Application of Unconstrained Optimization Techniques for Designing a Composite Conventional Propeller

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Abstract

Applying numerical optimization techniques, a propeller of conventional geometry is designed from composite materials. Ply orientation angles of the composite material have been used as the design variables, to achieve various design objectives. The design objectives in this work were maximizing the propeller coefficient of thrust, minimizing the coefficient of power, and maximizing the propeller efficiency. It is shown in this study that all the design objectives can be achieved by arranging the orientation angles and stacking sequence properly within the composite laminate. Improvements of up to 47% were obtained compared to a metallic propeller of the same geometry.

Keywords: composite materials, propellers, numerical optimization, finite element method, blade element theory

Introduction

Composite materials have been fully established as workable materials in the fields of aerospace, civil, marine, and mechanical engineering and in many other fields. Their main success is because of their high strength and stiffness and low specific gravity. Composite materials are fundamentally different from isotropic materials, as their properties are direction dependent. This anisotropy can be controlled to gain certain advantages which traditional materials are unable to provide. The primary advantages are strength and stiffness in the loading direction. In this study, the design of a composite propeller is attempted by varying the orientation angles of the plies within the composite laminate.

Many researchers have applied optimization techniques for designing composite structures. Schmit and Farsi [1] presented a minimum-weight design of symmetric laminates using the ply angles as the optimization variables. Fleury and Schmit [2] presented an efficient design technique in which the thickness was the only design variable. Fukunaga and Vanderplaats [3] considered an optimization problem under in-plane loading. Both ply orientation angle and ply thickness were used as design variables. Lee and Lin [4] developed regressions to determine the response of a composite marine propeller and rotor wing. The regression was used in the optimization algorithm to determine the best ply angle and stacking sequence to use within the composite laminate. The authors found that the response of the structure was dependant on the ply angles. Lin and Lee [5] reduced the calculation time of Lee and Lin [4] by applying local improvements in the optimization algorithm. Khan et al. [6], using blade element theory and the finite element method, performed a detailed study of conventional propellers made from composite materials. They showed that propeller characteristics are a function of the ply orientation angles within the composite laminates. The use of blade element theory is not new, and many investigators have used it in their work (e.g. [7] and [8]). Further, many researchers have successfully used the finite element method for stress, modal, and buckling analyses of structures made from composite materials. Reference [9] summarizes the details of various laminate theories, analytical solutions, and finite element models of composite structures. This reference demonstrates that the finite element method is an accurate method for predicting the response of composite structures to applied loads.

Objective of this work

In this study, the work of reference [6] is advanced. It was shown in reference [6] that the characteristics of composite propellers are functions of the ply orientation angle, and a composite propeller can provide better performance characteristics than a propeller of the same geometry but made from conventional metals. In reference [6], the coupling behavior obtainable between bending and twisting of composite laminates was utilized for improving propeller performance. Thus it is possible that a composite propeller can be designed for optimum performance by manipulating with the ply orientation angles. In this investigation, the design of a composite propeller is posed as a numerical optimization problem, whose objective function is the maximizing of selected performance parameters and whose design variables are the ply orientation angles.

Methodology and Theoretical Aspects

The candidate propeller for this work was propeller 4102 from reference [10]. This propeller was chosen because the experimental characteristics data was available. The procedure used for the design optimization as well as the

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necessary theoretical aspects of the development are overviewed in this section.

Procedure

The objective of this study was to design a composite propeller with the same geometry as that of propeller 4102. For this purpose, a simple algorithm was developed to calculate the propeller blade deformation and resulting changes in the loading conditions until equilibrium between deformation and loading was achieved. For calculating blade deformations, the finite element method was used. For calculating aerodynamic loadings, the blade element method was used. In order to apply the algorithm, the propeller was initially divided into a number of blade elements, and then further divided into a number of finite elements. Figure 1 shows the blade element and finite element discretization of the propeller 4102. Figure 2 shows the flow chart of the algorithm developed by the authors for calculating propeller characteristics using blade element and finite element methods.

