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On γ-S-closed Spaces

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Abstract. In this paper, we introduce and explore γ-S-closed spaces. It is evident by definition that γ-S-closed space coincides with γ-s-closed space, if the underlying space is γ-extremally-disconnected.

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1. Introduction

Topology is an important and interesting area of mathematics, the study of which will not only introduce you to new concepts and theorems but also put into context old ones like continuous functions. It is so fundamental that its influence is evident in almost every other branch of mathematics. This makes the study of topology relevant to all who aspire to be mathematicians whether their first love is algebra, analysis, category theory, chaos, continuum mechanics, dynamics, geometry, industrial mathematics, mathematical biology, mathematical economics, mathematical finance, mathematical modelling, mathematical physics, mathematics of communication, number theory, numerical mathematics, operation research or statistics.

S. Kasahara [10] introduced and discussed an operation γ of a topology τ into the power set P(X) of a space X . H. Ogata [15] introduced the concept of γ-open sets and investigated the related topological properties of the associated topology τ_γ and τ by using operation γ.

S. Hussain and B. Ahmad [1-9] continued studying the properties of γ-operations on topological spaces and investigated many interesting results. In 2007-08, they introduced and discussed γ-s-closed spaces and subspaces[6-7]. It is shown that the concept of γ-s-closed spaces generalized s-closed spaces [12]. It is interesting to note that γ-s-closedness is the generalization of γ_0-compactness (which generalized compactness) defined and investigated in [3].

In this paper, we introduce and explore γ-S-closed spaces. It is evident from definition that γ-S-closed space coincide with γ-s-closed space, if underlying space is γ-extremally-disconnected.

First, we recall some definitions and results used in this paper. Hereafter, we shall write a space in place of a topological space.

2. Preliminaries

Definition 2.1[9]. Let X be a space. An operation $\gamma : \tau \to P(X)$ is a function from τ to the power set of X such that $V \subseteq V^\gamma$, for each $V \in \tau$, where V^γ denotes the value of γ at V. The operations defined by $\gamma(G) = G$, $\gamma(G) = cl(G)$ and $\gamma(G) = intcl(G)$ are examples of operation γ.

Definition 2.2[15]. Let $A \subseteq X$. A point $x \in A$ is said to be γ-interior point of A, if there exists an open nbd N of x such that $N^\gamma \subseteq A$ and we denote the set of all such points by $int_\gamma(A)$. Thus

$$int_\gamma(A) = \{x \in A : x \in N \in \tau \text{ and } N^\gamma \subseteq A\} \subseteq A.$$

Note that A is γ-open [15] iff $A = int_\gamma$(A). A set A is called γ- closed [1] iff $X - A$ is γ-open.

Definition 2.3[15]. A point $x \in X$ is called a γ-closure point of $A \subseteq X$, if $U^\gamma \cap A \neq \phi$, for each open nbd U of x. The set of all γ-closure points of A is called γ-closure of A and is denoted by $cl_\gamma(A)$. A subset A of X is called γ-closed, if $cl_\gamma(A) \subseteq A$. Note that $cl_\gamma(A)$ is contained in every γ-closed superset of A.

Definition 2.4[1]. The γ-exterior of A, written $ext_\gamma(A)$ is defined as the γ-interior of $X - A$. That is, $ext_\gamma(A) = int_\gamma(X - A)$.

Definition 2.5 [1]. The γ-boundary of A, written $bd_\gamma(A)$ is defined as the set of points which do not belong to the γ-interior or the γ-exterior of A.

Definition 2.6[15]. An operation γ on τ is said be regular, if for any open nbds U,V of $x \in X$, there exists an open nbd W of x such that $U^\gamma \cap V^\gamma \supseteq W^\gamma$.

Definition 2.7[15]. An operation γ on τ is said to be open, if for any open nbd U of each $x \in X$, there exists γ-open set B such that $x \in B$ and $U^\gamma \supseteq B$.

Definition 2.8[15]. A space X is said to be γ-T_2 space, if for each disjoint points x, y of X, there exist open sets U and V such that $x \in U$, $y \in V$ and $U^\gamma \cap V^\gamma = \phi$.

