Numerical Modelling of Carbon Fibre Reinforced Polymer Composites for Hole Size Effect

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Abstract:

Carbon Fibre Reinforced Polymer (CFRP) composites are widely used in several high performance structures such as aeroplanes, automobiles and wind turbines. In these applications holes are made for access to view, cut-outs for weight reduction as well as joining of composite members. However, these hole possess a threat to the strength and damage tolerance of composites in service. The strength of composites varies with hole diameter known as hole size effect. The hole size effect, becomes more complex once associated with specimen size effect (strength variation with specimen size) as well as anisotropy and heterogeneity of composite material. The leading influencing factors linked with the hole size effect are in-plane and transverse plane thickness, stacking sequence and hole diameter of the specimen. Extensive experimental, analytical and Finite Element (FE) based studies have been done by the researchers in past on this account. Despite these studies, differences still persist among composite researchers on the extent of these influencing factors on hole size effect.

The current paper presents numerous FE models to investigate the stresses and stress concentrations influenced by the diameter to width ratios of rectangular plate subjected to axial loading. FE models are developed both for isotropic and anisotropic/orthotropic materials. FE models for anisotropic/orthotropic materials have been developed using different lamina stacking configurations to investigate their effect on stresses and stress concentrations in parallel with hole size effect. The effect of laminas orientations in case of anisotropic/orthotropic materials with global (reference or loading axes) orientations has also been investigated through coordinate transformation technique. The effect on the use of laminate effective material properties for designing of engineering structures is also evaluated. The comparison deduce significant considerations for designing of the composite materials containing central circular holes subjected to axial loadings.

Keywords-finite element method, rectangular plate, circular hole, stress concentration factor

1. Introduction

Applications of isotropic and orthotropic/anisotropic (fibre reinforced polymer composite) materials in the form of plates with circular holes have extensively been found in various engineering fields like automobile, aerospace, marine and mechanical structures[1, 2]. The presence of holes presents geometric discontinuities where crack initiation or propagation may starts in engineering structures upon loading[3]. Consequently, due to these geometric discontinuities high stresses and stress concentrations produced near the hole boundary[4]. For designing of such engineering structures precise knowledge and estimation of stresses and stress concentrations is needed[5, 6]. Evaluation of stress concentration factor (SCF) through analytical and experimental approaches is generally time consuming and challenging. FE modelling presents an alternative approach to deal with these challenges.

The top of the considerations for the adoption of numerical based investigations is that the FE method possess advantage to deal with complex geometric problems over the analytical and experimental approaches. Secondly, the two dimensional FE modelling is easier to develop and behaviour of the structure under loading can realistically be represented yet equally applicable to large variety of practical applications. Moreover, reliable information about the structural deformation, distribution of stresses and stress concentrations is achievable. The study focused on numerical based approach to estimate the SCF around central circular hole. Emphasis of the study is on the hole size effect with regards to SCF for a thin rectangular plate made up of isotropic (steel) and anisotropic/orthotropic (CFRP) composite materials[7].

The influence of material and laminas stacking sequences has already been investigated by various researchers [2, 8, 9]. The current study aimed at analysis of the effect of SCF upon diameter to width (d/w) ratios for isotropic (steel), orthotropic/anisotropic (CFRP) composite material plates. The effect of laminas material orientation and the transformed orientation with reference or loading orientation on the stresses and stress concentrations has also been investigated. The value of the SCF generally depends on the material, laminas stacking orientation for the case of fibre reinforced polymer composites, geometric parameters such as diameter to width (d/w) ratios in case of the central circular hole and the applied loads. Analysis has been performed for symmetric conditions for CFRP composite laminates. The variations of stresses and stress concentrations with respect to diameter to width (d/w) ratios are presented graphically for better visualization.

2. Theory

Any structure subjected to uniaxial loading experiences a stress generally known as normal stress (or gross stress)[10]. Whereas the stress is defined as the intensity of force per unit area as expressed in equation-1.

$$\boldsymbol{\sigma} = \frac{\boldsymbol{P}}{\boldsymbol{A}} = \frac{\boldsymbol{P}}{\boldsymbol{w * t}} \qquad \textbf{\textit{equ - 1}}$$

$\boldsymbol{\sigma}$ = stress (Pa) **w** = width of the member (mm) **P** = applied load (N) **t** = thickness of the member (mm) **A** = cross sectional area (mm^2)

However, when the structure possess discontinuity in the form of holes, notches or even abrupt changes in cross sections, results into high localized stresses near the region of discontinuity as shown in figure-1[11]. Figure shows that the high stresses are passing through the centre of the central circular hole of the rectangular plate specimen.

