regarding sensitive attributes. For many reasons, it might be necessary to gather information about prevalence of stigmatized attribute(s) in the population. Warner [34] was the first survey statistician who proposed a nifty method of survey to gather information regarding stigmatized attributes by ensuring confidentiality of the respondent. Up till now, a large number of improvements on Warner's Randomized Response Technique (RRT) have been proposed in several studies. For example, Greenberg et al. [14], Chaudhuri and Mukerjee [10], Mangat and Singh [23], Mangat et al. [24], Tracy and Mangat [32], Mahmood et al. [22], Bhargava and Singh [9], Singh et al. [31], Christofides [11], Kim and Warde [20], Zaizai [36], Hussain et al. [16], Perri [29], Diana and Perri [12,13] Hussain et al.[15], Hussain et al. [18,19], Abid et al. [6], Adebola and Adepetun [1], Adebola and Adepetun [2], Adepetun and Adebola [3] are some of the numerous to be mentioned. However, in situations where prior information is available, Bayesian technique of estimation can be adopted in order to estimate the unknown parameter by combining sample and prior information. Winkler and Franklin [35], Pitz [28], Spurrier and Padgett [30], O'Hagan [26], Oh [27], Migon and Tachibana [25], Unnikrishnan and Kunte [33], Bar-Lev and Bobovich [8], Barabesi and Marcheselli [7], and Kim et al. [21], Adepetun and Adewara [4], Adepetun and Adewara[5] are some of the researchers who made efforts in Bayesian analysis of randomized response techniques. Using the Mangat and Singh [23] Randomized Response Technique and applying the Bayesian estimation, we propose two Bayesian estimators of population proportion of respondents possessing stigmatized attribute(s). Before moving to the formal development of the proposed Bayesian estimators, we present the Mangat and Singh [23] Randomized Response Technique in the following section. The existing Bayes estimator is presented in section 3. The proposed Bayesian estimators are presented in section 4 and a comparative study is presented in section 5.

2. Mangat and Singh [23] Randomized Response Technique

The basic idea behind Mangat and Singh [23] Randomized Response Technique is to develop a random relationship between the stigmatized questions and individual's responses. In this technique, two randomization devices R_1 and R_2 are used. The device R_1 consists of the two statements: (i) "do you belong to sensitive group?" and (ii) "do you not belong to sensitive group?" presented with probabilities T and $1 - T$ respectively. The randomization device R_2 consists of the two statements: (i) "do you belong to sensitive group?" and (ii) "do you not belong to sensitive group?" presented with probabilities P and $1 - P$. A respondent is selected and the response is recorded as "yes" if the respondent's actual status matches with the selected question and "no" else wise. For a particular respondent, the probability for a "yes answer" is given by

$$P(yes) = \phi = \pi\{2(T + P - PT) - 1\} + (1 - P)(1 - T) \tag{2. 1}$$

where T is the probability of answering the stigmatized question according to Mangat and Singh [23], and P is the preset probability that the spinner (Randomized Device) points to stigmatized question A according to Warner. The Maximum Likelihood Estimator of π is given by

$$\hat{\pi}_{ML} = \frac{\hat{\phi} - \{(1 - P)(1 - T)\}}{\{2(T + P - PT) - 1\}} \tag{2. 2}$$

Where $\hat{\phi} = \frac{x}{n}$ and x is the number of "yes responses" in the sample of n respondents. Consequently,

$$\hat{\pi}_{ML} = \frac{\pi(1 - \pi)}{n} + \frac{(1 - P)(1 - T)\{1 - (1 - P)(1 - T)\}}{n\{(2P - 1) + 2T(1 - P)\}^2} \tag{2. 3}$$

3. Presentation of the Existing Bayesian Technique of Estimation

Hussain et al. [15] in their referred paper presented a Bayesian estimation to the Randomized Response Technique (RRT) put forward by Mangat and Singh[23] using a simple beta prior distribution to estimate the population proportion of respondents possessing sensitive attribute. Assume the simple beta prior is defined as follows

$$f(\pi) = \frac{1}{\beta(a,b)}\pi^{a-1}(1-\pi)^{b-1};\ 0 < \pi < 1 \tag{3. 4}$$

where (a, b) are the shape parameters of the distribution and π is the population proportion of respondents possessing the sensitive attribute.

Let $X = \sum_i^n x_i$, where X represents the total number of 'yes' responses in a sample size n drawn from the population with simple random sampling with replacement. Here, $x_i = 1$ with probability ϕ and $x_i = 0$ with probability $1 - \phi$, where ϕ is defined in (2. 1) above. Then the conditional distribution of X given π is

$$f(X|\pi) = \binom{n}{x}\phi^x(1-\phi)^{n-x}$$

$$f(X|\pi) = \begin{matrix} n \\ x \end{matrix} \ \{\pi(2T+2P-2PT-1)+(PT-P-T+1)\}^x \ \ 1 - \{\pi(2T+2P-2PT-1)+PT-P-T+1\}^{n-x}$$

Letting $f_1 = \frac{(T-1)(P-1)}{2(T+P-PT)-1}$ and $h_1 = \frac{3(1-PT)-2(T^2+P^2-2PT^2+T^2P^2)-T-P}{2(T+P-PT)-1}$ we have that

$$f(X|\pi) = \binom{n}{x}\{(2T+2P-2PT-1)\}^n(\pi+f_1)^x(1-\pi+h_1)^{n-x}$$

Letting

$$A = \binom{n}{x}\{(2T+2P-2PT-1)\}^n$$

then

$$f(X|\pi) = A\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}f_1^{x-i}h_1^{n-x-j}\pi^i(1-\pi)^j \tag{3. 5}$$

where $x = 0, 1, 2, ..., n$.