Definition 2.9[5]. A subset A of a space X is called γ-regular open, if $A = int_\gamma(cl_\gamma(A))$. The set of γ-regular open sets is denoted by $RO_\gamma(X)$.

Note that $RO_\gamma(X) \subseteq \tau_\gamma \subseteq \tau$. Where τ_γ denotes the set of all γ-open sets in X.

Definition 2.10[5]. A subset A of a space X is called γ-regular closed, denoted by $RC_\gamma(X)$, if one of the following conditions holds:

(i) $A = cl_\gamma(int_\gamma(A))$.

(ii) $X - A \in RO_\gamma(X)$.

Clearly A is γ-regular open if and only if $X - A$ is γ-regular closed.

Definition 2.11[3]. A subset A of a space X is said to be γ_0-compact relative to X, if every cover $\{V_i : i \in I\}$ of X by γ-open sets of X, there exists a finite subset I_0 of I such that $A \subseteq \bigcup_{i \in I_0} cl_\gamma(V_i)$. A space X is γ_0-compact, if $X = \bigcup_{i \in I_0} cl_\gamma(V_i)$

Definition 2.12[3]. A space X is said to be a γ-regular space, if for any γ-closed set A and any point $x \notin A$, there exist γ-open sets U and V such that $x \in U$, and $A \subseteq V$ and

$U \cap V = \phi$.

Definition 2.13[2]. Let X be a space and $x \in X$. Then x is called a γ-limit point of A if and only if $U^{\gamma} \cap (A - \{x\}) \neq \phi$, where U is open set in X. The set of all γ-limit points is called a γ-derived set and is denoted by A^d_{γ}.

Definition 2.14[1]. Let $A \subseteq X$. Then A is called γ-dense in itself if $A \subseteq A^d_{\gamma}$.

Definition 2.15[9]. A subset A of a space X is said to be a γ-semi-open set, if there exists a γ-open set O such that $O \subseteq A \subseteq cl_{\gamma}(O)$. The set of all γ-semi-open sets is denoted by $SO_{\gamma}(X)$. A is γ-semi-closed if and only if $X - A$ is γ-semi-open in X. Note that A is γ-semi-closed if and only if $int_{\gamma}(cl_{\gamma}(A)) \subseteq A$ [4].

It is shown that every γ-open sets is γ-semi-open but converse is not true in general [4].

Definition 2.16[4]. Let X be a space and $A \subseteq X$. The intersection of all γ-semi-closed sets containing A is called γ-semi-closure of A and is denoted by $scl_{\gamma}(A)$. A is γ-semi-closed iff $scl_{\gamma}(A) = A$.

Definition 2.17[4]. Let X be a space and $A \subseteq X$. The union of γ-semi-open subsets of A is called γ-semi-interior of A and is denoted by $sint_{\gamma}(A)$.

Definition 2.18[8]. A space X is said to be γ-s-regular, if for any γ-semi-regular set A and $x \notin A$, there exist disjoint γ-open sets U and V such that $A \subseteq U$ and $x \in V$.

Definition 2.19[9]. A subset A of a space X is said to be γ-semi-regular, if it is both γ-semi-open and γ-semi-closed. The class of all γ-semi-regular sets of X is denoted by $SR_{\gamma}(A)$. If γ is regular, then the union of γ-semi-regular sets is γ-semi-regular.

Definition 2.20[6]. A space X is γ-extremally disconnected space, if $cl_{\gamma}(U)$ is γ-open set, for every γ-open set U in X.

It is also shown [6] that $cl_{\gamma}(U) = scl_{\gamma}(U)$, if X is γ-extremally disconnected.

Definition 2.21[6]. A space X is said to be γ-s-closed , if for any cover $\{V_{\alpha} : \alpha \in I\}$ of X by γ-semi-open sets of X, there exists a finite subset I_0 of I such that $X = \bigcup_{\alpha \in I_0} scl_{\gamma}(V_{\alpha})$.

Definition 2.22[5]. A subset A of a space X is said to be γ-s-closed relative to X, if for any cover $\{V_{\alpha} : \alpha \in I\}$ of X by γ-semi-open sets of X, there exists a finite subset I_0 of I such that $A \subseteq \bigcup_{\alpha \in I_0} scl_{\gamma}(V_{\alpha})$.

