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Superquadratic method for generalized equations under relaxed conditions

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Abstract. We present a new approach to study the convergence of some superquadratic iterative method in Banach space for solving variational inclusions under different assumptions used in [12, 14, 2]. Here, we relax Lipschitz, Hölder or center–Hölder type conditions by introducing ω–type–conditioned second order Fréchet derivative. Under this conditions, we show that the sequence is locally superquadratically convergent if some Aubin continuity property is satisfied. In particular, we recover a quadratic and a cubic convergence.

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1. Introduction

Generalized equations [18, 19] are an abstract model of a wide variety of variational problems. They may characterize optimality or equilibrium and then have several applications economics and engineering (see for example [11]).

Throughout, X and Y are Banach spaces, we denote by $\mathbb{B}_r(x)$ the closed ball centered at x with radius r. The distance from a point x and a subset A of X will be denoted by $\text{dist}\,(x, A) = \inf_{a \in A} \|x - a\|$. A set–valued mapping Λ from X to Y is indicated by $\Lambda : X \longrightarrow 2^Y$ and its graph is the set gph $\Lambda := \{(x, y) \in X \times Y,\ y \in \Lambda(x)\}$ and $\Lambda^{-1}(y) = \{x \in X,\ y \in \Lambda(x)\}$. From now on $f : X \to Y$ denotes a twice (Fréchet) differentiable function while $G : X \longrightarrow 2^Y$ stands for a set–valued mapping with closed graph. We are concerned with the problem of approximating a solution x^* of the generalized equation of the form

$$0 \in f(x) + G(x), \tag{1.1}$$

and we consider the following iterative method for solving (1.1):

$$0 \in A(x_{k+1}, x_k) + G(x_{k+1}), \tag{1.2}$$

where,

$$A(y, x) = f(x) + \nabla f(x)(y - x) + \frac{1}{2}\nabla^2 f(x)(y - x)^2, \ \forall x, y \in X. \tag{1.3}$$

Algorithm (1.2) is based on the second–degree Taylor polynomial expansion A of f. The cubically convergence of method (1.2) is presented in [12] when the set–valued mapping $[A(\cdot, x^*) + G(\cdot)]^{-1}$ is Aubin continuous around $(0, x^*)$ (or pseudo–Lipschitz at $(0, x^*)$), and the function f is $\mathcal{C}^2$ and the second Fréchet derivative of f is L–Lipschitz in some neighborhood V of x^*

$$\| \nabla^2 f(x) - \nabla^2 f(y) \| \leq L \ \| x - y \|, \ x, y \in V. \tag{1.4}$$

Recall that a set-valued map $F : Y \longrightarrow 2^X$ is pseudo–Lipschitz at $(z, w) \in \text{gph}\, F$ if there exist constants a, b, M such that for every $y_1, y_2 \in \mathbb{B}_b(z)$ and for every $z_1 \in F(y_1) \cap \mathbb{B}_a(w)$ there exists $z_2 \in F(y_2)$ with

$$\| z_1 - z_2 \| \leq M \| y_1 - y_2 \| .$$

The pseudo–lipschitzian property is introduced in [5] and is tied to the concept of metric regularity; actually, the Aubin continuity of F around (z, w) is equivalent to the metric regularity of the inverse F^{-1} of F at w for z, i.e., $z \in F^{-1}(w)$ and there exists $\kappa \in [0, \infty[$ along with neighborhoods U of w and V of z such that

$$\text{dist}\ (x, F(y)) \leq \kappa \ \text{dist}\ (y, F^{-1}(x)), \ \ \forall x \in U, \ y \in V.$$

The infimum of the set of values κ for which this holds is the modulus of metric regularity. For more details on these topics one can refer to [7, 8, 9, 16, 17, 20, 21].

Geoffroy and Piétrus [14] showed that the sequence (1.2) is locally superquadratic convergent to the solution x^* whenever $\nabla^2 f$ satisfies some α–Hölder–type condition on some neighborhood V of x^* with constant K ($\alpha, \ K > 0$):

$$\| \nabla^2 f(x) - \nabla^2 f(y) \| \leq K \ \| x - y \|^\alpha, \ x, y \in V. \tag{1.5}$$

The stability of method (1.2) is investigated in [13] with respect to some perturbations; more precisely, if we consider the perturbed equation $y \in f(x) + G(x)$ (y is some parameter in Y) then the attraction region does not depend on small perturbations of the parameter y.

