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Two Torsion Free Prime Gamma Rings With Jordan Left Derivations

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Abstract. Let M be a 2-torsion free prime Γ-ring and X a nonzero faithful and prime ΓM-module. Then the existence of a nonzero Jordan left derivation $d : M \to X$ satisfying some appropriate conditions implies M is commutative. M is also commutative in the case that $d : M \to M$ is a derivation along with some suitable assumptions.

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1. Introduction

Let M and Γ be additive abelian groups. M is said to be a Γ-ring if there exists a mapping $M \times \Gamma \times M \to M$ (sending (x, α, y) into $x\alpha y$) such that
$(a)(x + y)\alpha z = x\alpha z + y\alpha z$,
$x(\alpha + \beta)y = x\alpha y + x\beta y$,
$x\alpha(y + z) = x\alpha y + x\alpha z$,
$(b)(x\alpha y)\beta z = x\alpha(y\beta z)$,
for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.
M is commutative if $a\alpha b = b\alpha a$, for all $a, b \in M$ and $\alpha \in \Gamma$. A subset A of a Γ-ring M is a left(right) ideal of M if A is an additive subgroup of M and $M\Gamma A$, the set of all $m\alpha a$ such that $m \in M, \alpha \in \Gamma$ and $a \in A$ $(A\Gamma M)$ is contained in A. An ideal P of a Γ-ring M is prime if $P \neq M$ and for any ideals A and B of M, $A\Gamma B \subseteq P$, then $A \subseteq P$ or$B \subseteq P$. M is prime if $a\Gamma M\Gamma b = 0$ with $a, b \in M$, then $a = 0$ or $b = 0$. M is semiprime if $a\Gamma M\Gamma a = 0$ with $a \in M$, then $a = 0$. M is of characteristic not equal to n (or, n-torsion free) if $nm = 0$, for $m \in M$ implies $m = 0$, where n is an integer. The commutator $a\alpha x - x\alpha a$, for all $a \in M$, $x \in X$ and $\alpha \in \Gamma$ is denoted by $[a, x]_\alpha$. An element a in a $\Gamma-$ring M is called nilpotent if $(a\alpha)^n a = 0$, for all $\alpha \in \Gamma$ and for some n. The kernel of a derivation d on a $\Gamma-$ring M is denoted by Ker$d$ and defined by Ker$d = \{a \in M : d(a) = 0\}$. The

subset $Z(M) = \{a \in M : a\alpha b = b\alpha a$, for any $b \in M$ and $\alpha \in \Gamma$ $\}$ is called the centre of a $\Gamma-$ring M.
Let M be a Γ-ring and let X be an additive abelian group. X is called a ΓM-module if there exists a mapping $M \times \Gamma \times X \to X$ (sending (m, α, x) into $m\alpha x)$ such that
$(a)(m_1 + m_2)\alpha x = m_1\alpha x + m_2\alpha x$
$m(\alpha + \beta)x = m\alpha x + m\beta x$
$m\alpha(x_1 + x_2) = m\alpha x_1 + m\alpha x_2$
$(b)(m_1\alpha m_2)\beta x = m_1\alpha(m_2\beta x)$,
for all $m, m_1, m_2 \in M, \alpha, \beta \in \Gamma, x, x_1, x_2 \in \Gamma$.
X is faithful if $X\Gamma a = 0$ forces $a = 0$. X is prime if $m\Gamma M\Gamma x = 0$, for $m \in M$ and $x \in X$ implies that either $x = 0$ or $m\Gamma X = 0$. An additive mapping $d : M \to M$ is a derivation if $d(a\alpha b) = a\alpha d(b) + d(a)\alpha b$, a left derivation if $d(a\alpha b) = a\alpha d(b) + b\alpha d(a)$, a Jordan derivation if $d(a\alpha a) = a\alpha d(a) + d(a)\alpha a$ and a Jordan left derivation if $d(a\alpha a) = 2a\alpha d(a)$, for all $a, b \in M$ and $\alpha \in \Gamma$.