3. γ-S-closed Spaces

Definition 3.1. A space X is said to be γ-S-closed, if for every γ-semi-open cover $\{U_{\alpha} : \alpha \in I\}$ of X there exists a finite subfamily $\{U_{\alpha_i} : i = 1, 2, ..., n\}$ such that $X = \bigcup_{\alpha_i} cl_{\gamma}(U_{\alpha_i})$.

It is apparent from the definition that γ-S-closed spaces and γ-s-closed spaces coincide, if X is γ-extremally disconnected space.

Example 3.2. Let X= $\{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. For $b \in X$, define an operation $\gamma : \tau \to P(X)$ by

$$\gamma(A) = \begin{cases} A, & \text{if } b \in A \\ cl(A), & \text{if } b \notin A \end{cases}$$

Calculations shows that $\{a, b\}, \{a, c\}, \{b\}, X, \phi$ are γ-open sets and $\{a, c\}, \{b\}, X, \phi$ are γ-semi-open sets. Clearly X is γ-S-closed.

Definition 3.3. A filter base $F = \{A_i : i \in I\}$ on X is said to γ-s-converges to $x \in X$, if for each $V \in SO_{\gamma}(X)$ containing x , there exists $A_i \in F$ such that $A_i \subseteq cl_{\gamma}(V)$.

Definition 3.4. A filter base $F = \{A_i : i \in I\}$ is said to be γ-s-accumulate at $x \in X$, if $A_i \cap cl_{\gamma}(V) \neq \phi$, for every $V \in SO_{\gamma}(X)$ containing x and every $A_i \in F$.

The following Theorem is easy consequence of definitions 3.3 and 3.4:

Theorem 3.5. Let F be a maximal filterbase in a space X. Then F γ-s-accumulates to a point $x \in X$ if and only if F γ-s-converges to x.

Theorem 3.6. Let X be a space. Then the following are equivalent.

(1) X is γ-S-closed.

(2) For each family of disjoint γ-semi-closed sets $\{F_\alpha, \alpha \in I\}$, there exists a finite subfamily $\{F_{\alpha_i}\}_{i=1}^n$ such that $\bigcap_{i=1}^n int_\gamma(F_{\alpha_i}) = \phi$.

(3) Each filterbase $F = \{A_\alpha : \alpha \in I\}$ γ-s-accumulates to some point $x_0 \in X$.

(4) Each maximum filterbase F γ-s-converges.

Proof. (1) $\Rightarrow$ (4). Let $F = \{A_\alpha : \alpha \in I\}$ be a maximum filterbase. Contrarily suppose that F does not γ-s-converge to any point; therefore, by Theorem 3.5, F does not γ-s-accumulate to any point. This implies that for every $x \in X$, there exists a γ-semi-open set $V(x)$ containing x and $A_{\alpha(x)} \in F$ such that $A_{\alpha(x)} \cap cl_\gamma(V(x)) = \phi$. Obviously $\{V(x) : x \in X\}$ is a γ-semi-open cover for X and by hypothesis there exists a finite subfamily such that $\bigcup_{i=1}^n cl_\gamma(V(x_i)) = X$. Since F is a filterbase, there exists a $A_0 \in F$ such that $A_0 \subseteq \bigcap_{i=1}^n A_{\alpha(x_i)}$. Hence, $A_0 \cap cl_\gamma(V(x_i)) = \phi$, $1 \leq i \leq n$, which implies $A_0 \cap (\bigcup_{i=1}^n cl_\gamma(V_{x_i})) = A_0 \cap X = \phi$, which contradics the essential fact that $A_0 \neq \phi$.

(4) $\Rightarrow$ (3). Each filterbase is contained in a maximal filterbase.