Figure-1: *stress distribution near centre circular hole of plate under tensile loading*

To determine geometric discontinuity (central circular hole in this case) the net stress value would be changed due to change in net unit area for which the expression is given in equation-2. The net cross-sectional area in this case is width minus diameter of the hole multiplied by the thickness of the rectangular plate.

$$\boldsymbol{\sigma_{net}} = \frac{\boldsymbol{F}}{\boldsymbol{(w-d)\,t}} \qquad \textbf{\textit{equ - 2}}$$

Where, d = diameter of the hole (mm)

For any material, the general state of stress at a point is represented by nine stress components as shown in figure-2[12].

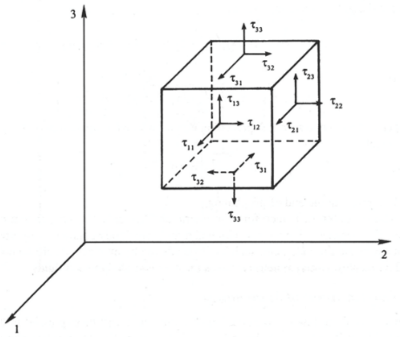


Figure-2: *stress state at a point*

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \qquad \textbf{\textit{equ - 3}}$$

However, for the case of anisotropic/orthotropic materials the stress state may differ due to lamina orientation or global orientation. In real scenario, the structural applications of composite materials are in the form of plates subjected to axial loadings. Thus materials behaviour is orthotropic and lamina is considered to be under plane stress condition considering all out of plane stresses as zero. Thus the in-plane stress state along the principle material axes is expressed as given in equation-4.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_6 \end{bmatrix} \textbf{\textit{equ – 4}}$$

Where

Q_{11}, Q_{22}, Q_{12} *and* Q_{66} are reduced stiffness's, and their values may be obtained by following expressions:

$$Q_{11} = \frac{E_1}{1-\mu_{12}\ \mu_{21}} \qquad \textbf{\textit{equ - 5}}$$

$$Q_{22} = \frac{E_2}{1-\mu_{12}\ \mu_{21}} \qquad \textbf{\textit{equ - 6}}$$

$$Q_{12} = Q_{21} = \frac{\mu_{21}E_1}{1-\mu_{12}\ \mu_{21}} = \frac{\mu_{12}\ E_2}{1-\mu_{12}\ \mu_{21}}$$
equ - 7

$$Q_{66} = G_{12} \qquad \textbf{\textit{equ – 8}}$$

Where

$\boldsymbol{E} = young's\ modulus$ $\boldsymbol{\gamma} = shear\ strain$

$\boldsymbol{\tau} = shear\ stress$ $\boldsymbol{G} = shear\ modulus$

Generally in practical situations the laminas principle axes does not coincide with reference plane or the loading axes. In this condition the stress or even the strain components in line with the principle axes are to be transformed in line with the loading axes by using expression given in equqtion-9.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_s \end{bmatrix} \qquad \textbf{\textit{equ – 9}}$$

Whereas the transformation matrix [T] is represented as given in equation-10. However, for laminate case each lamina will be having different state of stress.

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \textbf{\textit{equ – 10}}$$

Where

m = cos θ

n = sin θ

2.1 Stress Concentration Factor (SCF)

Various researchers including Inglis[13] defined SCF (Kt) as the ratio of maximum tensile stress to the average tensile stress expressed in equation-11 below.

$$\boldsymbol{K_t} = \frac{\sigma_{max}}{\sigma_{ave}} \qquad \textbf{\textit{equ - 11}}$$

The SCFs considered for the current study are evaluated by two distinct methods. The first method used for the SCF depends on the grass stress value as expressed below in equation-12.

$$\boldsymbol{K_{tgross}} = \frac{\sigma_{max}}{\sigma_{gross}} \qquad \textbf{\textit{equ - 12}}$$

Whereas $\boldsymbol{\sigma_{max}}$ is the maximum tensile stress at the boundary of the hole and $\boldsymbol{\sigma_{gross}}$ is the applied stress at the edge (far field stress) of the rectangular plate as shown in figure-3.