Since the stacking sequence of the plies within the composite laminate can affect the propeller characteristics, a composite propeller can be designed for one or more specific purposes by changing the ply stacking sequence. In this work, the process of designing a composite propeller is posed as an optimization problem. The objective function and design variables are defined, constraints on various parameters are identified, and the optimization problem is solved to arrive at a final design. For this purpose, the computational subroutine "CONMIN" [11] coupled with a finite element and blade element subroutine was used to obtain the optimized solution. The material used for the composite propeller was a unidirectional carbon/epoxy with material properties E_{11} = 153 GPa, E_{22} = 10.9 GPa, $G_{12} = G_{13}$ = 5.9 GPa, and υ_{12} = 0.3.

Since the algorithm developed for calculating propeller performance uses the blade element method, the finite element method, and numerical optimization techniques, some theoretical background about these topics are presented here. It is also mentioned here that for implementation of the algorithm, complete computer programs inclusive of subroutines for finite element and blade element calculations were written by the authors using the Fortran programming language.

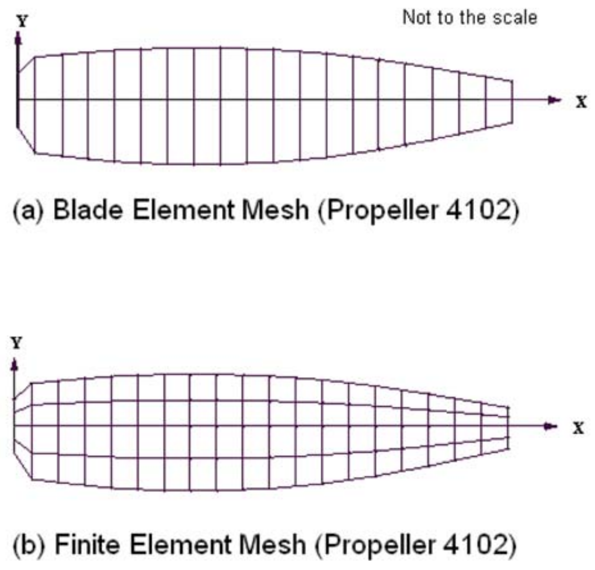

Fig. 1. Blade element and finite element mesh for propeller 4102

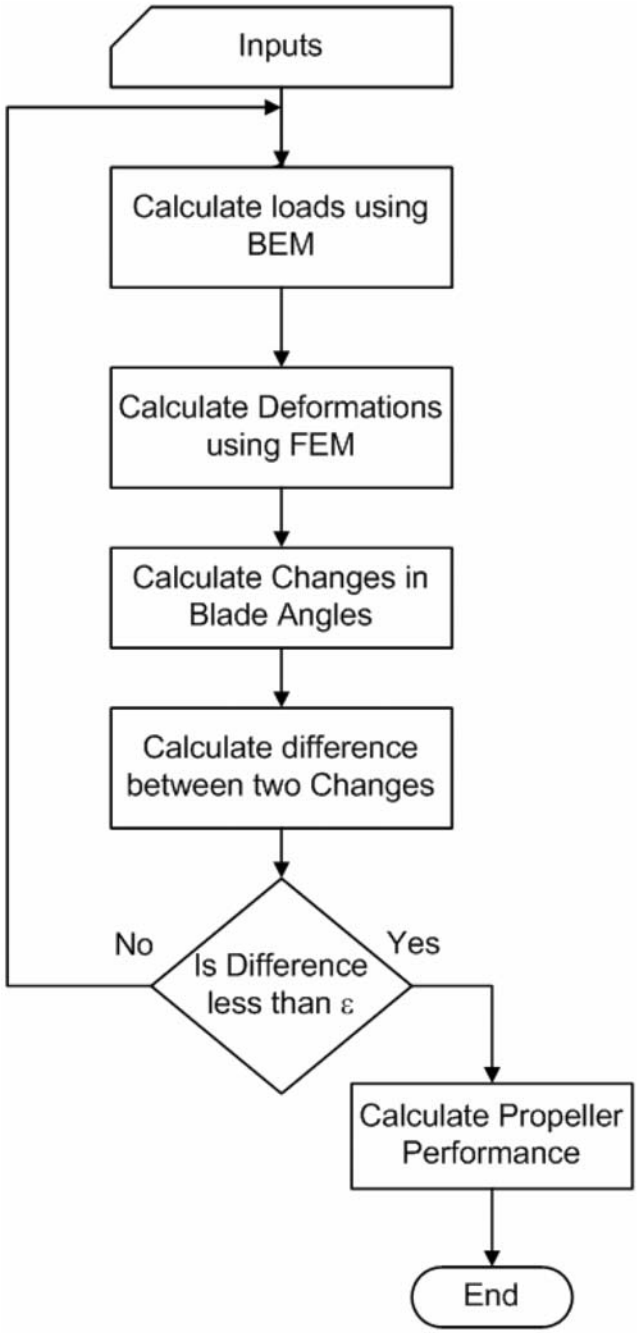

Fig. 2. Flow chart of algorithm for calculating propeller performance characteristics

Numerical Optimization

A typical optimization problem can be written as

minimize F(X) (objective function)

subject to:

$g_j\,(X) \leq 0$	j=1,…,m	(inequality constraints)
$h_k\,(X) = 0$	k=1,…,l	(equality constraints)
$X_i^l \leq X_i \leq X_i^u$	i=1,…,n	(side constraints)

where X are the design variables.

In structural optimization, a physical problem is converted into a mathematical problem in the above form and then optimization techniques are applied to find the optimal values of the design variables. Generally the optimization procedure is performed iteratively by starting from an initial set of design variables $\mathbf{X}^0$. This set is updated using the relation

$$X^p = X^{p1} + \alpha^*_p\, S^p, \tag{1}$$