Y. Ceven [4] worked on Jordan left derivations on completely prime Γ-rings. He investigated the existence of a nonzero Jordan left derivation on a completely prime Γ-ring M that makes M commutative if $a\alpha b\beta c = a\beta b\alpha c$, for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. With the same assumption, he showed that every Jordan left derivation on a completely prime Γ-ring is a left derivation on it. In this paper, he gave an example of Jordan left derivation for Γ-rings.

Mustafa Asci and Sahin Ceran [7] studied on a nonzero left derivation d on a prime Γ-ring M for which M is commutative with the conditions $d(U) \subseteq U$ and $d^2(U) \subseteq Z$, where U is an ideal of M and Z is the centre of M. They also proved the commutativity of M by the nonzero left derivation d_1 and right derivation d_2 on M with the conditions $d_2(U) \subseteq U$ and $d_1 d_2(U) \subseteq Z$.

In [9], Sapanci and Nakajima defined a derivation and a Jordan derivation on Γ-rings and investigated a Jordan derivation on a certain type of completely prime Γ-ring which is a derivation. They also gave examples of a derivation and a Jordan derivation of Γ-rings.

Bresar and Vukman [2] proved that a Jordan derivation on a prime Γ-ring is a derivation. Furthermore, in [3], Bresar and Vukman showed that the existence of a nonzero Jordan left derivation of R into X implies R is commutative, where R is a ring and X is 2-torsion free and 3-torsion free left R-module. In [6], Jun and Kim proved their results without the property 3-torsion free.

Qing Deng [5] worked on Jordan left derivations d of prime ring R of characteristic not 2 into a nonzero faithful and prime left R-module X. He proved the commutativity of R with the Jordan left derivation d.

Joso Vukman [10] studied on Jordan left derivations on semiprime rings.

In this paper, we prepare a note on the basis of the results of Qing Deng [5] in Γ-rings. We show that the existence of a nonzero Jordan left derivation d on a 2-torsion free prime Γ-ring M into a faithful and prime ΓM-module X gives the commutativity of M. We also obsereve the commutativity of M when $d : M \to M$ is derivation.

Throughout this paper, we shall treat $a\alpha b\beta c = a\beta b\alpha c$, for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, as $(*)$.

2. Jordan Left Derivations

In order to equare our main results, we use some steps as lemmas.

Lemma 1. *Suppose that X is a faithful and prime ΓM-module. Let $a, b \in M$ and $x \in X$. If (the prime Γ-ring) M is 2-torsion free satisfying (*) and $a\alpha m\beta b\gamma m\delta x = 0$, for all $m \in M$ and $\alpha, \beta, \gamma, \delta \in \Gamma$, then $a = 0$ or $b = 0$ or $x = 0$.*

Proof. We use the hypothesis

$$a\alpha m\beta b\gamma m\delta x \quad = 0,$$

for all $a, b, m \in M$, $x \in X$ and $\alpha, \beta, \gamma, \delta \in \Gamma$.
Replacing m by $u + v$ in the above equation and then putting $v = m\beta a\alpha m\beta b\gamma m$, we get $a\alpha u\beta b\gamma m\beta a\alpha m\beta b\gamma m\delta x + a\alpha m\beta a\alpha m\beta b\gamma m\beta b\gamma u\delta x = 0$.This gives $a\alpha m\beta a\alpha m\beta b\gamma m\beta b\gamma u\delta x = 0$, for all $a, b, m, u \in M$, $x \in X$ and $\alpha, \beta, \gamma, \delta \in \Gamma$. If $x = 0$, we are done. Suppose that $x \neq 0$. Since X is faithful and prime, then $(a\alpha m\beta a)\alpha m\beta(b\gamma m\beta b) = 0$, for all $a, b, m \in M$ and $\alpha, \beta, \gamma \in \Gamma$. Primeness of M gives $a\alpha m\beta a = 0$ or $b\gamma m\beta b = 0$, and consequently, $a = 0$ or $b = 0$. □

Lemma 2. *Let M be a $\Gamma-$ring satisfying (*) and of characteristic not 3. If X is a 2-torsion free $\Gamma M-$module and $d : M \to X$ is a Jordan left derivation, then for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$,*
(a) $d(a\alpha b + b\alpha a) = 2a\alpha d(b) + 2b\alpha d(a)$,
(b) $d(a\alpha b\beta a) = a\beta a\alpha d(b) - b\alpha a\beta d(a)$,
(c) $d(a\alpha b\beta c + c\alpha b\beta a) = (a\beta c + c\beta a)\alpha d(b) - b\alpha a\beta d(c) - b\alpha c\beta d(a)$.