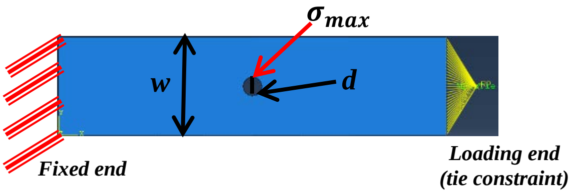


Figure-3: *base line specimen geometry*

The second method used for the SCF depends on the net stress value as expressed in equation-13 below.

$$K_{tnet} = \frac{\sigma_{max}}{\sigma_{net}} \qquad \textbf{\textit{equ - 13}}$$

Whereas the net stress is based on the reduction in cross sectional area due to the increase in size of the hole (diameter).

3. Model Description

Several FE models were modelled using base features shell and continuum shell for a rectangular plate containing central circular hole as shown in figure-4. The length, width and thickness of the rectangular plate (baseline specimen) is 128 mm, 32 mm and 2 mm respectively. For CFRP composite material models, 16 laminas are used having 0.125 mm thickness of each individual lamina to achieve overall 2 mm thickness of the rectangular plate. The size's (diameters) of hole considered to investigate the hole size effect are 1, 2, 4, 6, 8, 10, 12 and 14 mm. All the specimens are subjected to a constant displacement rate of 1 mm/min (1.6667 E^{-6} m/s).

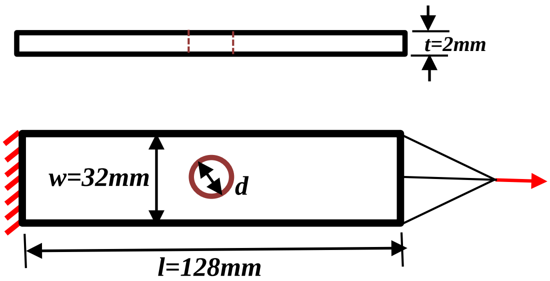


Figure-4: *dimensions of base line specimen*

Due to symmetry of the rectangular plate one fourth of the model is considered for the estimation of SCF as shown in figure-5, because it will save the computational time and other computer resources.

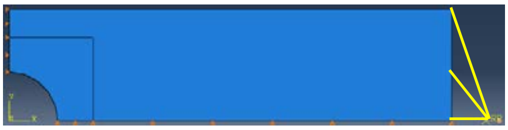


Figure-5: *quarter model with symmetric boundary conditions at the left and bottom and tie constraint with dummy node*

3.1 Mesh Convergence

Authenticity of FE values generally depends on the meshing level of the model. Extremely fine meshing of the entire part results into time consuming and bourdon on computational resources. Therefore, only required portion of the model geometry is fine meshed where FE values deemed necessary, rest of the model geometry is left with course mesh. For current models partition scheme provided in Abaqus is used to create different mesh schemes. Highly fine mesh is created using Quad, Free and Medial element shape, technique and algorithm respectively around the hole as shown in figure-6 & 7 and course mesh for rest of the parts geometry. Mesh convergence analysis is carried out using global element size against maximum tensile stress values as shown in figure-8 for shell elements of steel material. Also the mesh convergence analysis is performed using smallest element size verses SCF for shell elements of steel material as shown in figure-9.

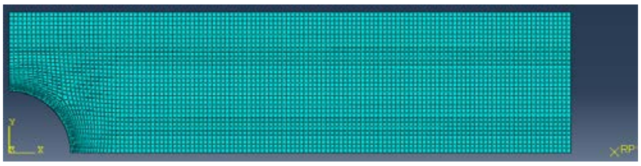


Figure-6: *Quad mesh of the model*

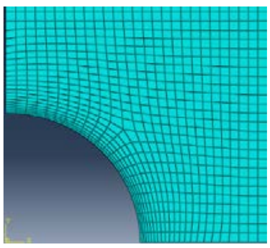


Figure-7: *fine Quad mesh near the hole boundary*

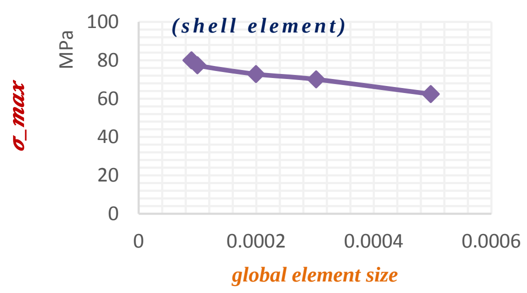


Figure-8: *mesh convergence using global element size for maximum tensile stress*