where p is the iteration number, X is the vector of design variables, S^p is the search direction, and α^*_p is a scalar factor which scales the amount of change in X for the p^{th} iteration. Thus the optimization procedure consists of two steps; the determination of search direction S^p and the interpolation of the parameter α^*_p which minimizes F(X) in the direction of S^p. Since the objective function F(X) is in general a non-linear function, the gradients need to be reevaluated at X^p and a new set of design variables obtained. This process is repeated until a converged solution is obtained.

An optimization problem having only side constraints on the design variables is known as an unconstrained optimization problem. An optimization problem having constraints other than the side constraints on the design variables is known as a constrained optimization problem. In this type of optimization problem, the optimized solution must be within the bounds of the imposed constraints.

Propellers and Blade Element Theory

A propeller blade can be considered as a twisted wing having an airfoil shaped cross section. The angle β, referred to as the blade setting angle, is defined as the angle that the chord of a blade section makes with the plane of rotation for the propeller. As shown in Figure 3, β is greater for blade sections near the hub than for those near the blade tip. This variation in angle β along the axis of the propeller is called the β distribution. The resulting twist of the blade described by the β distribution is necessary to ensure that each blade section operates at a favorable angle of attack. In this investigation, different β distributions for a given propeller design from the literature are distinguished by the blade setting angle at a specific radial position. For example, $\beta_{0.75r}$ = 15° refers to the specific β distribution with an initial blade setting angle of 15° at 75% of the blade radius.

A propeller undergoes two general motions; it moves forward and it rotates about its axis of rotation. The forward velocity component V is common to all the sections of propeller blade. The rotational velocity component u, is proportional to the distance r from the propeller axis (Figure 3). Angle γ is defined as the angle between the plane of the rotation and the resultant velocity. Thus,

$$\tan\gamma = \frac{V}{u} = \frac{V}{r\omega} = \frac{V}{2\pi rn}, \tag{2}$$

where ω is the rotational speed in radians per second and n is measured in revolution per second. The angle γ decreases with increasing r. The angle of attack α for any blade section is given by

$$\alpha = \beta - \gamma. \tag{3}$$

The propeller advance ratio J is a non-dimensional term defined as

$$J = \frac{V}{nd}, \tag{4}$$

where n is the rotational speed and d is diameter of the propeller.