The joint distribution of X and π is found by combining equations (3. 4) and (3. 5) which gives

$$f(X,\pi) = \frac{A}{\beta(a,b)}\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}f_1^{x-i}h_1^{n-x-j}\pi^i(1-\pi)^j\pi^{a-1}(1-\pi)^{b-1}$$

$$f(X,\pi) = \frac{A}{\beta(a,b)}\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}f_1^{x-i}h_1^{n-x-j}\pi^{a+i-1}(1-\pi)^{b+j-1} \tag{3. 6}$$

The marginal distribution of X can be obtained from (3. 6) by integrating it over π. Thus, the marginal distribution of X is

$$f(X,\pi) = \frac{A}{\beta(a,b)}\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}f_1^{x-i}h_1^{n-x-j}\int_0^1 \pi^{a+i-1}(1-\pi)^{b+j-1}d\pi \tag{3. 7}$$

$$f(X,\pi) = \frac{A}{\beta(a,b)}\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}f_1^{x-i}h_1^{n-x-j}\beta(a+1,b+j) \tag{3. 8}$$

Then, the posterior distribution of π given X is

$$f(\pi|X) = \frac{f(X,\pi)}{f(X)} = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j} f_1^{x-i} h_1^{n-x-j} \pi^{a+i-1}(1-\pi)^{b+j-1}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j} f_1^{x-i} h_1^{n-x-j} \beta(a+1,b+j)}$$

The Bayes estimator which is the posterior mean is

$$\hat{\pi}_{SM} = \int_0^1 \pi f(\pi|X) d\pi = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j} f_1^{x-i} h_1^{n-x-j} \beta(a+i+1,b+j)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j} f_1^{x-i} h_1^{n-x-j} \beta(a+1,b+j)} \tag{3. 9}$$

The Bias of $\hat{\pi}_{SM}$ is

$$B(\hat{\pi}_{SM}) = \hat{\pi}_{SM} - \pi \tag{3. 10}$$

Similarly, the mean square error (MSE) of $\hat{\pi}_{SM}$ is

$$MSE(\hat{\pi}_{SM}) = \sum_{x=0}^{n} (\hat{\pi}_{SM} - \pi)^2 \phi^x (1-\phi)^{n-x} \tag{3. 11}$$

4. The Proposed Bayesian Techniques of Estimation

In this section, we propose an alternative Bayesian estimation to Hussain et al [15] Randomized Response Technique using both the Kumaraswamy (KUMA) and the Generalised (GLS) beta prior distributions as our alternative beta prior distributions in addition to the simple beta prior distribution used by Hussain et al [15].

4.1. Estimation of π using Kumaraswamy prior.

The Kumaraswamy prior is defined as follows:

$$f_k(\pi) = bc\pi^{c-1}(1-\pi^c)^{b-1};\ b, c > 0 \tag{4. 12}$$

Then the conditional distribution of X given π according to equation(4. 12) is as follows

$$f(X|\pi) = A\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j} f_1^{x-i} h_1^{n-x-j} \pi^i (1-\pi)^j$$

By combining equations (4. 12) and (3. 5), the joint distribution of X and π is

$$f(X,\pi) = bcA\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j} f_1^{x-i} h_1^{n-x-j} \pi^i (1-\pi)^j \pi^{c-1}(1-\pi^c)^{b-1}$$

We know that using Binomial series expansion

$$(1-\pi^c)^{b-1} = \sum_{k=0}^{b-1} (-1)^k \binom{b-1}{k} \pi^{ck}$$

Therefore,

$$f(X,\pi) = bcA\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1} (-1)^k \binom{x}{i}\binom{n-x}{j}\binom{b-1}{k} f_1^{x-i} h_1^{n-x-j} \pi^{ck+c-1+i}(1-\pi)^j \tag{4. 13}$$

The marginal distribution of X by integrating (4. 13) over π is

$$f(X) = \int_0^1 f(X,\pi) d\pi$$

Thus,

$$f(X,\pi) = bcA\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\beta(ck+c+i,j+1) \tag{4. 14}$$

Then, the posterior distribution of π given X is

$$\begin{aligned} f(\pi|X) &= \frac{f(X,\pi)}{f(X)} \\ &= \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\pi^{ck+c-1+i}(1-\pi)^j}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\beta(ck+c+i,j+1)} \end{aligned}$$

