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Maximum Coverage Location Model for Rescue-15 Islamabad under Budgetary Conditions

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Abstract

In this study, we presented a Maximum Coverage Location (MCL) model for emergency services which are Rescue-15, Islamabad. We addressed this issue under fixed cost and allocation of different services of the emergency facility. The MCL model is calibrated in two Phases. Phase-I model marks the optimal facility sites that provide maximum coverage to all the demand sites under budgetary constraint. Phase-II model solves the allocation problems of the services to these facility sites that have been selected in Phase-I. Illustrative examples are given to show how the proposed model can be used to optimize the locations of emergency facilities of Rescue-15 Islamabad. We used General Algebraic Modeling System (GAMS) to solve these models.

Key Words: Facility location optimization, linear programming, Maximum coverage location, Rescue 15 Islamabad.

Introduction

In real life problems like an immediate risk to life, health, property, environment, medical emergencies, natural disaster and accidents etc., the world has to face emergencies. It is the necessity of life to deal all such emergencies. To deal extraordinary situations, the developed world has a number of emergency handling infrastructures.

Research work on facility location optimization problems is abundant. Many optimization models have been developed to formulate and solve various location problems. In these research works, models have been designed to cope the situations such as household fires or vehicle accidents, deployment of staff, equipment and logistics etc. For these frequent emergencies, many facility location problems have been investigated. These solutions, however, do not translate well into problems where a vast area has to be capture with a sufficient level of efficiency. The complete coverage to emergencies requires a modification in the definition of facility coverage to ensure an acceptable form of coverage of all demand areas.

Location theory has attracted significant research effort since the beginning of the 1960s. Facility location theory deals with a lot of mathematical models that are used to decide locations for facilities with maximum profit, minimum loss or minimum transportation cost. There is a lot of literature on emergency facility location models for large-scale emergencies and regular emergencies.

In location allocation theory, MCL problem searches for the maximum population which can be facilitated within a stated service time or distance, given a limited number of facilities. A general form of maximum coverage location model is discussed by Church and ReVelle [1]. They addressed the issue that a decision maker may focus on covering population within as much as possible desire service distance. The MCL problem and its different modifications or types have been widely used to solve emergency service location problems. Eaton et al. [2] plan the emergency medical service in Austin, Texas by using MCL problem. The generalized form of MCL model is given by Schilling et al. [3] to locate emergency fire-fighting servers and workshops in the city of Baltimore. Goldberg and Paz [4], ReVelle et al. [5] and Beraldi and Ruszczynski [6] studied stochastic and probabilistic characteristics of an emergency situation in emergency service covering models to capture the uncertainty and complexity of these problems. Daskin [7] formulate the Maximum Expected Covering Location Problem (MEXCLP) to allocate P facilities on a network with the aim to maximize the expected value of coverage location. A modified version of MEXCLP given by Bianchi and Church (MOFLEET) [8], ReVelle and Hogan (MALP) [9], Batta et al.(AMEXCLP) [10], Goldberg et al. [11] and Repede and Bernardo (TIMEXCLP) [12] to solve Emergency Medical Service (EMS) location problems. A review and summary about the chance constrained emergency service location model is given by ReVelle [13]. Schilling [14] proposed a MCL problem based on scenarios to maximize the covered location. Jarvis [15] developed a location model and descriptive model on the basis of the hypercube model. On the basis of results from queuing theory, Marianov and ReVelle [16] proposed a realistic location model for the emergency system. Some other theoretical and practical application of queuing models has also been reported by Barman and Larson [17], Batta [18], and Burwell et al. [19]. The generalization of maximum coverage models is presented by Revelle and Hogan [9], Revelle and Hogan [20] and Marianov and Revelle [16].

In the present study, we determine optimal rescue services locations to address the needs generated by vast coverage of emergencies. A facility location models are used to place any type of service in certain locations, from fire stations to triage areas. In this article, we discussed the problem of locating facility services geographically and then what services should be offered on these particular selected locations. In general, our facility location optimization problem is a two phase problem to decide the number and locations in the first stage and what services should be placed in a particular location in the second stage.

Moreover, due to potential impact of emergency events, this model considers

- an appropriate strategy for the facility deployment (facility location selection),
- the number of facilities assigned to each demand point (facility quantity) and
- the maximum distance up to which a facility should service a demand point (service quality).

The objective is to facilitate regardless of the solution. Therefore, care should be taken in prioritizing one solution over another. The other important aspect of this optimization exercise is that the resources in the nearby areas can be pooled to address a specific high need.