Argyros [2] provided a finer local superquadratic convergence of algorithm (1.2) using α–center–Hölder condition on some neighborhood V of x^* with constant K_0 ($\alpha, \ K_0 > 0$):

$$\| \nabla^2 f(x) - \nabla^2 f(x^*) \| \leq K_0 \ \| x - x^* \|^\alpha, \ x \in V. \tag{1.6}$$

In this paper, we use different conditions to the previous one to study the convergence of (1.2). We relax these usual Lipschitz and Hölder conditions by ω–conditioned second derivative. This condition is used in [10, 15] to study Newton's method for solving nonlinear equations ($G = \{0\}$ in (1.1)). The main conditions required are

$$\| \nabla^2 f(x) - \nabla^2 f(y) \| \leq \omega(\| x - y \|), \ \text{for } x, \ y \text{ in } V, \tag{1.7}$$

$$\| \nabla^2 f(x) - \nabla^2 f(y) \| \leq \sigma(\| x - y \|) \ \| x - y \|^\theta, \quad \text{for all } x, \ y \text{ in } V \text{ and } \theta \text{ is fixed in } (0, 1], \tag{1.8}$$

$$\| \nabla^2 f(x) - \nabla^2 f(x^*) \| \leq \mu(\| x - x^* \|), \ \text{for } x \text{ in } V, \tag{1.9}$$

$$\| \nabla^2 f(x) - \nabla^2 f(x^*) \| \leq \vartheta(\| x - x^* \|) \ \| x - x^* \|^\theta, \text{ for } x \text{ in } V, \tag{1.10}$$

where $\omega,\, \sigma,\, \mu,\, \vartheta \,:\, \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ are a continuous nondecreasing functions. When the condition (1.7) is satisfied, we say that $\nabla^2 f$ is ω–conditioned. The condition (1.9) is called μ–center–condition on the second derivative $\nabla^2 f$. Similar conditions to (1.7) and (1.9) on the Fréchet derivative ∇f are used in [3] to study of Newton's methods for solving (1.1). The inspiration for considering (1.8) comes from [22, 1].

Such a study can be of interest, for example, to variational inequalities for saddle points (see [21]). Let A and B be nonempty, closed and convex subsets of $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively, and let $\mathcal{L} : \mathbb{R}^n \times \mathbb{R}^m \longrightarrow \mathbb{R}$ be some $\mathcal{C}^1$ convex–concave on $A \times B$. The point $(\bar{x}$,$\bar{y})$ $\in A \times B$ is a saddle point if the following hold:

$$\mathcal{L}(x, \bar{y}) \ \geq \ \mathcal{L}(\bar{x}, \bar{y}) \ \geq \ \mathcal{L}(\bar{x}, y), \quad \text{for all } x \in A \text{ and } y \in B. \tag{1.11}$$

The saddle point condition (1.11) is equivalent to

$$0 \in f(\bar{x}, \bar{y}) + G(\bar{x}, \bar{y}), \tag{1.12}$$

where f and G are defined on $A \times B$ by $f(x, y) = (\nabla_x \mathcal{L}(x, y), -\nabla_y \mathcal{L}(x, y))$ and by $G(x, y) = N_A(x) \times N_B(y)$, with N_A (resp. N_B) the normal cone to the set A (resp. B). Hence, the variational problem (1.11) corresponds to generalized equation in formulation (1.1) and $(\bar{x}, \bar{y})$ can be approximated by the method (1.2).

This paper is organized as follows: In section 2 we have collected a fixed point theorem [6] and a number of necessary results, needed in our local analysis. In section 3, we give some convergence results using the different assumptions (1.7), (1.8) or (1.9) and the the Aubin continuity of $[A(\cdot, x^*) + G(\cdot)]^{-1}$.

2. Background material and assumptions

Let us begin with some basic results [4] that will be used throughout this paper. The first tool in our analysis is the fixed point theorem for set–valued maps proved by Dontchev and Hager [6].

Lemma 1. (see [6]) *Let ϕ a set–valued map from X into the closed subsets of X, let $\eta_0 \in X$ and let r and λ be such that $0 \leq \lambda < 1$ and the following conditions hold:*

(a) dist $(\eta_0, \phi(\eta_0)) \leq r(1 - \lambda)$.

(b) $e(\phi(x_1) \cap \mathbb{B}_r(\eta_0), \phi(x_2)) \leq \lambda \ \| x_1 - x_2 \|, \ \forall x_1, x_2 \in \mathbb{B}_r(\eta_0)$.

Then ϕ has a fixed–point in $\mathbb{B}_r(\eta_0)$. That is, there exists $x \in \mathbb{B}_r(\eta_0)$ such that $x \in \phi(x)$. If ϕ is single–valued, then x is the unique fixed point of ϕ in $\mathbb{B}_r(\eta_0)$.