Therefore, the Bayes estimator which is the posterior mean under square loss function is

$$\hat{\pi}_{KM} = \int_0^1 \pi f(\pi|X)d\pi \tag{4. 15}$$

Therefore, we have

$$\hat{\pi}_{KM} = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\beta(ck+c+i+1,j+1)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\beta(ck+c+i,j+1)} \tag{4. 16}$$

The bias as well as the mean square error (MSE) of $\hat{\pi}_{KM}$ is

$$B(\hat{\pi}_{KM}) = \hat{\pi}_{KM} - \pi \tag{4. 17}$$

$$MSE(\hat{\pi}_{KM}) = \sum_{x=0}^{n}(\hat{\pi}_{KM}-\pi)^2\phi^x(1-\phi)^{n-x} \tag{4. 18}$$

4.2. **Estimation of π using Generalised Beta prior.** A Generalized Beta Prior with shape parameters a, b, and c, is as follows:

$$f_g(\pi) = \frac{c\pi^{ac-1}(1-\pi^c)^{b-1}}{\beta(a,b)}; a,b,c>0; 0<\pi<1 \tag{4. 19}$$

Recalling the conditional distribution of X given π according to equation (3. 5) as follows

$$f(X|\pi) = A\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}f_1^{x-i}h_1^{n-x-j}\pi^i(1-\pi)^j$$

By combining equations (3. 5) and (4. 19), the joint distribution of X and π is

$$f(X,\pi) = \frac{c\pi^{ac-1}(1-\pi^c)^{b-1}}{\beta(a,b)}A\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}f_1^{x-i}h_1^{n-x-j}\pi^i(1-\pi)^j$$

Using the fact that

$$(1-\pi^c)^{b-1} = \sum_{k=0}^{b-1}(-1)^k\binom{b-1}{k}\pi^{ck}$$

The joint distribution of X and π is then

$$f(X,\pi) = B\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\pi^{ac+ck+i-1}(1-\pi)^j \tag{4. 20}$$

where $B = \frac{c}{\beta(a,b)}\binom{n}{x}\{(2T + 2P - 2PT - 1)\}^n$
The marginal distribution of X can be obtained by integrating the joint distribution of X and π over π

$$f(X) = \int_0^1 f(X, \pi)d\pi$$

$$f(X) = B\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\beta(ac + ck + i, j + 1) \tag{4. 21}$$

Then the posterior distribution of π given X is

$$f(\pi|X) = \frac{f(X, \pi)}{f(X)}$$

Therefore,

$$f(\pi|X) = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\pi^{ac+ck+i-1}(1-\pi)^j}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\beta(ac + ck + i, j + 1)} \tag{4. 22}$$

Under the square error loss function, the Bayes estimator which is the posterior mean is

$$\hat{\pi}_{GM} = \int_0^1 \pi f(\pi|X)d\pi \tag{4. 23}$$

$$\hat{\pi}_{GM} = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\beta(ac + ck + i + 1, j + 1)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}f_1^{x-i}h_1^{n-x-j}\beta(ac + ck + i, j + 1)} \tag{4. 24}$$

The bias and the mean square error (MSE) of $\hat{\pi}_{GM}$ are respectively given as

$$B(\hat{\pi}_{GM}) = \hat{\pi}_{GM} - \pi \tag{4. 25}$$

$$MSE(\hat{\pi}_{GM}) = \sum_{x=0}^{n}(\hat{\pi}_{GM} - \pi)^2\phi^x(1-\phi)^{n-x} \tag{4. 26}$$

We have the following remarks:

(i) If $c = 1, b > 1$ in equation (4. 24), it reduces to equation (3. 9), (i.e. Posterior mean of simple beta prior).
(ii) If $a = 1$ in equation (4. 24), it reduces to equation (4. 16), (i.e. Posterior mean of Kumaraswamy prior) respectively.

5. Numerical consideration and comparison of Results

Here, we present the numerical consideration as well as comparative study of our results with the existing Hussain et al [15] using life data on induced abortion with sample sizes 25, 100 and 250 respectively. To overcome the associated computational complexity, we wrote computer programs using R statistical software to generate our results. To minimize spaces, we present few results in tables and figures as follows:

TABLE 1. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=25, x=11, P=0.6, T=0.1

π	MSE BETA	MSE KUMA	MSE GLS
0.1	3.522838E-09	1.595930E-08	2.118868E-08
0.2	1.637035E-09	1.154269E-08	1.604532E-08
0.3	4.651384E-10	7.839988E-09	1.161586E-08
0.4	7.149482E-12	4.851191E-09	7.900309E-09
0.5	2.630678E-10	2.576301E-09	4.898665E-09
0.6	1.232893E-09	1.015318E-09	2.610929E-09
0.7	2.916626E-09	1.682427E-10	1.037100E-09
0.8	5.314267E-09	3.507456E-11	1.771778E-10
0.9	8.425814E-09	6.158137E-10	3.116323E-11

TABLE 2. Absolute Bias for Mangat and Singh [23]; RRT at n=25, x=11, P=0.6, T=0.1