The propeller thrust T is the force in the direction of the propeller axis and is given by

$$T = \rho n^2 d^4 C_T, \tag{5}$$

where C_T is the thrust coefficient.

The propeller power P is the power required to drive the propeller and is given by

$$P = \rho n^3 d^5 C_P, \tag{6}$$

where P is the power that must be transmitted to the propeller to obtain the desired angular velocity. C_P is the coefficient of power of the propeller. On the other hand,

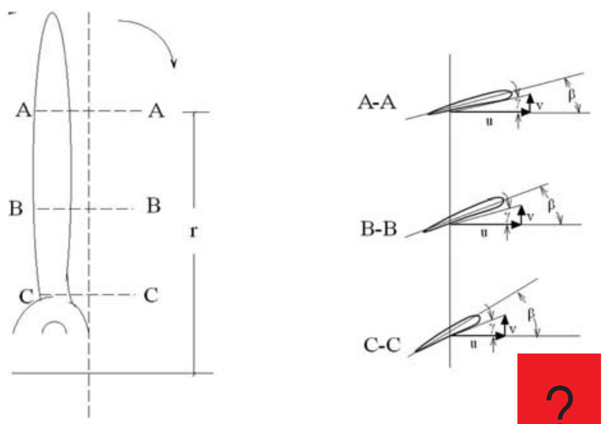

Fig. 3. A typical propeller blade and its cross sections

the product of thrust T and velocity V defines the power that is available for propulsion. Thus, propeller efficiency η can be defined as the ratio of the power output to the power input.

$$\eta = \frac{TV}{P} = \frac{TV}{2\pi nQ} \tag{7}$$

From equation 5 and 6,

$$\eta = \frac{\rho n^2 d^4 C_T V}{\rho n^3 d^5 C_P} = \frac{C_T}{C_P}.\frac{V}{nd} = \frac{C_T}{C_P} J \tag{8}$$

The coefficients of thrust and power of a given propeller design depend on the advance ratio J. The curves of C_T and C_P as a function of J are called propeller characteristics. A typical plot of propeller characteristics is shown in Figure 4.

Blade element theory is used to determine the propeller characteristics numerically where the propeller blade is divided into a number of elements. Two-dimensional aerodynamic theory is used to calculate the lift and drag forces on each element. These forces are then transformed into thrust and power for each element. Summing up the contribution from all the elements gives thrust and power for the propeller itself.

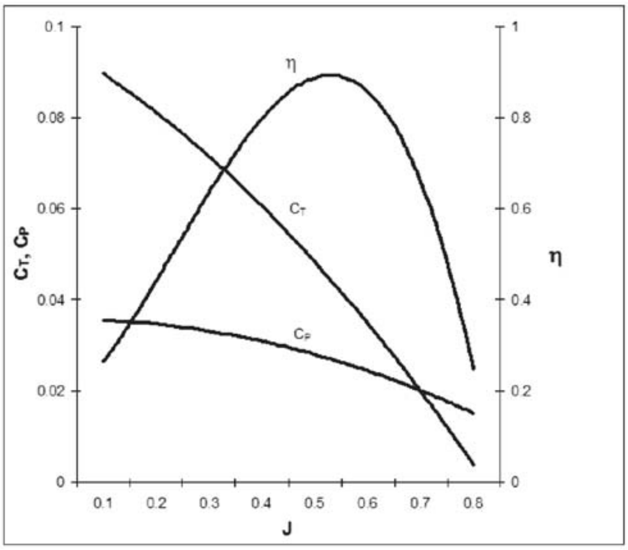

Fig. 4. A typical propeller characteristics plot

The Shear Deformation Theory of Composite Laminates

In the shear deformation theory, one of the assumptions of classical theory is dropped i.e. a straight line normal to the mid-plane does not have to remain normal after deformation. This means that the rotations φ_1 and φ_2 are no longer equal to $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.