By the second order Taylor expansion of f at $y \in V$ with the remainder is given by integral form, the following lemmas are obtained directely.

Lemma 2. *We suppose that the assumption (1.7) is satisfied on a convex neighborhood V. Then for all x and y in V we have the following*

$$\| f(x) - f(y) - \nabla f(y)\, (x - y) - \frac{1}{2}\nabla^2 f(y)\, (x - y)^2 \| \leq \\ \| x - y \|^2 \int_0^1 (1 - t)\, \omega(t \parallel x - y \parallel)\, dt.$$

In particular, if the assumption (1.9) is satisfied then for all x in V we have the following

$$\| f(x) - f(x^*) - \nabla f(x^*)\, (x - x^*) - \frac{1}{2}\nabla^2 f(x^*)\, (x - x^*)^2 \| \leq \\ \| x - x^* \|^2 \int_0^1 (1 - t)\, \mu(t \parallel x - x^* \parallel)\, dt.$$

Lemma 3. *We suppose that the assumption (1.8) is satisfied on a convex neighborhood V. Then for all x and y in V we have the following*

$$\| f(x) - f(y) - \nabla f(y)\,(x-y) - \frac{1}{2}\nabla^2 f(y)\,(x-y)^2 \| \leq$$
$$\| x-y \|^{2+\theta} \int_0^1 t^\theta\,(1-t)\,\sigma(t \| x-y \|)\,dt.$$

Remark 4. $\int_0^1 (1-t)\,\omega(t \| x-x^* \|)\,dt$ and $\int_0^1 t^\theta\,(1-t)\,\sigma(t \| x-x^* \|)\,dt$ given in the previous lemmas are bounded by $\omega(\text{diam } (V))$ and $\sigma(\text{diam } (V))$ respectively where diam (V) is the diameter of neighborhood V.

Before stating the main results on this study, we need to introduce some notations. First, for $k \in \mathbb{N}$ and (x_k) defined in (1.2), let us define the set–valued mappings $Q : X \longrightarrow 2^Y$ and $\psi_k : X \longrightarrow 2^X$ by the following

$$Q(.) := A(., x^*) + G(.); \quad \phi_k(.) := Q^{-1}(Z_k(.)), \tag{2.1}$$

where Z_k is defined from X to Y by

$$Z_k(x) := A(x, x^*) - A(x_k, x). \tag{2.2}$$

Let us mention that x_1 is a fixed point of ϕ_0 if and only if $0 \in A(x_1, x_0) + G(x_1)$.

We will make the following assumptions in a open convex neighborhood V of x^*:

- $(\mathcal{H}0)$ ∇f is L–Lipschitz on V with $L > 0$, and there exists $L_0 > 0$ such that $\| \nabla^2 f(x^*) \| < L_0$.
- $(\mathcal{H}1)$ The condition (1.7) is satisfied on V.
- $(\mathcal{H}1)^\star$ The condition (1.8) is satisfied on V.
- $(\mathcal{H}2)$ The set–valued map $[A(., x^*) + G(.)]^{-1}$ is pseudo–Lipschitz around $(0, x^*)$ with constants M, a and b (these constants are given by definition of Aubin continuity).

3. Convergence analysis

The main theorems of this study read as follows:

Theorem 5. *Let x^* be a solution of* (1.1)*. We suppose that assumptions ($\mathcal{H}0$)–($\mathcal{H}2$) are satisfied and we denote by $\beta = M \int_0^1 (1-t)\,\omega(t\,a)\,dt$. Then for every $C > \beta$, there exist $\delta > 0$ such that for every starting point $x_0 \in \mathbb{B}_\delta(x^*)$, and a sequence (x_k) for (1.1), defined by* (1.2)*, which satisfies*

$$\| x_{k+1} - x^* \| \leq C \| x_k - x^* \|^2 . \tag{3.1}$$

That is, (1.2) *generates (x_k) with second order.*

Theorem 6. *Let x^* be a solution of* (1.1)*. We suppose that assumptions ($\mathcal{H}0$), ($\mathcal{H}1)^\star$ and ($\mathcal{H}2$) are satisfied and we denote by $\beta' = M \int_0^1 t^\theta\,(1-t)\,\sigma(t\,a)\,dt$. Then for every $C' > \beta'$, there exist $\gamma > 0$ such that for every starting point $x_0 \in \mathbb{B}_\gamma(x^*)$, and a sequence (x_k) for (1.1), defined by* (1.2)*, which satisfies*

$$\| x_{k+1} - x^* \| \leq C' \| x_k - x^* \|^{2+\theta} . \tag{3.2}$$

That is, (1.2) *generates (x_k) with superquadratic convergence. In particular, if $\theta = 1$ then the convergence is cubic.*