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.31415244	0.66865342	0.77045287
0.2	0.21415244	0.56865342	0.67045287
0.3	0.11415244	0.46865342	0.57045287
0.4	0.01415244	0.36865342	0.47045287
0.5	0.08584756	0.26865342	0.37045287
0.6	0.18584756	0.16865342	0.27045287
0.7	0.28584756	0.06865342	0.17045287
0.8	0.38584756	0.03134658	0.07045287
0.9	0.48584756	0.13134658	0.02954713

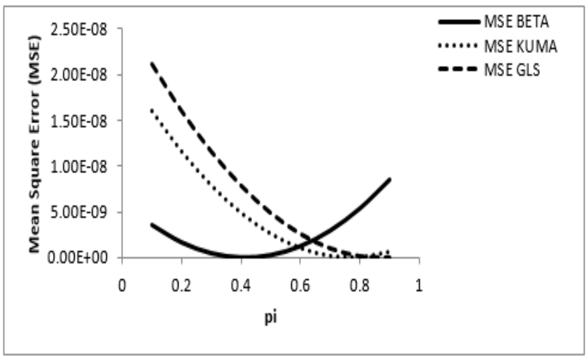

FIGURE 1. Graph showing the Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=25, x=11, P=0.6, T=0.1

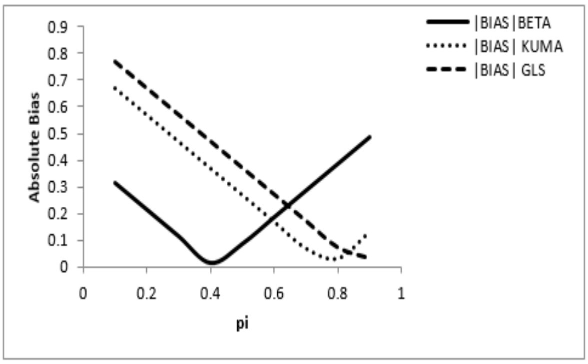

FIGURE 2. Graph showing the Absolute Bias for Mangat and Singh [23]; RRT at n=25, x=11, P=0.6, T=0.1

TABLE 3. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=25, x=11, P=0.6, T=0.2

π	MSE BETA	MSE KUMA	MSE GLS
0.1	3.684814E-09	1.509758E-08	2.052966E-08
0.2	1.748030E-09	1.081163E-08	1.547251E-08
0.3	5.251540E-10	7.239592E-09	1.112926E-08
0.4	1.618470E-11	4.381457E-09	7.499926E-09
0.5	2.211227E-10	2.237229E-09	4.584495E-09
0.6	1.139968E-09	8.069088E-10	2.382971E-09
0.7	2.772721E-09	9.049572E-11	8.953539E-10
0.8	5.119381E-09	8.798990E-11	1.216443E-10
0.9	8.179948E-09	7.993914E-10	6.184202E-11

TABLE 4. Absolute Bias for Mangat and Singh [23]; RRT at n=25, x=11, P=0.6, T=0.2

π	$\lvert$BIAS$\rvert$ BETA	$\lvert$BIAS$\rvert$ KUMA	$\lvert$BIAS$\rvert$ GLS
0.1	0.32129347	0.65035100	0.75837676
0.2	0.22129347	0.55035100	0.65837676
0.3	0.12129347	0.45035100	0.55837676
0.4	0.02129347	0.35035100	0.45837676
0.5	0.07870653	0.25035100	0.35837676
0.6	0.17870653	0.15035100	0.25837676
0.7	0.27870653	0.05035100	0.15837676
0.8	0.37870653	0.04964900	0.05837676
0.9	0.47870653	0.14964900	0.04162324

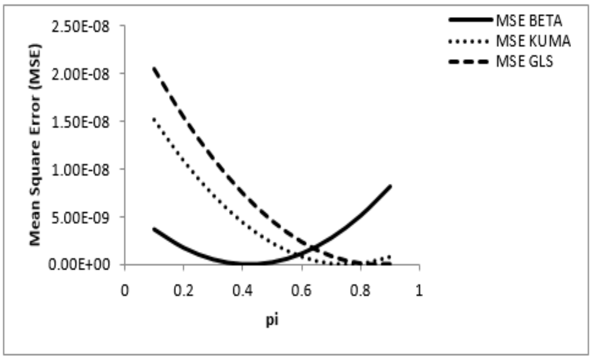

FIGURE 3. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=25, x=11, P=0.6, T=0.2

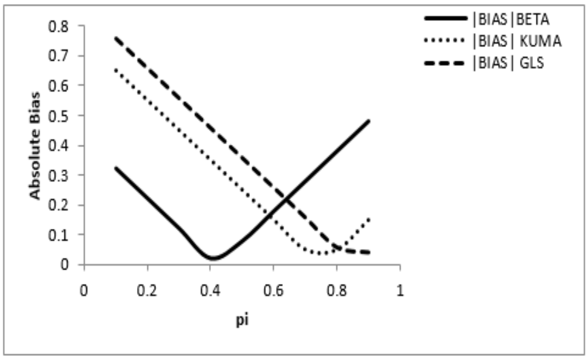

FIGURE 4. Absolute Bias for Mangat and Singh [23]; RRT at n=25, x=11, P=0.6, T=0.2

Comment: When $n = 25$, $P = 0.6$, $T = 0.1$ and 0.2, the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 < \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.8 < \pi < 1$ respectively.