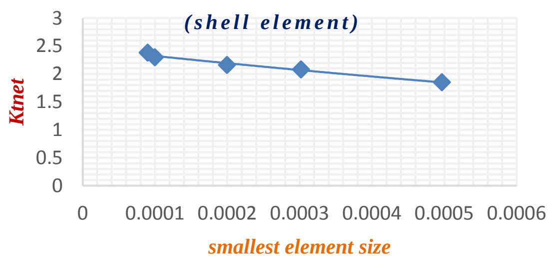


Figure-9: *mesh convergence using smallest element size*

3.2 Materials and Methods

Numerical models are made for both isotropic (metal) and orthotropic/anisotropic (fibre reinforced polymer composite) materials. Isotropic material selected for the analysis is steel. The material properties used for the analysis are as E = 209 GPa (modulus of elasticity) and $\mu = 0.3$ (poisson's ratio). For the case of anisotropic/orthotropic material, the CFRP composite (IM7/8552) material is selected for the analysis. The material properties of IM7/8552 are given in table-1.

ρ	E_{11}	E_{22}	μ_{12}	G_{12}	G_{13}	G_{23}
1610Kg/m^3	161GPa	11.4 GPa	0.32	5.17 GPa	5.17 GPa	3.98 GPa

Table-1: *material properties of CFRP composite (IM7/8552) material*

The analysis for the case of CFRP composite material is performed by using different lamina stacking configurations such as unidirectional (UD), cross-ply, angle-ply and quasi-isotropic.

Analyses are performed in Abaqus/Standard by using commercial software Abaqus/CAE 6.13-1. One end of the rectangular plate is fixed by using boundary condition ENCASTRE (U1=U2=U3=UR1=UR2=UR3=0). The opposite end of the rectangular plate is loaded with a constant displacement rate of 1 mm/min (1.6667 E^{-6} m/s) through a Dummy Node.

The values of maximum tensile stresses and in certain cases values of transverse stresses are obtained through FE mesh. Values of reaction forces are also obtained using unique nodal point for the evaluation of SCFs. For the case of CFRP composite materials the average maximum stress value is calculated by averaging the individual laminas maximum stress values of the laminate through FE mesh. The maximum stress values both for the laminas (material) and loading axes orientations are obtained through transformation method available in Abaqus. The values of stresses and stress concentrations are also evaluated using effective laminate properties. The effective laminate properties were obtained by using online software computer aided design environment for composites (CADEC).

4. Results

4.1 Isotropic Materials

The maximum tensile stress valuses are obtained by FE mesh at the boundary of the hole. Similarly the reaction force value is obtained by using unique nodal point at the dummy node through FE mesh. Based on these values and the net cross sectional area of the rectangular plate on loading region, the SCF is calculated. The SCFs for the different hole sizes obtained from numerical models are given in table-2. A steep increase is observed in stress concentration from hole diameter 1 mm to 2 mm and then nominal decrease till hole diameter 14 mm as also shown in figure-10. However for maximum tensile stress, an exponential increase is found from hole diameter 1 mm to 2 mm and then gradual increase in tensile stress value is seen. Whereas sudden decrease in transverse stress value is found from 1 mm to 2 mm hole diameter, then very nominal decrease in transverse stress values is observed as shown in figure-11.

d (mm)	σ_{max}	RF	A_{net}	σ_{net}	Kt
1	1.02E+08	1.75E+03	3.10E-05	5.64E+07	1.808
2	1.40E+08	1.74E+03	3.00E-05	5.82E+07	2.408
4	1.51E+08	1.73E+03	2.80E-05	6.19E+07	2.447
6	1.59E+08	1.71E+03	2.60E-05	6.58E+07	2.417
8	1.65E+08	1.68E+03	2.40E-05	7.01E+07	2.356
10	1.70E+08	1.64E+03	2.20E-05	7.46E+07	2.277
12	1.77E+08	1.59E+03	2.00E-05	7.96E+07	2.222
14	1.85E+08	1.53E+03	1.80E-05	8.50E+07	2.172