This model achieves these objectives:

i. To improve the present coverage model for regular emergency services under the restrictions of full coverage, budget and distance.

ii. Reconstruct the maximum coverage model for the purpose to use for those facilities that provides more than one services.

The article unfolds in 4 main parts. Section 2 analyzes the characteristics of proposed maximum coverage location model for regular emergency services. In Section 3, we present the formulation of proposed model. In Section 4, examples illustrate the use of the proposed model for regular emergency services. Conclusions and future research work discussed in Section 5.

Characteristics of Proposed Maximum Coverage Model

In this section, we analyze the unique characteristics that are inherent in the proposed maximum coverage location model.

Full Coverage Model: In this study, we try to solve the issues of coverage by taking help from the maximum coverage location model under partial coverage proposed by Karasakal and Karasakal [21]. The main problem in using the idea of partial coverage is that the maximum numbers of demand sites are partially covered. But if we use a single point as a critical distance instead of range, then we can achieve the task of full coverage and all the demand sites can fully cover. This situation is explained in Figure.1.

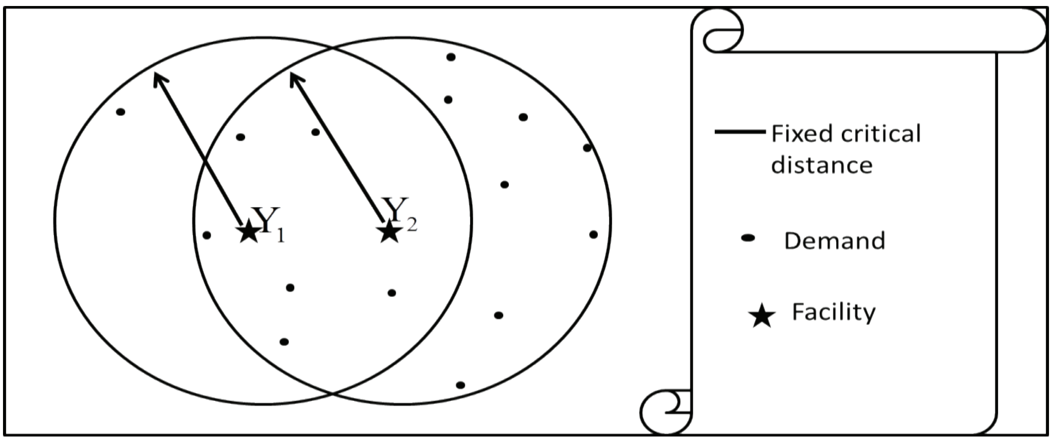

Fig-1: A possible situation for MCL problem under full coverage.

In Figure 1, the location Y1 can cover 7 demand points and location Y2 can cover 14 demand points within the full coverage range. Thus, a solution of MCL problem would choose location Y2 as the maximal coverage location.

Coverage Level: The purpose of this study is to maximise the coverage to all the demand sites under full coverage range. Thus, a solution of MCL problem would choose location Y2 as the maximal coverage location.

Fixed Cost or Budget: There are different coverage models that have been formulated under the concept of budgeted maximum coverage and a lot of research has been conducted related to this concept [22] and [23]. Similarly, we have also tried to solve this problem by using a budgetary constraint for a maximum coverage location model.

Services Allocation: In this proposed study, a maximum coverage location model has been developed for location and allocation of emergency services.

Optimization solver: In this study, to solve our MCL problem, we have used the optimization solver General Algebraic Modeling System (GAMS) and found the results of proposed MCL models by using the real data.

Formulation of Proposed Model

Phase-I (Selection of Facility Sites):

In Phase-I we select those facility sites for the emergency facility from the set of all potential facility sites that provide coverage to all the demand sites under two conditions, fixed budget and full coverage. The data flow diagram for the process of Phase-I MCL problem is given in Figure 2.

Decision variables: The variables y_j (Facility location) and x_{ij} (Facility coverage) are the decision variables of Phase-I model of MCL problem and define as:

$$y_j = \begin{cases} 1 & \text{if the facility located at } j \text{ site,} \\ 0 & \text{otherwise,} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if the facility at } j \text{ fully covere the demand point } i, \\ 0 & \text{otherwise.} \end{cases}$$

Where,

$i \in I$ is the Index set of all demand points

$j \in J$ is The Index set for all potential facility sites

Objective function: It maximizes the overall coverage level and defined as:

$$\text{Max Z} = \sum_{i \in I} \sum_{j \in M_i} c_{ij} x_{ij} \qquad (1)$$

Where,

M_i is the set of facilities site that are fully covered the demand point i.