The strain-displacement relationship for the shear deformation theory is given as follows;

$$\varepsilon_1^0 = \frac{\partial u}{\partial x}, \varepsilon_2^0 = \frac{\partial v}{\partial y}, \varepsilon_6^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \varepsilon_3^0 = 0, \varepsilon_4^0 = \frac{\partial w}{\partial y} + \varphi_2, \varepsilon_5^0 = \frac{\partial w}{\partial x} + \varphi_1 \tag{9}$$

$$\kappa_1 = \frac{\partial \varphi_1}{\partial x}, \kappa_2 = \frac{\partial \varphi_2}{\partial y}, \kappa_6 = \frac{\partial \varphi_1}{\partial y} + \frac{\partial \varphi_2}{\partial x}, \quad \kappa_3 = \kappa_4 = \kappa_5 = 0 \tag{10}$$

Equations of Motion

The equations of motion for shear deformation theory are given as

$$\frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0 \tag{11}$$

$$\frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} = 0 \tag{12}$$

$$\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + q = 0 \tag{13}$$

$$\frac{\partial M_1}{\partial x} + \frac{\partial M_6}{\partial y} - Q_1 = 0 \tag{14}$$

$$\frac{\partial M_6}{\partial x} + \frac{\partial M_2}{\partial y} - Q_2 = 0, \tag{15}$$

where N_i, Q_i, and M_i are given as

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \int_{-h/2}^{+h/2} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} dz \tag{16}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \int_{-h/2}^{+h/2} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} zdz \tag{17}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \int_{-h/2}^{+h/2} \begin{Bmatrix} \sigma_5 \\ \sigma_4 \end{Bmatrix} dz \cdot \tag{18}$$

Also note that N_i, M_i, and Q_i are functions of displacements and curvatures through the constitutive equations.

Finite Element Formulation

The finite element formulation is developed using the weak sense of weighted integral form. The displacements and rotations (u, v, w, ϕ_1, ϕ_2), are approximated as

$$u = \sum_{j=1}^{n} u_j \psi_j \quad v = \sum_{j=1}^{n} v_j \psi_j \quad w = \sum_{j=1}^{n} w_j^1 \psi_j$$

$$\varphi_1 = \sum_{j=1}^{n} w_j^2 \psi_j \quad \varphi_2 = \sum_{j=1}^{n} w_j^3 \psi_j \ , \tag{19}$$

where u_j, v_j, w_j^1 w_j^2 and w_j^3 are the nodal values of u, v, w, φ_1 φ_2 respectively at the *j*th node, and ψ_i are the interpolation functions or shape functions. These expressions are substituted back in the weak form of the differential equations, and the final finite element form of the governing equation is

$$\sum_{l=1}^{5}\sum_{j=1}^{n} K_{ij}^{kl}\Delta_j^l - F_i^k = 0 \quad , \qquad i = 1,2,...,n \quad k = 1,2,...,5 \tag{20}$$

where Δ_j^l are the nodal values of displacements and rotations. The terms K_{ij}^{kl} and F_i^k expressed in terms of displacements and rotations are provided in reference [12]. For more details of the application of the finite element method using the shear deformation theory of composite laminates, the reader is referred to [13] and [14]. In order to evaluate the expressions for K_{ij}^{kl}, a Gauss quadrature was used for the purpose of integration. For this work, nine-nodded quadrilateral elements were chosen.