Table-2: *hole size effect on SCF for steel*

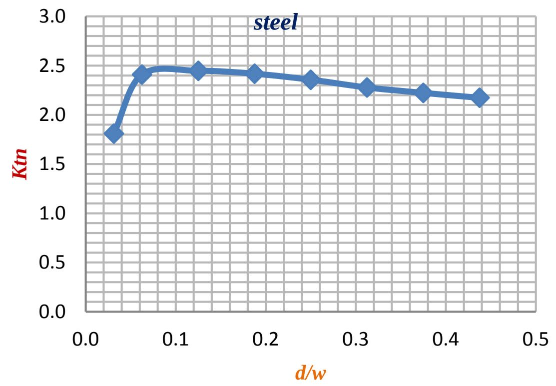


Figure-10: *d/w ratio effect on SCFs for steel material*

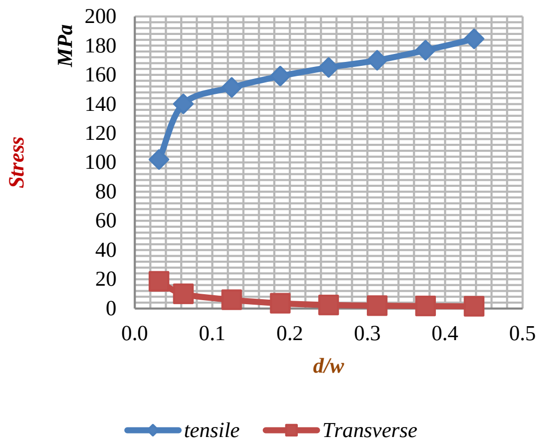


Figure-11: *d/w ratio effect on tensile & transverse stresses for steel material*

4.2 Anisotropic/orthotropic material

4.2.1Unidirectional Configuration

In this stacking configuration all the plies are layed at 0°. the layup sequence is $[0^0]_{16s}$. The SCFs for varying hole sizes for UD stacking sequence of CFRP composite material obtained from the numerical models are given in table-3. The effect on the tensile and transverse stresses with varying d/w ratios is also shown in figure-12. The variation in SCF against varying d/w ratios is shown in figure-13.

D (mm)	σ_{max}	RF	A_{net}	σ_{net}	Kt
1	6.52E+07	1.34E+03	3.10E-05	4.32E+07	1.509
2	1.26E+08	1.33E+03	3.00E-05	4.45E+07	2.827
4	1.69E+08	1.31E+03	2.80E-05	4.69E+07	3.611
6	1.99E+08	1.28E+03	2.60E-05	4.92E+07	4.045
8	2.15E+08	1.23E+03	2.40E-05	5.13E+07	4.203
10	2.20E+08	1.17E+03	2.20E-05	5.33E+07	4.132
12	2.22E+08	1.10E+03	2.00E-05	5.52E+07	4.021
14	2.22E+08	1.03E+03	1.80E-05	5.72E+07	3.878

Table-3: *hole size effect on SCF for UD CFRP material*

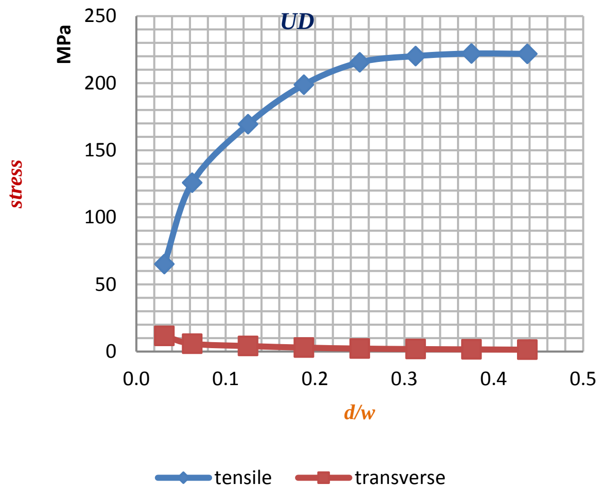


Figure-12: *d/w ratio effect on tensile & transverse stresses for UD CFRP material*