C_{ij} is a level of coverage that provided to the demand point i by facility j.

The presented C_{ij} (coverage level) can be defined as:

$$C_{ij} = \begin{cases} 1 & \text{if } d_{ij} \le D, \\ 0 & \text{if } d_{ij} > D. \end{cases}$$

Where,

d_{ij} is the travel distance between demand point i and facility j.

D is the maximum full coverage distance

Number of facilities constraint: This constraint confirms that the total number of sited facilities equals where, is defined as:

$$\sum_{j \in J} y_j = K \tag{2}$$

Coverage constraint:

$$x_{ij} \le y_j \qquad \forall i \in I, j \in M_i \tag{3}$$

It requires that we have limited x_{ij} with respect to the sited facilities i.e. if facility j is sited then demand point i is facilitated by that facility and if that is not then facility j will not be provide any coverage, so all x_{ij}'s related to the facility j will be required to become '0'.

Capacity Constraint:

$$\sum_{j \in M_i} x_{ij} \le 1 \qquad \forall i \in I \tag{4}$$

This constraint ensures that demand point covered by at most one of the facility locations that have been sited. If there are more than one facilities sited that can provide coverage to the demand point then only that facility would be selected which give maximum coverage.

Maximum coverage constraint:

$$\sum_{j \in M_i} x_{ij} y_j = 1 \qquad \forall i \in I \tag{5}$$

This constraint restricts the facility sites to cover all the demand sites. A demand site must be cover by one of the facility sites.

Budget constraint:

$$\sum_{i\in I}\sum_{j\in M_i} b_{ij}x_{ij} \le B \tag{6}$$

Where,

B is the limit of the budget for the whole project.

b_{ij} is the total cost to cover the demand point i by facility j.

This constraint requires the limit on the expenses that the sited facilities that provide coverage having cost under the limited budget B.

Binary restriction constraints:

$$y_j \in \{0,1\} \qquad \forall j \in J \tag{7}$$

This constraint imposes a binary restriction on the decision variable 'Facility location'.

$$x_{ij} \in \{0,1\} \quad \forall i \in I, j \in M_i \tag{8}$$

This constraint imposes a binary restriction on the decision variable 'Facility coverage'.

Phase-II (Allocation of Emergency Services)