Details of the Adopted Methodology

It has been discussed earlier that the ply orientation angles in the stacking sequence can strongly influence the propeller characteristics. Thus, a propeller can be designed for one or more specific purposes by changing the ply orientation angles and stacking sequence of the composite laminate. The task of designing the structure can be attacked by one of the two approaches. In the first approach, a preliminary design is considered as a starting point. This design is then improved by trial and error to reach a final design. This approach can be time and effort consuming and may not lead to the best design. The second approach focuses on the use of an optimization technique. In this investigation, the process of designing a composite propeller is set up as an optimization problem. The objective function and design variables are defined, constraints on various parameters are identified, and a complete optimization problem is solved to come up with a final design. For the purpose of searching for the optimized solution, the computational subroutine "CONMIN" [11] was coupling with an algorithm developed by the authors using finite element and blade element subroutines.

For the application of this algorithm and the optimization code, the blade of the propeller was divided into 19 blade elements. Each blade element was further divided into four finite elements. Figure 1 shows the finite element and blade element mesh. All of the initial values of the problem (propeller geometry, material properties, RPM, forwards velocity, etc.) were then identified and provided to the algorithm. Subsequently, all of the remaining required data for running the optimization code is identified as initial values of the optimization variables and is provided for the optimization. The outputs of a successful optimization run are the values of the objective function and the design variables for one particular value of advance ratio J.

Results and Discussion

Before the results of the optimization are presented, the effects of ply orientation angle on propeller characteristics are discussed. For this task, propeller 4102 was selected because the geometry and all required data were available. Additionally, this propeller geometry was used for design optimization runs. Propeller 4102 was modeled as a variable thickness plate consisting of six composite plies with a stacking sequence of [0/90/θ]s, where θ was varied from -90 degrees to +90 degrees in intervals of 7.5 degrees. The thickness values were taken from ref [10]. Thus a total of 24 computer runs were made, and for each run the laminate stacking sequence was changed. Performance characteristics of propeller 4102 (made from composite material) were calculated and recorded. The characteristics were plotted against θ for various values of β and J. Figures 5 - 7 show the behavior of C_T, C_P, and η as function of θ. Three different values of J were evaluated (J = 0.45, 0.5, and 0.55) as well as two different values of β (β = 15^o and 25^o). Here for brevity we have shown the plot for β = 15 degrees only. In Figure 5, the coefficient of thrust C_T is plotted against θ for three different values of advance ratio, J (J = 0.45, J = 0.5, and J = 0.55). For these three advance ratios, the values of C_T for the conventional metallic propeller are 0.041, 0.035, and 0.02 respectively. However, for the composite propeller, the value of C_T varies from 0.04 to 0.05 as a function of θ for J = 0.45. Similarly for J = 0.5, the value for C_T varies from 0.036 to 0.044; and for J = 0.55, the value of C_T varies from 0.03 to 0.0375. Similar behavior is observed for plots of C_P and η vs θ for various values of advance ratio J. These results show that ply orientation angle has a significant effect on propeller characteristics and propeller performance can be enhanced through the proper stacking sequence of the composite laminate. These results may also be used to identify the optimal values of ply angle θ that produce the best propeller performance characteristics.

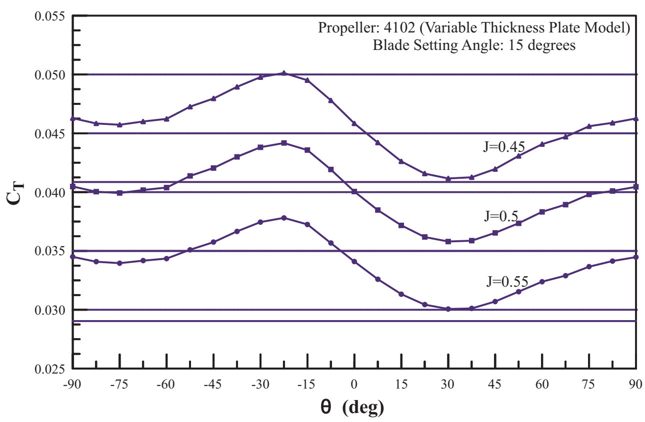

Fig. 5. Plot of CT versus (variable thickness plate model)