(3) $\Rightarrow$ (2). Let $\{F_\alpha : \alpha \in I\}$ be a collection of γ-semi-closed sets such that $\bigcap_\alpha F_\alpha = \phi$. Contrarily suppose that for every finite subfamily $\bigcap_{i=1}^n int_\gamma(F_{\alpha_i}) \neq \phi$. Therefore for any positive integer n, $F = \{\bigcap_{i=1}^n int_\gamma(F_{\alpha_i}), F_{\alpha_i} \in \{F_\alpha\}\}$ forms a filterbase. By hypothesis, F γ-s-accumulates to some point $x_o \in X$. This implies that for every γ-semi-open set $V(x_0)$ containing x_0, $int_\gamma(F_\alpha) \cap cl_\gamma(V(x_0)) \neq \phi$, for every $\alpha \in I$. Since $x_0 \notin \cap F_\alpha$ there exists a point $\alpha_0 \in I$ such that $x_0 \notin F_{\alpha_0}$. Hence, x_0 is contained in the γ-semi-open set $X - F_{\alpha_0}$. Therefore, $int_\gamma(F_{\alpha_0}) \cap cl_\gamma(X - F_{\alpha_0}) = int_\gamma(F_{\alpha_0}) \cap (X - int_\gamma(F_{\alpha_0})) = \phi$, which contradicts the fact that F γ-s-accumulates to x_0.

(2) $\Rightarrow$ (1). Let $\{V_\alpha : \alpha \in I\}$ be a γ-semi-open covering of X. Then $\bigcap_\alpha (X - V_\alpha) = \phi$. By hypothesis, there exists a finite subfamily such that $\bigcap_{i=1}^n int_\gamma(X - V_{\alpha_i}) = \bigcap_{i=1}^n (X - cl_\gamma(V_{\alpha_i})) = \phi$. Therefore, $\bigcap_{i=1}^n (cl_\gamma(V_{\alpha_i})) = X$, and consequently X is γ-S-closed. This completes the proof.

The proof of the following Lemma directly follows from the definitions and is thus omitted:

Lemma 3.7. If Y is a γ-regularly-closed subset in a γ-S-closed space X, then Y is γ-S-closed.

Theorem 3.8. Each γ-extremally disconnected, γ_0 compact space is γ-S-closed.

Proof. If X is γ-extremally disconnected, then $cl_\gamma(U)$ is γ-open, for every γ-open set U. The γ-interior of a γ-semi-open set is γ-dense in it. We take $\{cl_\gamma(int_\gamma(U_i) : i \in I\}$ instead of given γ-semi-open cover. Hence the proof.

Theorem 3.9. If X is a γ-S-closed, γ-regular space, then X is γ-extremally disconnected.

Proof. Contrarily suppose that X is not γ-extremally disconnected. Then there exists a γ-regular-open set U in X such that $cl_\gamma(U) - U$ and $X - cl_\gamma(U)$ are non empty. Let $x \in cl_\gamma(U) - U$. Then for every nbd V of x, $V \cap O \neq \phi$. Therefore, $F = \{(V \cap U)\}$ forms a filterbase in $cl_\gamma(U)$. By Lemma 3.7, $cl_\gamma(U)$ is γ-S-closed. Therefore F γ-s-accumulates to some point x_0 in $cl_\gamma(U)$. Hence the filterbase F also converges to x in the usual sense. We claim that $x_0 \notin (cl_\gamma(O) - O)$; for if it were, then $x_0 \in (X - O)$ and every member of F would have to intersect $(X - O)$, an impossibility. Thus $x_0 \in O$. There now exists a γ-open set U such that $x_0 \in U \subseteq cl_\gamma(U) \subseteq O$ and $x \in X - cl_\gamma(U)$. But since F converges to x, there must exist a nbd of x, say V, such that $(V \cap O) \subseteq X - cl_\gamma(U)$. This then would imply that $(V \cap O) \cap cl_\gamma(U) = \phi$, which contradicts the fact that F γ-s-accumulates to x_0. Therefore, our assumption that X is not γ-extremally disconnected is false, and the Theorem follows.

The proof of the following theorem is easy and is thus omitted:

Theorem 3.10. If X is a γ-T_2 γ-S-closed space, then X is γ-extremally disconnected.

4. γ-S-CLOSED SUBSPACES

Definition 4.1. A subset S of a space X is said to be γ-S-closed relative to X, if for every cover $\{U_\alpha : \alpha \in I\}$ of S by γ-Semi-open sets in X, there exists a finite subfamily I_0 of I such that $S \subseteq \bigcup\{cl_\gamma(U_\alpha : \alpha \in I_0\}$.