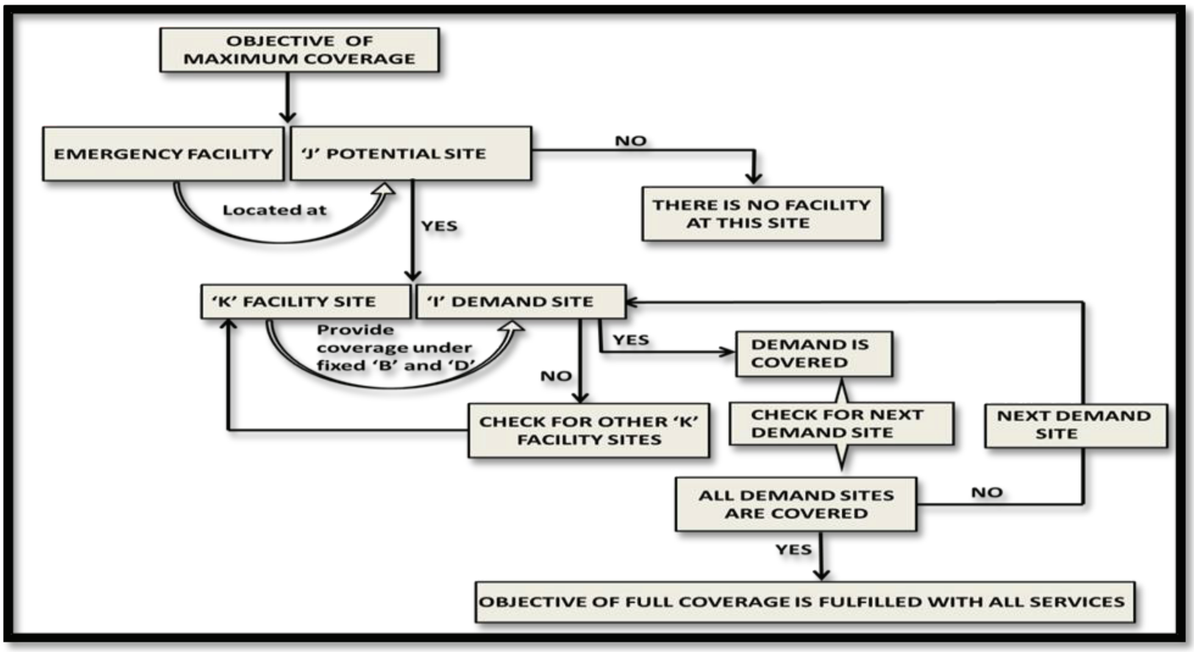

Fig-2: DFD for Phase-I model of MCL problem

The selection of facility sites has been done by using Phase-I model. Phase-II model decides about the allocation of services to the selected sites of Phase-I Model in such a way that these services provide maximum coverage under fixed budget. In Phase-II model, one more subscript "$h \in H$" introduced to denote the services index that could be coupled with facility location j. The explanation of the variables and constraints in Phase-II is different from the Phase-I model because here our concentration is on the allocation of services at selected facility sites. The data flow diagram for Phase-II model of MCL problem is given in figure 3

Decision variables: The variables y_{jh} and x_{ijh} are the decision variables of Phase-II model of MCL problem, which are defined below;

$$y_{jh} = \begin{cases} 1 & \text{if the } h \text{ service is allocate} \\ & \text{to sited facility } j, \\ 0 & \text{otherwise,} \end{cases}$$

Where,

$$x_{ijh} = \begin{cases} 1 & \text{if the } h \text{ service of sited} \\ & \text{facility } j \text{ fully covered} \\ & \text{the demand point } i, \\ 0 & \text{otherwise.} \end{cases}$$

$i \in I$ is the set of all demand points.

$j \in J$ is the set of all selected facility site.

$h \in H$ is the set of all services that would be provided by selected facility site.

Objective function: The objective function maximizes the overall coverage level of different services at a time.

$$\text{Max } Z = \sum_{i \in I} \sum_{j \in M_i} \sum_{h \in H} C_{ijh} x_{ijh} \qquad (1)$$

Where,

M_i is the set of facilities site that are fully covered the demand point i.

C_{ijh} is the coverage level that provided to the demand point i by h service of sited facility j.

The variable C_{ijh} is the function off d_{ijh} because the coverage level of facilities locations to each demand points depends on the travel distance from sited facility j having h service to demand point i. C_{ijh} is defined

$$C_{ijh} = \begin{cases} 1 & \text{if } d_{ijh} \le D_h, \\ 0 & \text{if } d_{ijh} > D_h. \end{cases}$$

Number of facilities constraint:

$$\sum_{j\in J} y_{jh} = K_h, \qquad \forall h \in H \tag{2}$$

Where,

K_h is the no. of sited facilities that provide h service.

This constraint confirms that the total number of sited facilities that provide h service is equal to K_h.

Coverage constraint:

$$x_{ijh} \le y_{jh} \qquad \forall i \in I,\, j \in M_i,\, h \in H \tag{3}$$

It requires that we have limited x_{ijh} with respect to the sited facilities with different services. That is if sited facility j provide h service then demand point i is facilitated by that facility otherwise facility j with h service will not be provide any coverage, so all x_{ijh}'s related to the facility j will be required to become '0'.

Capacity constraint:

$$\sum_{j\in M_i} x_{ijh} \le 1 \qquad \forall i \in I,\, h \in H \tag{4}$$

This constraint ensures that a demand point covered by at most one of the facility locations with h service, which have been sited. If there are more than one sited facilities that can provide coverage to the demand point with h service then only that facility would be selected whose service give maximum coverage. This condition is forced by the objective function that the selection of the facility having maximum coverage.

Maximum coverage constraint:

$$\sum_{j\in M_i} x_{ijh} y_{jh} = 1 \qquad \forall i \in I,\, h \in H \tag{5}$$

This constraint ensures that h service at j sited facility, must provide coverage to all the demand sites.

Budget constraint:

$$\sum_{i \in I} \sum_{j \in M_i} b_{ijh} x_{ijh} \leq B_h \qquad \forall h \in H \qquad (6)$$

Where,

b_{ijh} is the total cost to cover the demand point i by h service of sited facility j.

B_h is the limit of budget for h service.

This constraint requires the limit on the expenses that the h service of sited facility that provide coverage having cost under the limited budget.