The Proof is obtained in Y. Ceven [4] by using the condition that M is of characteristic not 3.
Define $D_\alpha(x) = [a, x]_\alpha$, for all $a, x \in M$ and $\alpha \in \Gamma$.

Lemma 3. *Let M be a Γ-ring which satisfies (*) and let $a \in M$ be a fixed element. Then*
(a) $D_\alpha(x)$ *is a derivation,*
(b) $D_\alpha D_\beta(x) = a\alpha D_\beta(x) - D_\beta(x)\alpha a$,
(c) $D_\alpha D_\beta(x) = D_\beta D_\alpha(x)$,
(d) $D_\alpha D_\beta(x\gamma y) = D_\alpha D_\beta(x)\gamma y + 2D_\alpha(x)\beta D_\gamma(y) + x\gamma D_\alpha D_\beta(y)$,
for all $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$.

Proof. (a) For all $x, y \in M$ and $\alpha, \beta \in \Gamma$ and using (*), we have

$$\begin{aligned} D_\alpha(x\beta y) \quad &= [a, x\beta y]_\alpha \\ &= [a, x]_\alpha \beta y + x\alpha[a, y]_\beta \\ &= D_\alpha(x)\beta y + x\alpha D_\beta(y). \end{aligned}$$

(b) By definition, we have

$$\begin{aligned} D_\alpha D_\beta(x) \quad &= D_\alpha([a, x]_\beta) \\ &= [a[a, x]_\beta]_\alpha \\ &= a\alpha[a, x]_\beta - [a, x]_\beta \alpha a \\ &= a\alpha D_\beta(x) - D_\beta(x)\alpha a, \end{aligned}$$

for all $a, x \in M$ and $\alpha, \beta \in \Gamma$.
(c) Using (*), we get

$$\begin{aligned} D_\alpha D_\beta(x) &= D_\alpha([a,x]_\beta) \\ &= [a,[a,x]_\beta]_\alpha \\ &= a\alpha(a\beta x - x\beta a) - (a\beta x - x\beta a)\alpha a \\ &= a\beta(a\alpha x - x\alpha a) - (a\alpha x - x\alpha a)\beta a \\ &= [a,[a,x]_\alpha]_\beta \\ &= D_\beta([a,x]_\alpha) \\ &= D_\beta D_\alpha(x), \end{aligned}$$

for all $a, x \in M$ and $\alpha, \beta \in \Gamma$.
(d) By (b) and (*), we have

$$\begin{aligned} D_\alpha D_\beta(x\gamma y) &= a\alpha a\beta x\gamma y - a\alpha x\gamma y\beta a - a\beta x\gamma y\alpha a + x\gamma y\beta a\alpha a \\ &= (a\alpha a\beta x - a\alpha x\beta a - a\beta x\alpha a + x\beta a\alpha a)\gamma y \\ &+2a\alpha x\beta(a\gamma y - y\gamma a) - 2x\alpha a\beta(a\gamma y - y\gamma a) \\ &+x\gamma(a\alpha a\beta y - a\alpha y\beta a - a\beta y\alpha a + y\beta a\alpha a) \\ &= D_\alpha D_\beta(x)\gamma y + 2(a\alpha x - x\alpha a)\beta(a\gamma y - y\gamma a) + x\gamma D_\alpha D_\beta(x) \\ &= D_\alpha D_\beta(x)\gamma y + 2D_\alpha(x)\beta D_\gamma(y) + x\gamma D_\alpha D_\beta(y), \end{aligned}$$

for all $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$. □

Lemma 4. *Let M be a Γ-ring satisfying (*) and of characteristic not 3, and $d : M \to X$ a Jordan left derivation, where X is a faithful and prime $\Gamma M-$module. If $d(a) \neq 0$, for some $a \in M$, then $[a,[a,b]_\beta]_\alpha\gamma[a,[a,b]_\beta]_\alpha = 0$, for all $b \in M$ and $\alpha, \beta, \gamma \in \Gamma$.*