Let $B \subseteq X$, $\gamma : \tau \to P(X)$ be an operation. We define $\gamma_B : \tau_B \to P(X)$ as $\gamma_B(U \cap B) = \gamma(U) \cap B$. From here γ_B is an operation and satisfies that $cl_{\gamma_B}(U \cap B) \subseteq cl_\gamma(U \cap B)$.

Theorem 4.2. Let γ be a regular operation then a γ-open set B of a space X is γ-S-closed if and only if it is γ-S-closed relative to X.

Proof. Necessity: Let $\{U_i : i \in I\}$ be a cover of B and $U_i \in SO_\gamma(X)$, for each $i \in I$. Since B is γ-open in X and γ is regular, for each $i \in I$, $B \cap U_i \in SO_\gamma(X)$[4] and hence $B \cap U_i \in SO_\gamma(B)$. Since B is γ-S-closed, there exists a finite subfamily I_0 of I such that $B = \bigcup\{cl\gamma_B(B \cap U_i) : i \in I_0\}$. Therefore, we have $B \subseteq \bigcup\{cl_\gamma(U_i) : i \in I_0\}$.

Strong sufficiency. Suppose that $B \in SO_\gamma(X)$ and B is γ-S-closed relative to X. Let $\{U_i : i \in I\}$ be a cover of B and $U_i \in SO_\gamma(B)$ for each $i \in I$. Since B is γ-S-closed relative to X, there exists a finite subfamily I_0 of I such that $B \subseteq \{cl_\gamma(U_i) : i \in I_0\}$. Therefore, we obtain $B = \bigcup\{cl_{\gamma_B}(U_i) : i \in I_0\}$. This completes the proof.

The following Example shows that the regularity of operation γ is necessary for above Theorem.

Example 4.3. Let X= $\{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. For $b \in X$, define an operation $\gamma : \tau \to P(X)$ by

$$\gamma(A) = \begin{cases} A, & \text{if } b \in A \\ cl(A), & \text{if } b \notin A \end{cases}$$

Then the operation γ is not regular. Calculations shows that $\{a, b\}, \{a, c\}, \{b\}, X, \phi$ are γ-open sets and $\{a, c\}, \{b\}, X, \phi$ are γ-semi-open sets. Clearly γ-open set $\{a, b\}$ does not satisfy above Theorem.

Theorem 4.4. Let X be a space and γ is a regular operation then X is γ-S-closed if and only if every proper γ-regularly-open set of X is γ-S-closed.

Proof. Necessity: Suppose that X is γ-S-closed and let G be a proper γ-regularly-open set of X. By Theorem 4.2, we show that G is γ-S-closed relative to X. Let $\{U_i : i \in I\}$ be a cover of G and $U_i \in SO_\gamma(X)$ for each $i \in I$. Then, since $X - G$ is γ-regularly-closed, we have $X - G \in SO_\gamma(X)$[6] and $X = (X - G) \bigcup (\cup\{U_i : i \in I\}]$. Since X is γ-S-closed and γ is regular, there exists a finite subfamily I_0 of I such that $X = cl_\gamma(X - G) \bigcup (\cup\{cl_\gamma(U_i) : i \in I_0\})$. Therefore, we obtain $G \subseteq \{cl_\gamma(U_i) : i \in I_0\}$.

Sufficiency: Let $\{U_i : i \in I\}$ be a cover of X and $U_i \in SO_\gamma(X)$ for each $i \in I$. Suppose that $X \neq cl_\gamma(U_{i_0})$ and $U_{i_0} \neq \phi$. Then since $U_{i_0} \in SO_\gamma(X)$, $cl_\gamma(U_{i_0})$ is γ-regularly-closed and hence $X - cl_\gamma(U_{i_0})$ is a proper γ-regularly-open set. Therefore, by Theorem 4.2, there exists a finite subfamily I_0 of I such that $X - cl_\gamma(U_{i_0}) \subseteq \bigcup\{cl_\gamma(U_i) : i \in I_0\}$.