Binary restriction constraints:

$$y_{jh} \in \{0,1\} \qquad \forall j \in J,\, h \in H \qquad (7)$$

This constraint imposes a binary restriction on the decision variables 'Service allocation'.

$$x_{ijh} \in \{0,1\} \qquad \forall i \in I,\ j \in M_i,\, h \in H \qquad (8)$$

This constraint imposes binary a restriction on the decision variable 'Service coverage'.

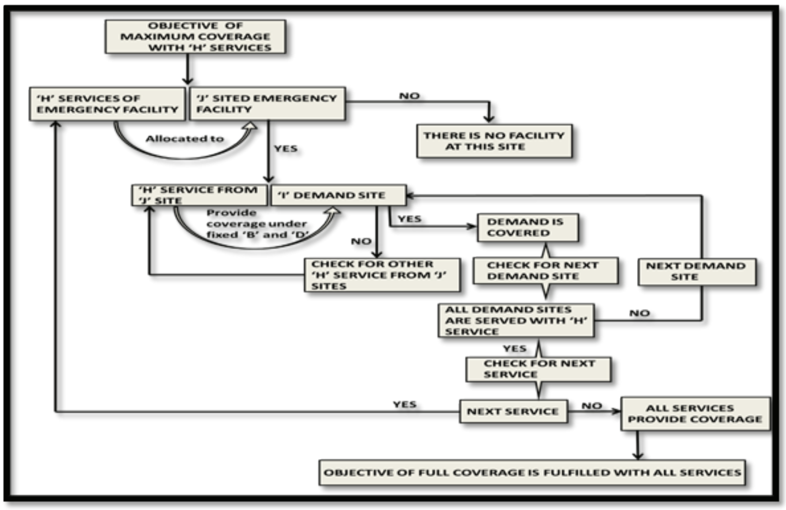

Figure 3: DFD for Phase-II model of MCL problem

Application

The proposed models can be used to optimize the facility locations for regular emergencies. The maximum coverage model is used to formulate the location problems for emergency services. An organization have different demand and facility sites where each demand point is required to be covered by at most single facility site (based on the weights of the demand points) in order to receive adequate emergency supplies.

The organization considers the area of Islamabad (the capital of Pakistan) to locate facilities. The area is divided into square zones using the center of each zone as an aggregated demand point. This example considers twenty-two demand points (zones). Furthermore, a number of eligible sites in which the emergency facilities might be placed are identified in red color in Figure.4. The potential facility sites are taken according to the conditions and requirements of distance, locations and population. We used the data on three variables. These are distance in kilometers between each demand site and facility site (in Phase-I, in Phase-II), coverage level for each facility site to cover a demand site (in Phase-I, in Phase-II) and the budget in million for each facility site to provide coverage to the demand site (in Phase-I, in Phase-II). The illustrative emergency service in this study is Rescue-15. It provides different services in the area of Islamabad to provide help in emergencies. The main 7 services of Rescue-15 are Help line (HL), Ambulance coordination (AC), Emergency help (EH), Theft reporting counter (TRC), Vehicle verification center (VVC), Security alarm system (SAS) and Child lost/found system (CL). The main office of Rescue-15 is in the G-8 .

Results under Phase-I model

We run Phase-I model to determine the facility locations for Rescue-15, under fixed budget for fixed distance. The locations for emergency facility are decided by the value of decision variable y_j (facility location) and demand coverage by the value of decision variable x_{ij} (facility coverage). To provide the input parameters and variables for the model, the data on distance (d_{ij}) and budget (b_{ij})are collected from Rescue-15. The limits of budget and distance for the given example are B=190 million rupees and D=6 km. Based on this information the locations and coverage for Rescue-15 in the area of Islamabad are shown in Figur.5

The optimum solution of MCLP for this problem is:

Model Status: Optimal with Objective Value = 22

This value can be explained by the table 1.

The objective value of the proposed MCL model of Phase-I is 22, which is an optimal value because the total number of demand sites are 22. It gives that, all the demand sites are covered under fixed budget for fixed distance.

Results obtained from the model suggest the selection of sites E-9,F-7, F-8, F-10, G-11, H-9 and I-11 as the facility locations. Table.1 and Figure.5 shows that E-9 provides coverage to E-8, E-9 and F-9. Similarly, we can see the result for other facility sites.

The required limit of budget and coverage distance is the minimum threshold under which the task of full coverage to all the demand areas is achieved. The coverage level will decrease as well as the threshold level of budget is decrease. Therefore, the two possible conditions are either increase budget threshold with the constraint of full coverage to all the demand areas or to decrease budget threshold without such constraint. The level of coverage with the different threshold of the budget is given below.