Proof. Let $a \in M$ be a fixed element.
By Lemma 3, we have

$$D_\alpha D_\beta(x) = a\alpha(a\beta x - x\beta a) - (a\beta x - x\beta a)\alpha a, \quad (2.\ 1)$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$.
Using (*) in $(a\alpha b - b\alpha a)\beta a\alpha D(a) = a\alpha(a\alpha b - b\alpha a)\beta D(a)$, for all $a, b \in M$ and $\alpha, \beta \in \Gamma$ [Y. Ceven, Lemma 2.2(i)], we obtain

$$(a\alpha(a\beta x - x\beta a) - (a\beta x - x\beta a)\alpha a)\alpha d(a) = 0, \quad (2.\ 2)$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$.
From (2.1) and (2.2), we get

$$D_\alpha D_\beta(x)\alpha d(a) = 0, \quad (2.\ 3)$$

for all $x \in M$ and $\alpha, \beta \in \Gamma$.
By Lemma 3(d) and (2.3), we have

$$(D_\alpha D_\beta(x)\gamma y + 2D_\alpha(x)\beta D_\gamma(y))\alpha d(a) = 0, \quad (2.\ 4)$$

for all $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$.
Replacing y by $D_\alpha(y\beta z)$ in (2.4) and by Lemma 3(a), we obtain

$$(D_\alpha D_\beta(x)\gamma(D_\alpha(y)\beta z + y\alpha D_\beta(z)) + 2D_\alpha(x)\beta D_\gamma(D_\alpha(y\beta z)))\alpha d(a) = 0 \quad (2.\ 5)$$

Using Lemma 3(c) in (2.5), and then using (2.3), we get

$$(D_\alpha D_\beta(x)\gamma(D_\alpha(y)\beta z + y\alpha D_\beta(z) + D_\alpha D_\beta(x)\gamma y\alpha D_\beta(z)))\alpha d(a) \quad = 0 \quad (2.\ 6)$$

Replacing $D_\alpha(z)$ for z in (2.6) and then by Lemma 3(c) and (2.3), we obtain

$$(D_\alpha D_\beta(x)\gamma D_\alpha(y)\alpha D_\beta(z))\alpha d(a) \quad = 0 \qquad (2.\ 7)$$

Replacing y by $D_\alpha(y)$ in (2.6), and then by Lemma 3(c) and (2.7), we get

$$(D_\alpha D_\beta(x))\gamma(D_\alpha D_\beta(y))\alpha z\alpha d(a) = 0 \qquad (2.\ 8)$$

Since (2.8) holds for all $z \in M$, we are forced to conclude that $d \neq 0$ implies

$$(D_\alpha D_\beta(x))\gamma(D_\alpha D_\beta(y)) = 0$$

for all $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$.
In particular, $(D_\alpha D_\beta(b))\gamma((D_\alpha D_\beta(b)) = 0$, for all $b \in M$ and $\alpha, \beta, \gamma \in \Gamma$.
This gives $[a, [a, x]_\beta]_\alpha\gamma[a, [a, x]_\beta]_\alpha = 0$, for all $x \in M$ and $\alpha, \beta, \gamma \in \Gamma$. □

Lemma 5. *Let M be a prime Γ-ring satisfying (*) and of characteristic not 3. Suppose that X is a faithful and prime $\Gamma M-$module. If there exists a nonzero Jordan left derivation $d : M \to X$, then M has no nonzero nilpotent elements(more precisely, M has no nonzero zero divisors).*