TABLE 5. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=100, x=43, P=0.6, T=0.1

π	MSE BETA	MSE KUMA	MSE GLS
0.1	2.490621E-31	7.174576E-31	1.042948E-30
0.2	1.252086E-31	4.925454E-31	7.674376E-31
0.3	4.352874E-32	3.098068E-31	5.341012E-31
0.4	4.022418E-33	1.692417E-31	3.429383E-31
0.5	6.689663E-33	7.085024E-32	1.939490E-31
0.6	5.153048E-32	1.463233E-32	8.713334E-32
0.7	1.385449E-31	5.879997E-34	2.249120E-32
0.8	2.677328E-31	2.871723E-32	2.263645E-35
0.9	4.390943E-31	9.902004E-32	1.972764E-32

TABLE 6. Absolute Bias for Mangat and Singh [23]; RRT at n=100, x=43, P=0.6, T=0.1

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.34367555	0.58330127	0.70327642
0.2	0.24367555	0.48330127	0.60327642
0.3	0.14367555	0.38330127	0.50327642
0.4	0.04367555	0.28330127	0.40327642
0.5	0.05632445	0.18330127	0.30327642
0.6	0.15632445	0.08330127	0.20327642
0.7	0.25632445	0.01669873	0.10327642
0.8	0.35632445	0.11669873	0.00327642
0.9	0.45632445	0.21669873	0.09672358

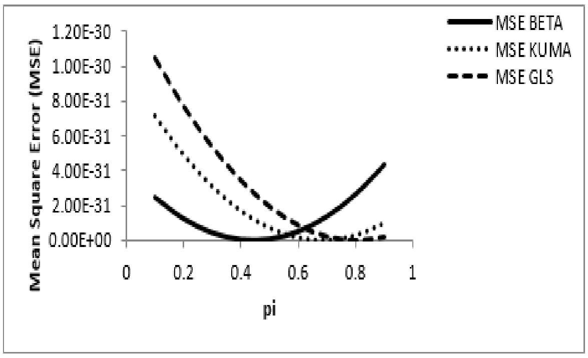

FIGURE 5. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=100, x=43, P=0.6, T=0.1

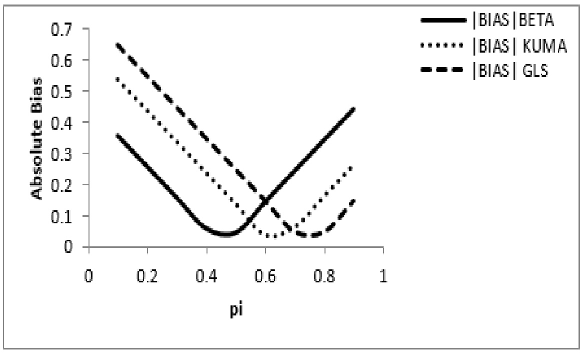

FIGURE 6. Absolute Bias for Mangat and Singh [23]; RRT at n=100, x=43, P=0.6, T=0.1

TABLE 7. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=100, x=43, P=0.6, T=0.2

π	MSE BETA	MSE KUMA	MSE GLS
0.1	2.687366E-31	6.095046E-31	8.920355E-31
0.2	1.392672E-31	4.038537E-31	6.388220E-31
0.3	5.197138E-32	2.403764E-31	4.277821E-31
0.4	6.849123E-33	1.190727E-31	2.589157E-31
0.5	3.900436E-33	3.994249E-32	1.322230E-31
0.6	4.312532E-32	2.985888E-33	4.770374E-32
0.7	1.245238E-31	8.202854E-33	5.358087E-33
0.8	2.480958E-31	5.559339E-32	5.186008E-33
0.9	4.138414E-31	1.451575E-31	4.718750E-32

TABLE 8. Absolute Bias for Mangat and Singh [23]; RRT at n=100, x=43, P=0.6, T=0.2

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.35699179	0.53762977	0.65040803
0.2	0.25699179	0.43762977	0.55040803
0.3	0.15699179	0.33762977	0.45040803
0.4	0.05699179	0.23762977	0.35040803
0.5	0.04300821	0.13762977	0.25040803
0.6	0.14300821	0.03762977	0.15040803
0.7	0.24300821	0.06237023	0.05040803
0.8	0.34300821	0.16237023	0.04959197
0.9	0.44300821	0.26237023	0.14959197

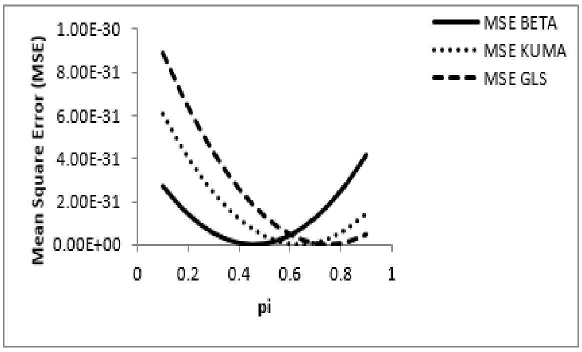

FIGURE 7. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=100, x=43, P=0.6, T=0.2