Results of Phase-II model

The objective of proposed Phase-II model of MCL problem is to allocate the services of the facility at selected sites, where a selection of sites has been done in Phase-1the model. All services have a different budget and distance threshold to cover a demand point and a different number of facility sites that provide these services.

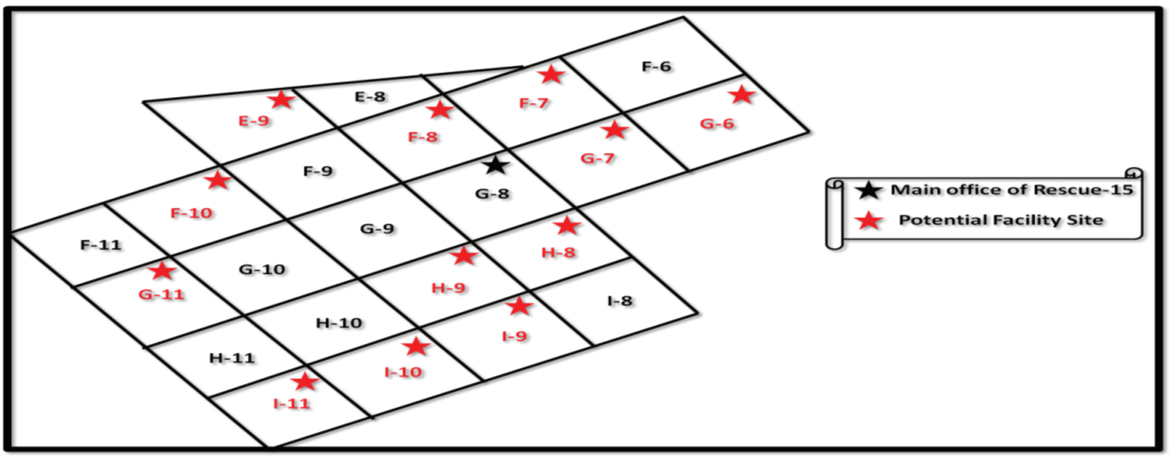

Fig-4: Sectors of Islamabad

Table 1: Facility Coverage

Facility site	E-9	F-7	F-8	F-10	G-11	H-9	I-11	Total
Demand site	3	4	3	2	3	5	2	22

Table 2: Limits of the budget, distance and no. of facility sites for each service.

Services	HL	AC	EH	TRC	VVC	SAS	CL
Budget (million)	20	40	30	10	15	20	15
Distance (km)	7	6	5	9	10	7	9
No. of Sites	6	7	7	3	3	5	4

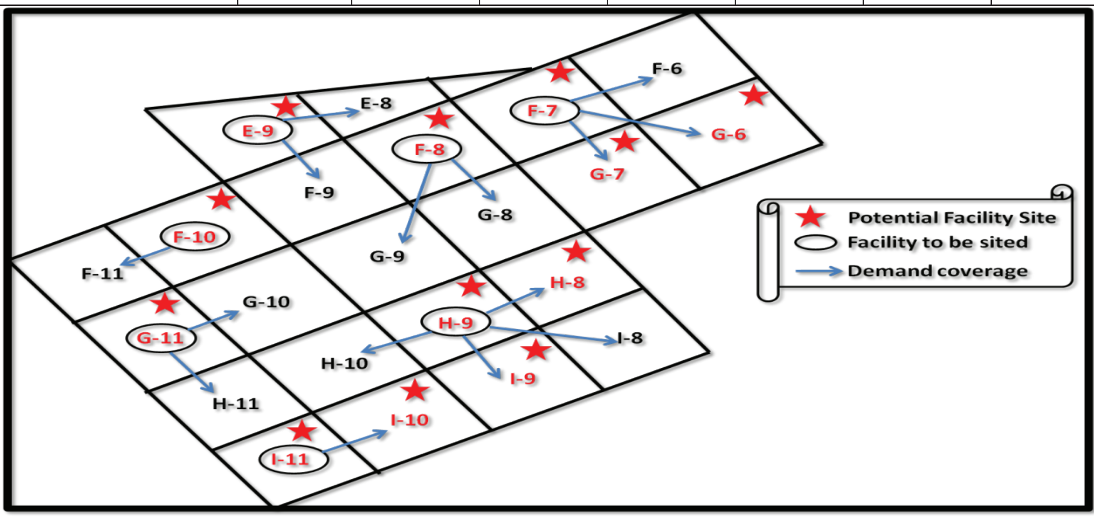

Fig-5: Facility location and coverage.