Theorem 5 is showed as follows. Once x_k is computed, we show that the function ϕ_k has a fixed point x_{k+1} in X. This process allows us to prove the existence of a sequence (x_k) satisfying (1.2). Now, we state a result which is the starting point of our algorithm . It will be very usefull to prove theorem 5 and reads as follows:

Proposition 7. *Under the hypotheses of theorem 5, there exists* $\delta > 0$ *such that for all* $x_0 \in \mathbb{B}_\delta(x^*)$ $(x_0 \neq x^*)$*, the map* ϕ_0 *has a fixed point* x_1 *in* $B_\delta(x^*)$ *satisfying* $\| x_1 - x^* \| \leq C \ \| x_0 - x^* \|^2$.

Proposition 8. *Under the hypotheses of theorem 6, there exists* $\gamma > 0$ *such that for all* $x_0 \in \mathbb{B}_\gamma(x^*)$ $(x_0 \neq x^*)$*, the map* ϕ_0 *has a fixed point* x_1 *in* $B_\gamma(x^*)$ *satisfying* $\| x_1 - x^* \| \leq C' \ \| x_0 - x^* \|^{2+\theta}$.

Proof of Proposition 7. By hypothesis $(\mathcal{H}2)$ we have

$$e(Q^{-1}(y') \cap \mathbb{B}_a(x^*), Q^{-1}(y'')) \leq M \ \| y' - y'' \|, \ \forall y', y'' \in \mathbb{B}_b(0). \tag{3.3}$$

Fix $\delta > 0$ such that

$$\delta < \min\left\{a;\ \frac{1}{C};\ \sqrt{\frac{b}{5\,\beta}}\right\}. \tag{3.4}$$

To prove Proposition 7 we intend to show that both assertions (a) and (b) of lemma 1 hold; where $\eta_0 : = x^*$, ϕ is the function ϕ_0 defined at the very begining of this section and where r and λ are numbers to be set.

According to the definition of the excess e, we have

$$\text{dist }(x^*, \phi_0(x^*)) \leq e\bigg(Q^{-1}(0) \cap \mathbb{B}_\delta(x^*), \phi_0(x^*)\bigg). \tag{3.5}$$

Moreover, for all $x_0 \in B_\delta(x^*)$ such that $x_0 \neq x^*$ we have by $(\mathcal{H}1)$ and Lemma 2

$$\| Z_0(x^*) \| = \| A(x_0, x^*) \| \leq \beta \ \| x_0 - x^* \|^2 . \tag{3.6}$$

Then (3.4) yields, $\| Z_0(x^*) \| < b$. Hence from (3.3) one has

$$e\bigg(Q^{-1}(0) \cap \mathbb{B}_\delta(x^*), \phi_0(x^*)\bigg) = e\bigg(Q^{-1}(0) \cap \mathbb{B}_\delta(x^*), Q^{-1}[Z_0(x^*)]\bigg) \leq M\,\beta \ \| x^* - x_0 \|^2 .$$

By (3.5), we get

$$\text{dist }(x^*, \phi_0(x^*)) \leq M\,\beta \ \| x^* - x_0 \|^2 . \tag{3.7}$$

Since $C > M\,\beta$ there exists $\lambda \in]0,1[$ such that $C(1-\lambda) \geq M\,\beta$. Hence,

$$\text{dist }(x^*, \phi_0(x^*)) \leq C\,(1-\lambda) \ \| x_0 - x^* \|^2 . \tag{3.8}$$

By setting $\eta_0 := x^*$ and $r := r_0 = C \ \| x^* - x_0 \|^2$ we can deduce from the last inequalities that assertion (a) in lemma 1 is satisfied.

Now, we show that condition (b) of Lemma 1 is satisfied. Since $\dfrac{1}{C} \geq \delta$ and $\| x^* - x_0 \| \leq \delta$, we have $r_0 \leq \delta \leq a$. Moreover by Lemma 2, we have for $x \in \mathbb{B}_\delta(x^*)$,

$$\begin{array}{rcl} \| Z_0(x) \| & = & \| A(x, x^*) - A(x_0, x) \| \\ & \leq & \| A(x, x^*) \| + \| A(x_0, x) \| \\ & \leq & \beta \ \| x - x^* \|^2 + \beta \ \| x - x_0 \|^2 \leq 5\,\beta\,\delta^2 \end{array} \tag{3.9}$$