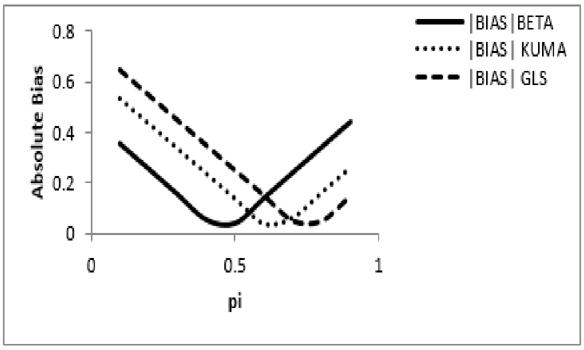

FIGURE 8. Absolute Bias for Mangat and Singh [23]; RRT at n=100, x=43, P=0.6, T=0.2

Comment: When $n = 100$, $P = 0.6$, $T = 0.1$ and 0.2, the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.6 < \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.6 < \pi < 1$ respectively.

TABLE 9. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=250, x=106, P=0.6, T=0.1

π	MSE BETA	MSE KUMA	MSE GLS
0.1	2.687366E-31	6.095046E-31	8.920355E-31
0.1	1.370491E-75	2.458503E-75	3.452974E-75
0.2	7.290805E-76	1.565361E-75	2.375915E-75
0.3	2.884169E-76	8.729657E-76	1.499603E-75
0.4	4.849995E-77	3.813171E-76	8.240377E-76
0.5	9.329721E-78	9.041524E-77	3.492192E-76
0.6	1.709062E-76	2.600564E-79	7.514735E-77
0.7	5.332293E-76	1.108516E-76	1.822210E-78
0.8	1.096299E-75	4.221898E-76	1.292438E-76
0.9	1.860116E-75	9.342747E-76	4.574120E-76

TABLE 10. Absolute Bias for Mangat and Singh [23]; RRT at n=250, x=106, P=0.6, T=0.1

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.36951227	0.49490992	0.58652620
0.2	0.26951227	0.39490992	0.48652620
0.3	0.16951227	0.29490992	0.38652620
0.4	0.06951227	0.19490992	0.28652620
0.5	0.03048773	0.09490992	0.18652620
0.6	0.13048773	0.00509008	0.08652620
0.7	0.23048773	0.10509008	0.01347380
0.8	0.33048773	0.20509008	0.11347380
0.9	0.43048773	0.30509008	0.21347380

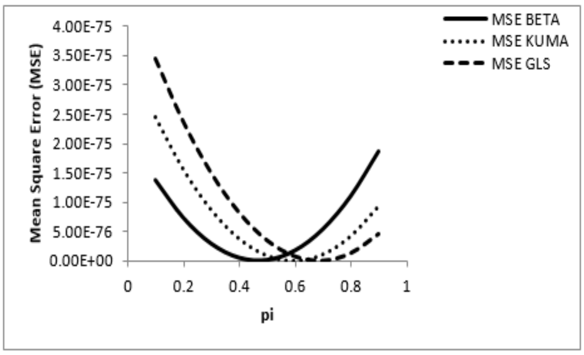

FIGURE 9. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=250, x=106, P=0.6, T=0.1

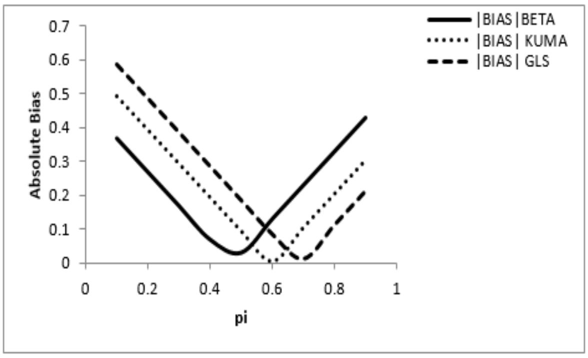

FIGURE 10. Absolute Bias for Mangat and Singh [23]; RRT at n=250, x=106, P=0.6, T=0.1

TABLE 11. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=250, x=106, P=0.6, T=0.2

π	MSE BETA	MSE KUMA	MSE GLS
0.1	1.444141E-75	2.152002E-75	2.816811E-75
0.2	7.830597E-76	1.322851E-75	1.853732E-75
0.3	3.227252E-76	6.944477E-76	1.091400E-75
0.4	6.313739E-77	2.667906E-76	5.298141E-76
0.5	4.296276E-78	3.988017E-77	1.689751E-76
0.6	1.462019E-76	1.371647E-77	8.882735E-78
0.7	4.888541E-76	1.882995E-76	4.953708E-77
0.8	1.032253E-75	5.636292E-76	2.909381E-76
0.9	1.776399E-75	1.139706E-75	7.330859E-76

TABLE 12. Absolute Bias for Mangat and Singh [23]; RRT at n=250, x=106, P=0.6, T=0.1