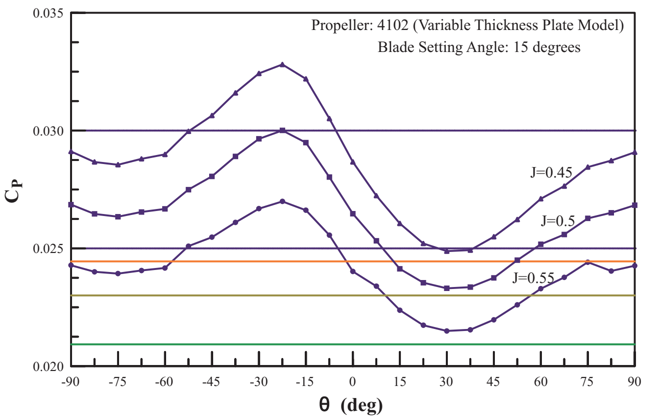

Fig. 6. Plot of CP versus (variable thickness plate model)

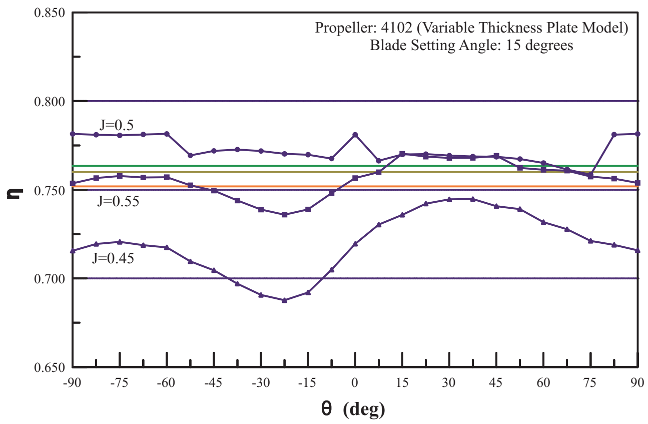

Fig. 7. Plot of propeller efficiency (variable thickness plate model)

Three different optimization analyses were performed for the composite propeller: (a) maximum C_T, (b) minimum C_P, and (c) maximum η. Note that none of the geometric or aerodynamic parameters of Propeller 4102 were changed, and thus all of the geometric dimensions and airfoil sections of the optimized composite propeller will be same as that of Propeller 4102. Initially the only design variable considered was the ply orientation angle θ of a $[0/90/\theta]_s$ laminate. Thus, the composite propeller under consideration consisted of six variable thickness plies, out of which the orientation of four plies (0 and 90 degrees) cannot be changed during the design optimization process. Thus, only one two-ply layer of the six ply laminate could be changed to achieve the design objectives, i.e. the number of design variables was one for this case. The design variable θ was bounded between -90 degrees to +90 degrees. This one variable optimization problem can be written as

maximize $C_T(\theta)$ or minimize $-C_T(\theta)$
subject to: $-90^0 \leq \theta \leq 90^0$

where C_T is an implicit function of θ. The problems of minimizing C_P and maximizing η can be posed in the same manner. The goal of each optimization problem was to identify the particular value of ply orientation angle θ which met the objective of the design.

Results for the three optimization problems are presented in Table 1. Additionally, the three parameters to be optimized, C_T, C_P, and η, are plotted versus the ply orientation angle θ in Figures 5-7, respectively. These optimization problems used an advance ratio of 0.8, and blade setting angle of 15 degrees. Results show that the value of θ for maximizing C_T is -22.9 degrees, for minimizing C_P the value of θ is 27 degrees, and for maximizing propeller efficiency η the value of θ is 26.8 degrees.