For input parameters and variables of Phase-II model, we collect the data on budget ( ), distance ( ) and coverage level ( ).According to the information of location, the population of required area, the limits of budget (B_h), distance (D_h) and a number of sites (K_h) for each service are given in table 2.

Based on all this information the results of Phase-II model of MCLP are given as:

The solution is optimal with value 154. Where 154 is the maximum coverage point (22demand sites * 7services=154). The mission of full covering to all the demand sites for each service is accomplished and all services are provided to each demand site. A service can be allocated to fixed number of facility sites under the limits of distance and budget. The decision variable 'Service allocation' (y_{jh}) decides the allocation of services of Rescue-15 to selected sites of Phase-I model of MCLP. The allocation of these services under the above limits, to the selected 7 sites of Phase-I model is shown in figure 7.

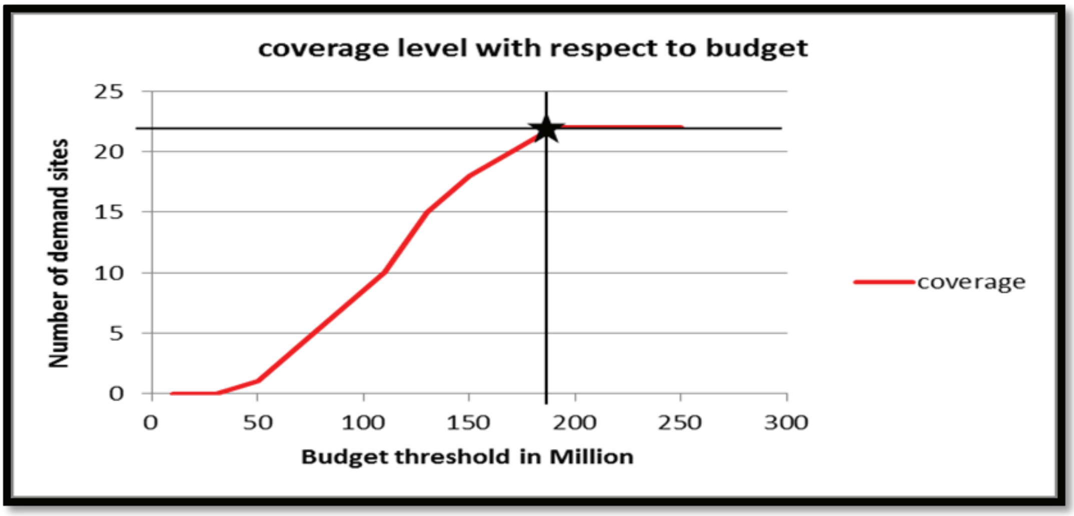

Fig-6: The coverage level with respect to the budget threshold.

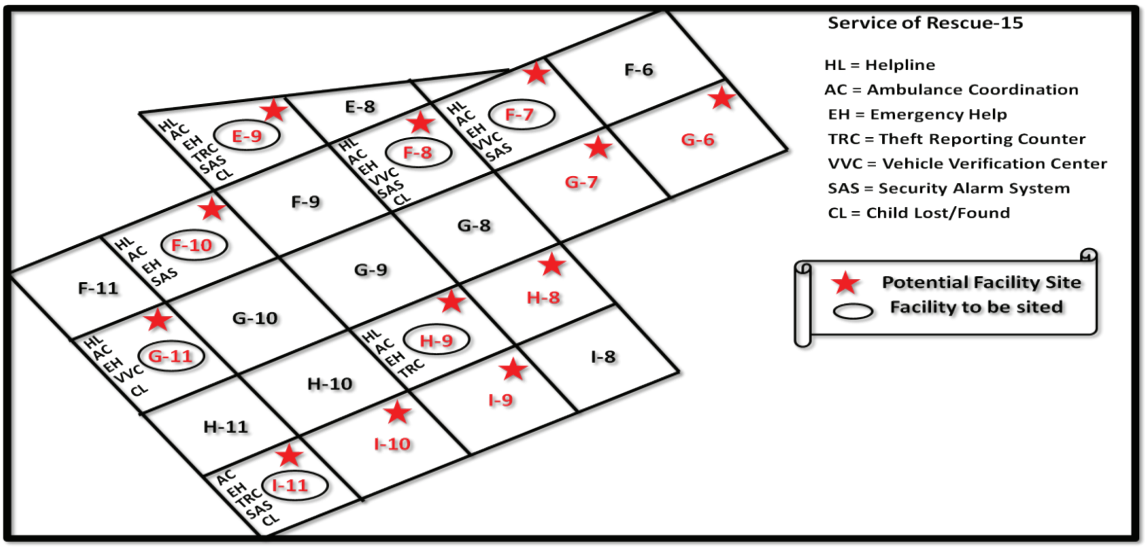

Fig-7: Services allocation

The figure 8 shows that Helpline (HL) service can be provided at 6 facility locations, Ambulance Coordination (AC) at all facility locations and so on.