π	|BIAS| BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.37931113	0.46303319	0.52974844
0.2	0.27931113	0.36303319	0.42974844
0.3	0.17931113	0.26303319	0.32974844
0.4	0.07931113	0.16303319	0.22974844
0.5	0.02068887	0.06303319	0.12974844
0.6	0.12068887	0.03696681	0.02974844
0.7	0.22068887	0.13696681	0.07025156
0.8	0.32068887	0.23696681	0.17025156
0.9	0.42068887	0.33696681	0.27025156

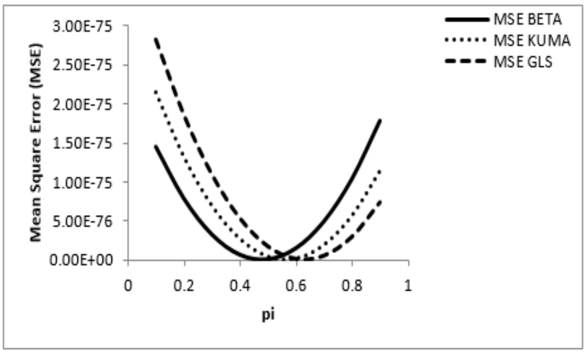

FIGURE 11. Mean Square Errors (MSEs) for Mangat and Singh [23]; RRT at n=250, x=106, P=0.6, T=0.2

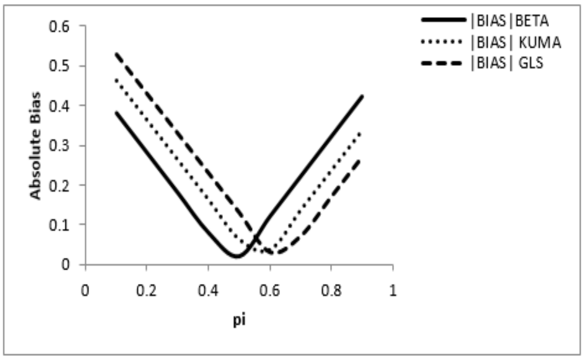

FIGURE 12. Absolute Bias forMangat and Singh [23]; RRT at n=250, x=106, P=0.6, T=0.2

Comment: When $n = 250$, $P = 0.6$, $T = 0.1$ and 0.2, the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 < \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.6 < \pi < 1$ respectively.

DISCUSSIONS OF RESULTS

From the results presented in tables and figures 1 to 12 respectively, when $n = 25$, $P = 0.6, T = 0.1$ and 0.2, the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 < \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.8 < \pi < 1$. When $n = 100$, $P = 0.6, T = 0.1$ and 0.2, the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.6 < \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.6 < \pi < 1$.

When $n = 250$, $P = 0.6$, $T = 0.1$ and 0.2, the conventional estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi < 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 < \pi < 1$. However, the proposed estimator assuming Generalised beta prior is the best in obtaining higher responses from respondents when π lies within the range $0.6 < \pi < 1$ respectively.

6. Conclusion

We have proposed alternative Bayesian estimators of the population proportion when data are gathered through the Randomized Response Technique (RRT) proposed by Hussain et al.[15]. We observed clearly from the results presented in tables and figures above, that for small, intermediate as well as large sample sizes, the proposed Bayesian estimators outperformed that of Hussain et al [15].

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References

[1] M. Abid, S. A. Cheema and Z. Hussain, *An Efficiency Comparison of Certain Randomized Response Strategies*, International Journal of Business and Social Sciences. **2**, No. 7 (2011) 188-198.

[2] F. B. Adebola and A. O. Adepetun, *A New Tripartite Randomized Response Technique*, Journal of the Nigerian Association of Mathematical Physics. **19**, (2011) 119-122.

[3] F. B. Adebola and A. O. Adepetun, *On a Qualitative Comparison of the Proposed Randomized Response Technique with Hussain and Shabbir (2007)*, International Journal of Mathematical Theory and Modeling. **2**, (2012) 61-67.

[4] A. O. Adepetun and F. B. Adebola, *On the Relative Efficiency of the Proposed Reparametized Randomized Response Model*, International Journal of Mathematical Theory and Modeling. **4**, (2014) 58-67.

[5] A. O. Adepetun and A. A. Adewara, *Bayesian Estimation of Population Proportion of a Stigmatized Attribute Using a Family of Alternative Beta Priors*, International Journal of Scientific and Engineering Research. **6**, No. 2 (2015) 1634-1650.

[6] A. O. Adepetun and A. A. Adewara, *Bayesian Analysis of Kim and Warde Randomized Response Technique Using Alternative Priors*, American Journal of Computational and Applied Mathematics. **4**, No. 4 (2014) 130-140.

[7] L. Barabesi and M. Marcheselli, *A Practical Implementation and Bayesian Estimation in Franklin's Randomized Response Procedure*, Communication in Statistics- Computation and Simulation. **35**, (2006) 563-573.

[8] S. K. Bar-Lev, E. Bobovich, and B. Boukai, *A Common Conjugate Prior Structure for Several Randomized Response Models*, Test. 12, No. 1 (2003) 101-113.

[9] M. Bhargava and R. Singh, *A Modified Randomization Device for Warner?s Model*, Statistica. **60**, (2000) 315-321.