Proof. We shall prove this lemma by contradictory supposition. Suppose that M contains a nonzero element a with $a\alpha a = 0$, for all $\alpha \in \Gamma$. Then $0 = d(a\alpha a) = 2a\alpha d(a)$ and so

$$a\alpha d(a) = 0, \qquad (2.\ 9)$$

for all $\alpha \in \Gamma$.
Replacing c by $b\beta a$ in Lemma 2(c) and then using (*) and $a\alpha d(a) = 0$, we have

$$\begin{aligned} d(a\alpha b\beta b\beta a) + d(b\beta a\alpha b\beta a) &= d(a\alpha b\beta b\beta a + b\beta a\alpha b\beta a) \\ &= (a\beta b\beta a + b\beta a\beta a)\alpha d(b) - b\alpha a\beta d(b\beta a) - b\alpha b\beta a\beta d(a). \end{aligned}$$

Thus

$$d(a\alpha b\beta b\beta a) + d(b\beta a\alpha b\beta a) = a\beta b\beta a\alpha d(b) \qquad (2.\ 10)$$

Lemma 2(b) with $a\alpha a = 0$ and $a\alpha d(a) = 0$ gives $d(a\alpha b\beta a) = 0$. Replacing b by $b\beta b$, we obtain $d(a\alpha b\beta b\beta a) = 0$. Again $d(b\beta a\alpha b\beta a) = 2b\beta a\alpha d(b\beta a) = 0$. Thus using $d(a\alpha b\beta b\beta a) = 0$ and $d(b\beta a\alpha b\beta a) = 0$ in (10), we get

$$a\beta b\beta a\alpha d(b) = 0 \qquad (2.\ 11)$$

Replacing b by $b + c$ in (2.11), and using (*), we obtain

$$a\alpha b\beta a\beta d(c) + a\alpha c\beta a\beta d(b) = 0, \qquad (2.\ 12)$$

for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$.
Replacing $a\gamma c + c\gamma a$ for c in (2.12), after that by Lemma 2(a), $a\alpha a = 0$ and (*), we get $a\alpha b\beta a\beta c\gamma d(a) = 0$, for all $a, b, c \in M$ and $\alpha, \beta, \gamma \in \Gamma$ and so by Lemma 1,

$$d(a) = 0 \qquad (2.\ 13)$$

Interchanging a and b in Lemma 2(b) and then by (2.13) and $a\alpha d(a) = 0$, we get $d(b\alpha a\beta b)$ $= 0$. This implies that

$$a\alpha c\beta a\beta d(b\alpha a\beta b) \quad = 0, \qquad (2.\ 14)$$

for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. Replacing b by $b\alpha a\beta b$ in (2.12) and using (2.14), we get $a\alpha b\alpha a\beta b\beta a\beta d(c) = 0$, for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. This implies that $a\beta d(c) = 0$, by Lemma 1. Replacing c by $c\alpha c$ in $a\beta d(c) = 0$, and then using (*), we have

$$a\alpha c\beta d(c) = 0, \tag{2. 15}$$

for all $a, c \in M$ and $\alpha, \beta \in \Gamma$.
Replacing c by $b + c$ in (16), we get $a\alpha b\beta d(c) + a\alpha c\beta d(b) = 0$. Again, replacing c by $a\alpha c$ in $a\alpha b\beta d(c) + a\alpha c\beta d(b) = 0$ and using $a\alpha a = 0$, we get $a\alpha b\beta d(a\alpha c) = 0$, for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. This implies that $d(a\alpha c) = 0$, by the faithfulness and the primeness of X.
Applying $d(a) = 0$ and $a\alpha d(c) = 0$, we obtain $d(a\alpha c) = d(c\alpha a + a\alpha c) = 2c\alpha d(a) + 2a\alpha d(c)$. Replacing a by $b\beta a$ in $d(c\alpha a) = 0$ and c by $b\beta c$ in $d(a\alpha c) = 0$ and adding the obtained results, we have

$$d(a\alpha b\beta c + c\alpha b\beta a) = 0, \tag{2. 16}$$

for all $a, b \in M$ and $\alpha, \beta \in \Gamma$.
The faithfulness and primeness of X and (*) in (2.11) gives $a\beta d(b) = 0$. With the help of the Lemma 2(c) and $a\beta d(b) = 0$, (2.16) gives $a\alpha c\beta d(b) = 0$, for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. Hence $d(b) = 0$, for all $b \in M$. But this is a contradiction. □

We now state and prove our main result.