The results of decision variable 'Service coverage' (x_{ijh}) decides the coverage of all demand sites by facility sites having different services. For example, the coverage of Helpline service can examine in figure 8.

The above figure shows that the Helpline (HL) service at the selected 6 facility sites provided coverage to all the demand areas. Similarly, each service location decided by phase-II model provides coverage to their nearest demand sites. The nearest demand site is one which is possible to cover while satisfying their limits of budget, distance and capacity of coverage.

Conclusion

- The minimum threshold of the budget is 190 million for stated problem of Rescue-15 (Islamabad) to cover all the twenty-two demand sites for satisfying the full coverage constraint.

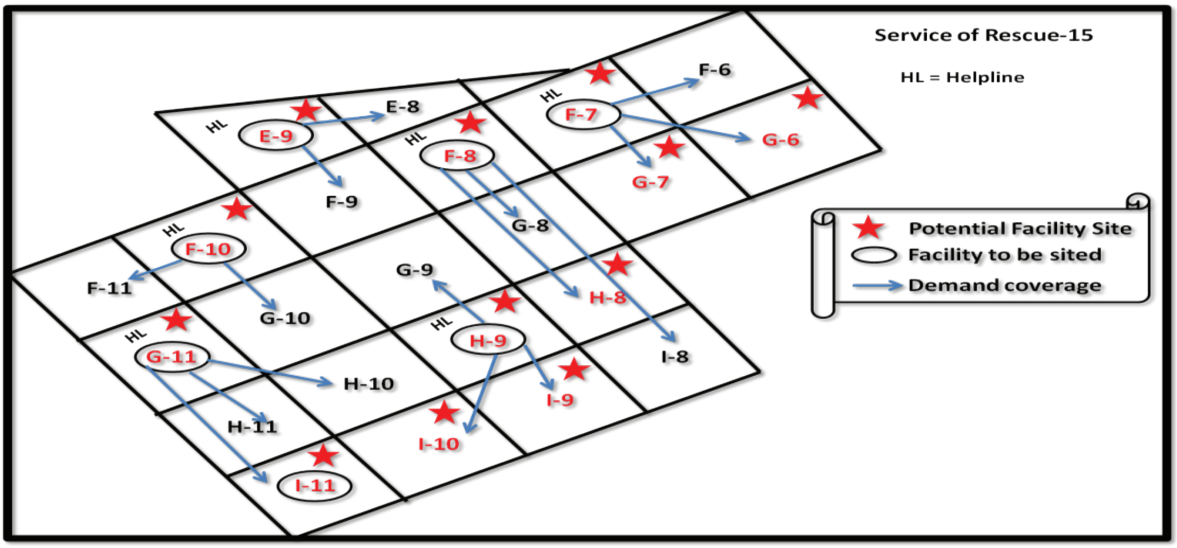

Figure 8: Helpline service coverage.

In general we conclude that the task of full coverage can be achieved under the desirable limits of budget and distance by using the proposed two phase model of MCL problem.

- If the purpose is to decrease the threshold of budget then we have to give relaxation to the full coverage constraint. In that case, the proposed model provides coverage to only those demand sites that are feasible under the limited budget.
- In this study, the allocation of different services of Rescue-15 is also carried out, so it is concluded that we can solve the location problems for those emergency facilities that provide different services, with the help of proposed Phase-II model of MCL problem.
- Computational results show that incorporating the full coverage has a substantial effect on the solution of the MCL problem and all the demand sites get coverage.

Future Prospective

- Several extensions to our method of budgetary restriction can be further investigated in the context of regular emergency facility location problems.
- The MCL problem can make more reliable by using time variable with the restrictions of budget and distance.
- The most efficient formulation can be obtained by using the expected form of maximum covering location model (MEXCLP) and its modified version.
- The interchange models i.e. *P*-median and *P*-center etc. ([24]) can reconstruct under our proposed structure for large-scale emergencies.
- Services allocation can be done by using different facility location model, to solve the different problems of emergency services.
- The task of services allocation can also be achieved under partial coverage.

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