[10] A. Chaudhuri and R. Mukerjee, *Randomized Response: Theory and Techniques*, Marcel Dekker, New York, 1988

[11] T. C. Christofides, *A Generalized Randomized Response Technique*, Metrika. **57**, (2003) 195-200.

[12] G. Diana and P. F. Perri, *Estimating a Sensitive Proportion through Randomized Response Procedures Based on Auxiliary Information*, Statistical Papers. **50**, (2009) 661-672.

[13] G. Diana and P. F. Perri, *New Scrambled Response Models for Estimating the Mean of a Sensitive Character*, Journal of Applied Statistics. **37**, No. 11 (2010) 1875-1890.

[14] B. G. Greenberg, A. A. Abul-Ela Abdel-Latif, W. R. Simmons, and D. G. Horvitz, *The Unrelated Question RR Model: Theoretical Framework*, Journal of the American Statistical Association. **64**, (1969) 529-539.

[15] Z. Hussain, S. A. Cheema and S. Zafar *Extension of Mangat Randomized Response Model*, International Journal of Business and Social Science. **2**, (2011) 261-266.

[16] Z. Hussain, J. Shabbir and S. Gupta *An Alternative to Ryu et al. Randomized Response Model*, Journal of Statistics & Management Systems. **10**, No. 4 (2007) 511-517.

[17] Z. Hussain and J. Shabbir, *An Estimation of Sensitive Proportion Utilizing Higher Order Moments of Auxiliary Variable*, International Journal of Business and Social Sciences. **2**, No. 2 (2011) 121-125.

[18] Z. Hussain, M. Hamraz and J. Shabbir, *An improved Quantitative Randomized Response Model*, International Journal of Business and Social Sciences. **2**, No. 4 (2011a) 200-205.

[19] Z. Hussain, S. Anjum, and J. Shabbir, *Improved Logit Estimation through Mangat Randomized Response Model*, International Journal of Business and Social Sciences. **2**, No. 5 (2011b) 179-188.

[20] J. M. Kim and D. W. Warde, *A Stratified Warner's Randomized Response Model*, J. Statist. Plann. Inference. **120**/1-2, (2004) 155-165.

[21] J. M. Kim, J. M. Tebbs and S. W. An, *Extensions of Mangat's Randomized Response Model*, J. Statist. Plann. Inference. **36**, No. 4 (2006) 1554-1567.

[22] M. Mahmood, S. Singh and S. Horn, *On the Confidentiality Guaranteed under Randomized Response Sampling: A Comparison with Several New Techniques*, Biometrical Journal. **40**, (1998) 237-242.

[23] N. S. Mangat and R. Singh, *An Alternative Randomized Response Procedure*, Biometrika. **77**, (1990) 439-442.

[24] N. S. Mangat, S. Singh and R. Singh, *A Note on the Inverse Binomial Randomized Response Procedure*, Journal of the Indian Society of Agricultural Statistics. **47**, (1995) 21-25.

[25] H. Migon and V. Tachibana, *Bayesian Approximations in Randomized Response Models*, Comput. Statist. Data Anal. **24**, (1997) 401-409.

[26] A. O'Hagan, *Bayes Linear Estimators for Randomized Response Models*, J. Amer. Statist. Assoc. **82**, (1987) 580-585.

[27] M. Oh, *Bayesian Analysis of Randomized Response Models: A Gibbs Sampling Approach*, J. Korean. Statist. Soc. **23**, (1994) 463-482.

[28] G. Pitz *Bayesian Analysis of Randomized Response Models*, J. Psychological Bull. **87**, (1980) 209-212.

[29] P. F. Perri, *Modified Randomized Devices for Simmons' Model*, Model Assisted Statistics and Applications. **3**, (2008) 233-239.

[30] J. Spurrier and W. Padgett, *The Application of Bayesian Techniques in Randomized Response*, Sociological Methodol. **11**, (1980) 533-544.

[31] S. Singh, S. Horn, R. Singh and N.S. Mangat, *On the Use of Modified Randomization Device for Estimating the Prevalence of a Sensitive Attribute*, Statistics in Transition. **6**, No. 4 (2003) 515-522.

[32] D. Tracy and N. Mangat, *Some Developments in Randomized Response Sampling during the Last Decade-A Follow Up of Review by Chaudhuri and Mukerjee*, Journal of Applied Statistical Sciences. **4**, (1996) 533-544.

[33] N. Unnikrishnan and S. Kunte, *Bayesian Analysis for Randomized Response Models*. Sankhya Ser. B. **61**, (1999) 422-432.

[34] S. L. Warner, *Randomized Response: A Survey Technique for Eliminating Evasive Answer Bias*, Journal of the American Statistical Association. **60**, (1965) 63-69.

[35] R. Winkler and L. Franklin, *Warner?s Randomized Response Model: A Bayesian Approach*, J. Amer. Statist. Assoc. **74**, (1979) 207-214.

[36] Y. Zaizai, *Ratio Method of Estimation of Population Proportion using Randomized Response Technique*, Model Assisted Statistics and Applications. **1**, (2005-2006) 125-130.