Therefore, we obtain $X = \bigcup\{cl_\gamma(U_i) : i \in I_0 \cup \{i_0\}\}$. This completes the proof.
Theorem 4.5. Let A and B be subsets of a space X. If A is γ-S-closed relative to X and B is γ-regular-open in X, then $A \cap B$ is γ-S-closed relative to X. Where γ is a regular operation.
Proof. Let $\{V_i : i \in I\}$ be a cover of $A \cap B$ and $V_i \in SO_\gamma(X)$ for each $\alpha \in I$. Since $X - B$ is γ-regular-closed[6], we have $X - B \in SO_\gamma(X)$ and $A \subseteq [\cup\{V_i : i \in I\}] \bigcup (X - B)$. Since A is γ-s-closed relative to X, there exists a finite subfamily I_0 of I such that $A \subseteq [\cup\{cl_{\gamma_X}(V_i) : i \in I_0\}] \bigcup (X - B)$. Therefore, we obtain $A \cap B \subseteq \cup\{cl_{\gamma_X}(V_i) : i \in I_0\}$. This shows that $A \cap B$ is γ-S-closed relative to X. This completes the proof.
Theorem 4.6. Let γ be an open operation and A is γ-S-closed relative to a space X, then $cl_{\gamma_X}(A)$ and $int_{\gamma_X}(cl_{\gamma_X}(A))$ are γ-S-closed relative to X.
Proof. Let $V = \{V_i : i \in I\}$ be a cover of $cl_{\gamma_X}(A)$ and $V_i \in SO_\gamma(X)$, for each $i \in I$. Then V is a cover of A and A is a γ-S-closed relative to X. Therefore, there exists a finite subfamily I_0 of I such that $A \subseteq \cup\{cl_{\gamma_X}(V_i) : i \in I_0\}$. Since γ is open, we have $cl_{\gamma_X}(A) \subseteq cl_{\gamma_X}(\cup\{cl_{\gamma_X}(V_i) : i \in I_0\}) = \cup cl_{\gamma_X}(\{cl_{\gamma_X}(V_i) : i \in I_0\}) = \cup\{cl_{\gamma_X}(V_i) : i \in I_0\}$ [15]. This shows that $cl_{\gamma_X}(A)$ is γ-S-closed relative to X. Moreover, $int_{\gamma_X}(cl_{\gamma_X}(A))$ is γ-regular-open in X and hence by Theorem 4.5, $int_{\gamma_X}(cl_{\gamma_X}(A))$ is γ-S-closed relative to X. Hence the proof.
Theorem 4.7. If A_1 and A_2 are sets γ-S-closed relative to a space X, then $A_1 \cup A_2$ is γ-S-closed relative to X.
Proof. Let $V = \{V_i : i \in I\}$ be a cover of $A_1 \cup A_2$ and $V_i \in SO_\gamma(X)$ for each $i \in I$. Then V is a γ-semi-open cover of A_j for $j = 1, 2$. Therefore, there exists a finite subfamily I_j of I such that $A_j \subseteq \cup\{cl_{\gamma_X}(V_i) : i \in I_j\}$. Thus, we have $A_1 \cup A_2 \subseteq \cup\{cl_{\gamma_X}(V_i) : i \in I_1 \cup I_2\}$. This shows that $A_1 \cup A_2$ is γ-S-closed relative to X.
Theorem 4.8. Let X be a γ-S-closed space and A is a γ-closed set of X. If $bd_\gamma(A)$ is γ-S-closed relative to X, then A is γ-S-closed relative to X. Where γ is a regular operation.
Proof. Since A is γ-closed, $int_{\gamma_X}(A)$ is γ-regular-open[6] and hence by Theorem 4.4, it is γ-S-closed relative to X. Therefore, by Theorem 4.7, $A = int_{\gamma_X}(A) \cup bd_\gamma(A)$ is γ-S-closed relative to X. This completes the proof.

The following Theorem follows from Theorem 4.6:
Theorem 4.9. Let X be a space and γ is an open operation. If each point $x \in X$ has a γ-open nbd which is γ-S-closed relative to X then it has a γ-open nbd V such that $cl_{\gamma_X}(V)$ is γ-S-closed relative to X.

Using Theorem 4.5, we can prove the following Theorem:
Theorem 4.10. Let X be a space and γ is a regular operation. If each point $x \in X$ has a γ-open nbd V such that $cl_{\gamma_X}(V)$ is γ-S-closed relative to X then X has a γ-open nbd V such that $int_{\gamma_X}(cl_{\gamma_X}(V))$ is γ-S-closed relative to X.

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