Table 1. Results of one variable optimization

Objective	Opt. θ (deg.)	$C_T/C_P/\eta$	initial $C_T/C_P/\eta$	% change
Maximize C_T	-22.9	0.06434	0.05233	22.95
Minimize C_P	27.0	0.05315	0.054	1.57
Maximize η	26.8	0.782	0.774	1.034

The next design optimization problem considered two design variables, θ_1 and θ_2, ply orientation angles for an eight-ply $[0/90/\theta_1/\theta_2]_s$ laminate. This problem can be formulated as

maximize $C_T(\theta_1, \theta_2)$ or minimize $-C_T(\theta_1, \theta_2)$
subject to:
$-90^0 \leq \theta_1 \leq 90^0$
$-90^0 \leq \theta_2 \leq 90^0$

where C_T is an implicit function of θ_1 and θ_2. This particular problem was solved for J=0.5 with $\beta_{0.75r}$ = 15 degrees, and J = 0.8 with $\beta_{0.75r}$ = 25 degrees. The results are presented in Table 2 (J = 0.5) and Table 3 (J = 0.8).

The results presented in Tables 2 and 3 show that the optimized composite laminate improves the efficiency of the propeller by 3% for J = 0.5 and 7% for J = 0.8 compared to the initial values of propeller 4102. Also, improvement of about 47 % for J = 0.5 and 32% for J = 0.8 is possible for coefficient of thrust C_T. Similarly for the case of minimizing C_P, the improvement was 14% for J = 0.5 and more than 23% for J = 0.8. It is also observed that the improvement is greater for C_T for J = 0.5 as compared to the J = 0.8 case. However, the reduction in C_P and improvement in propeller efficiency is observed to be greater for J = 0.8 as compared to J = 0.5. These results can be explained by observing the typical behavior of C_T, C_P, and η in Figure 4. The drop in the typical value of C_T is much more rapid as J increases compared to C_P and η. Note that J is the measure of forward velocity, and higher values of J imply higher angles of attack faced by the propeller. At very high angles of attack, the propeller airfoil sections are close to stalling and some of the sections near the root of the propeller have in fact stalled.

Conclusion

Both one-variable and two-variable design optimization was performed for a composite propeller. Results show that various design objectives can be achieved by changing the ply orientation angle within a composite laminate and without changing the geometric or aerodynamic properties

Table 2. Results of two variable optimization (J=0.5)

Objective	Optimum θ_1 (deg.)	Optimum θ_2 (deg.)	Optimum $C_T/C_P/\eta$	Initial $C_T/C_P/\eta$	% improvement
Max C_T	-31.1	-7.59	0.05136	0.035	46.75
Min C_P	30.0	22.96	0.01976	0.023	14.0
Max η	29.06	65.03	0.77	0.75	2.67

Table 3. Results of two variable optimization (J=0.8)

Objective	Optimum θ_1 (deg.)	Optimum θ_2 (deg.)	Optimum $C_T/C_P/\eta$	Initial $C_T/C_P/\eta$	% improvement
Max C_T	-57.54	-16.6	0.06714	0.05233	32.12
Min C_P	24.06	38.66	0.04122	0.054	23.66
Max η	25.06	41.58	0.827	0.774	6.84

of the propeller. The propeller was modeled as a variable thickness plate. The loads on the propeller blades were calculated using the Blade Element Method and resulting deformation was calculated using the Finite Element Method. Three design objectives were considered: maximizing the coefficient of thrust, minimizing the coefficient of power, and maximizing the propeller efficiency. Results show that improvements of up to 47% in coefficient of thrust are possible as compared to a metallic propeller. Similarly improvements in propeller efficiency and reduction in propeller power are possible.

List of Symbols

α	Angle of Attack
β	Blade Setting Angle
γ	Angle between plane of rotation and resultant velocity
J	Advance Ratio
n	Rotation Speed
ω	Angular Velocity
C_T	Coefficient of Thrust
C_P	Coefficient of Power
η	Blade Efficiency
ε^o_i	Mid-Plane Strains
κ_i	Mid-Plane Curvatures
u, v, w	Displacement components
ϕ_1, ϕ_2	Rotation about x, y axis
ψ_i	Interpolation functions

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