Theorem 6. *Let M be a prime Γ-ring satisfying (*) and of characteristic not 2, and X a nonzero left ΓM-module. Suppose that X is faithful and prime. If there exists a nonzero Jordan left derivation $d : M \to X$, then M is commutative.*

Proof. In order to develop [1, Theorem 2.2], it suffices to consider the case that M is of characteristic not 3. Consider an element $a \in M$ such that $d(a) \neq 0$. Then by Lemma 4, we get $[a, [a, b]_\beta]_\alpha \gamma [a, [a, b]_\beta]_\alpha = 0$, for all $b \in M$ and $\alpha, \beta, \gamma \in \Gamma$. By Lemma 5, we get $[a, [a, b]_\beta]_\alpha = 0$, for all $b \in M$ and $\alpha, \beta \in \Gamma$. This implies that $a\alpha[a, b]_\beta = [a, b]_\beta \alpha a$, for all $b \in M$ and $\alpha, \beta \in \Gamma$ and so $a \in Z(M)$. Thus $M = Z(M) \cup \mathrm{Ker} d$. Since d is nonzero, we conclude that $M = Z(M)$, by Brauer's trick (which states that a group cannot be the union of its two proper subgroups). Therefore, M is commutative. □

Finally, keeping relation with Theorem 6, we developed [11, Theorem 2] as follows.

Theorem 7. *Let M be a prime Γ-ring satisfying (*) and of characteristic not 2. If there exists a nonzero derivation $d : M \to M$ such that $[a, [a, d(a)]_\beta]_\alpha \in Z(M)$, for all $a \in M$ and $\alpha, \beta \in \Gamma$, then M is commutative.*

Proof. In view of [12, Theorem 2], we consider the case that M is of characteristic not 3. Then for any $a \in M$ and $\alpha, \beta \in \Gamma$, and using (*) and the condition that M is of characteristic not 3, we have

$$\begin{aligned} d(a\alpha a\beta a) &= a\alpha a\beta d(a) + d(a\alpha a)\beta a \\ &= a\alpha a\beta d(a) + a\alpha d(a)\beta a + d(a)\alpha a\beta a - a\beta d(a)\alpha a + d(a)\beta a\alpha a \\ &+ 3a\alpha d(a)\beta a = a\alpha[a, d(a)]_\beta - [a, d(a)]_\beta \alpha a \\ &= [a, [a, d(a)]_\beta]_\alpha \in Z(M). \end{aligned}$$

With the same conditions above and we get

$$\begin{aligned} d((a\alpha a\beta a)\gamma(a\alpha a\beta a)\delta(a\alpha a\beta a)) &= (a\alpha a\beta a)\gamma(a\alpha a\beta a)\delta d(a\alpha a\beta a)+(a\alpha a\beta a)\gamma \\ &\quad d(a\alpha a\beta a)\delta(a\alpha a\beta a)+d(a\alpha a\beta a)\gamma(a\alpha a\beta a) \\ &\quad \delta(a\alpha a\beta a) \\ &= (a\alpha a\beta a)\gamma(d(a\alpha a\beta a)\delta(a\alpha a\beta a)-(a\alpha a\beta a)\delta \\ &\quad d(a\alpha a\beta a))+d(a\alpha a\beta a)\gamma((a\alpha a\beta a)\delta(a\alpha a\beta a)) \\ &\quad -((a\alpha a\beta a)\delta(a\alpha a\beta a)) \\ &\quad \gamma d(a\alpha a\beta a)+3(a\alpha a\beta a)\gamma(a\alpha a\beta a)\delta d(a\alpha a\beta a) \\ &= 0 \end{aligned}$$

and hence M is commutative. □

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