Then by (3.4) we deduce that for all $x \in \mathbb{B}_\delta(x^*)$, $Z_0(x) \in \mathbb{B}_b(0)$. Then it follows that for all $x', x'' \in \mathbb{B}_{r_0}(x^*)$, we have

$$I = e(\phi_0(x') \cap \mathbb{B}_{r_0}(x^*), \phi_0(x'')) \leq e(\phi_0(x') \cap \mathbb{B}_\delta(x^*), \phi_0(x'')), \tag{3.10}$$

which yields by (3.3)

$$\begin{array}{rcl}
I & \leq & M \parallel Z_0(x') - Z_0(x'') \parallel \\
& \leq & M \parallel \nabla f(x^*)(x'-x'') - \nabla f(x_0)(x'-x'') \\
& & + \frac{1}{2}\nabla^2 f(x^*)(x'-x^*)^2 - \frac{1}{2}\nabla^2 f(x^*)(x''-x^*)^2 \\
& & + \frac{1}{2}\nabla^2 f(x_0)(x''-x_0)^2 - \frac{1}{2}\nabla^2 f(x_0)(x'-x_0)^2 \parallel \\
& \leq & M \parallel \nabla f(x^*)(x'-x'') - \nabla f(x_0)(x'-x'') \\
& & + \frac{1}{2}\nabla^2 f(x^*)(x'-x''+x''-x^*)^2 - \frac{1}{2}\nabla^2 f(x^*)(x''-x^*)^2 \\
& & + \frac{1}{2}\nabla^2 f(x_0)(x''-x_0)^2 - \frac{1}{2}\nabla^2 f(x_0)(x'-x''+x''-x_0)^2 \parallel \\
& = & M \parallel \nabla f(x^*)(x'-x'') - \nabla f(x_0)(x'-x'') \\
& & + \frac{1}{2}\Big(\nabla^2 f(x^*)(x'-x'')^2 - \nabla^2 f(x_0)(x'-x'')^2\Big) \\
& & + \nabla^2 f(x^*)(x''-x_0+x_0-x^*)(x'-x'') - \nabla^2 f(x_0)(x''-x_0)(x'-x'') \parallel \\
& \leq & M \Bigg(\parallel \nabla f(x^*) - \nabla f(x_0) \parallel \parallel x'-x'' \parallel \\
& & + \frac{1}{2} \parallel \nabla^2 f(x^*) - \nabla^2 f(x_0) \parallel \parallel x'-x'' \parallel^2 \\
& & + \parallel \nabla^2 f(x^*) - \nabla^2 f(x_0) \parallel \parallel x''-x_0 \parallel \parallel x'-x'' \parallel \\
& & + \parallel \nabla^2 f(x^*) \parallel \parallel x_0 - x^* \parallel \parallel x'-x'' \parallel \Bigg)
\end{array} \tag{3.11}$$

By Assumptions ($\mathcal{H}0$)–($\mathcal{H}1$) and (3.4) we deduce that

$$\begin{array}{rcl}
I & \leq & M(L\,\delta + \omega(a)\,\delta + 2\,\omega(a)\,\delta + L_0\,\delta) \parallel x'-x'' \parallel \\
& = & M\,\delta\,(L + L_0 + 3\,\omega(a)) \parallel x'-x'' \parallel
\end{array} \tag{3.12}$$

Without loss of generality we may assume that $\delta < \dfrac{\lambda}{M\,(L+L_0+3\,\omega(a))}$. Then condition (b) of Lemma 1 is satisfied. Since both conditions of Lemma 1 are fulfilled, we can deduce the existence of a fixed point $x_1 \in \mathbb{B}_{r_0}(x^*)$ for the map ϕ_0. Then the proof of Proposition 7 is complete. ⊠

Idea of the proof of Proposition 8. The proof of Proposition 8 is the same one as that of the proof of Proposition 7. It is enough to make some modifications by choosing the constant γ such that

$$\gamma < \min\left\{a;\ \left(\frac{1}{C'}\right)^{\frac{1}{1+\theta}};\ \left(\frac{b}{(1+2^{2+\theta})\,\beta'}\right)^{\frac{1}{2+\theta}}\right\}. \tag{3.13}$$

⊠

Now that we proved Proposition 7 and Proposition 8, the proof of Theorem 5 and Theorem 6 is straightforward as it is shown below.

Proof of Theorems 5 and 6. Proceeding by induction, keeping $\eta_0 = x^*$ and setting $r_k = C \parallel x_k - x^* \parallel^2$ and $r'_k = C' \parallel x_k - x^* \parallel^{2+\theta}$, the application of Proposition 7 and Proposition 8 to the map ϕ_k respectively gives the desired results. ⊠

Remark 9. Theorem 5 and Theorem 6 remain true under (1.9) and (1.10).

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