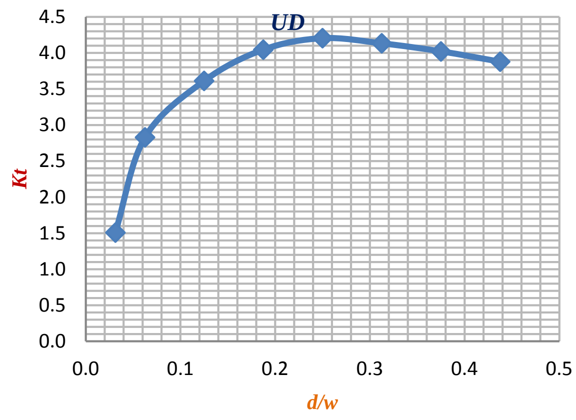


Figure-13: *d/w ratio effect on SCF for UD CFRP material*

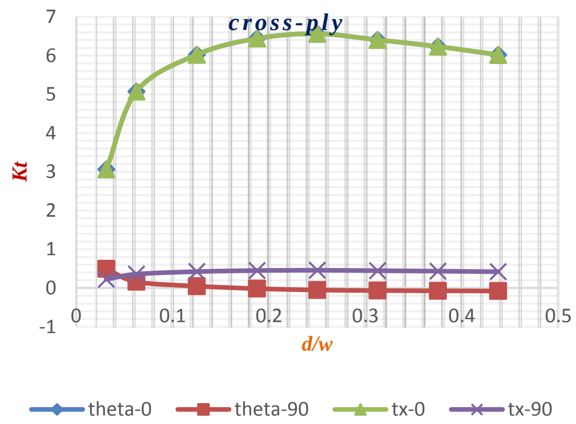


Figure-14: *d/w ratio effect on SCF for cross-ply CFRP against material and global orientations*

4.2.2 Cross-ply Configuration

The layup sequence for cross-ply configuration is $[0^0 / 90^0]_{8S}$. The SCFs for a 4 mm diameter hole using CFRP composite material obtained from the FE model are given in table-4. Three values of SCFs represents maximum for 0^0 lamina, minimum for 90^0 lamina and the third value is an aggregate value for both maximum and minimum values of 0^0 and 90^0 laminae. The overall effect on the SCFs both with respect to lamina material orientation (theta) and transformed orientation (tx) in loading direction against varying d/w ratios are shown in figure-14.

layup	σ_{max}	RF	A_{net}	σ_{net}	Kt
0	1.53E+08	7.11E+02	2.80E-05	2.54E+07	6.023
90	1.08E+07	7.11E+02	2.80E-05	2.54E+07	0.425
avg	8.18E+07	7.11E+02	2.80E-05	2.54E+07	3.224

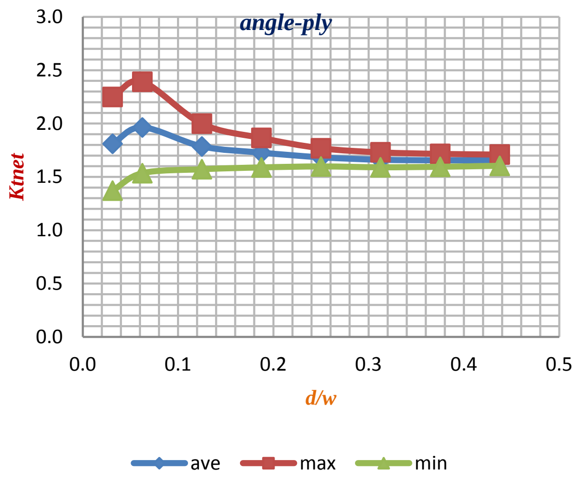
Table-4: *hole size effect on SCF on each lamina of 4 mm diameter hole for cross-ply CFRP material*

4.2.3 Angle-ply Configuration

The layup sequence of angle-ply configuration is $[45^0 / -45^0]_{8S}$. The SCFs for a hole size of 4 mm for this configuration using CFRP composite material obtained from the FE model is given in table-5. The 4 mm diameter hole size values are shown being mid size of the hole. Two values of the SCFs pertains to 45^o lamina and -45^o lamina and the third value is an average value for both 45^o and -45^o laminas. The overall effect on the SCF upon varying d/w ratios is shown in figure-15. Figure shows that with the increase of the hole size both values of 45^0 and -45^0 laminas come closer to the aggregate value of SCF and this occurs because of the shearing dominance.

layup	σ_{max}	RF	A_{net}	σ_{net}	Kt
45	9.07E+06	1.62E+02	2.80E-05	5.77E+06	1.572
-45	1.15E+07	1.62E+02	2.80E-05	5.77E+06	1.999
avg	1.03E+07	1.62E+02	2.80E-05	5.77E+06	1.785

Table-5: *hole size effect on SCF on each lamina of 4 mm diameter hole for angle-ply CFRP material*



Graph-15: *d/w ratio effect on SCF for cross-ply CFRP material*

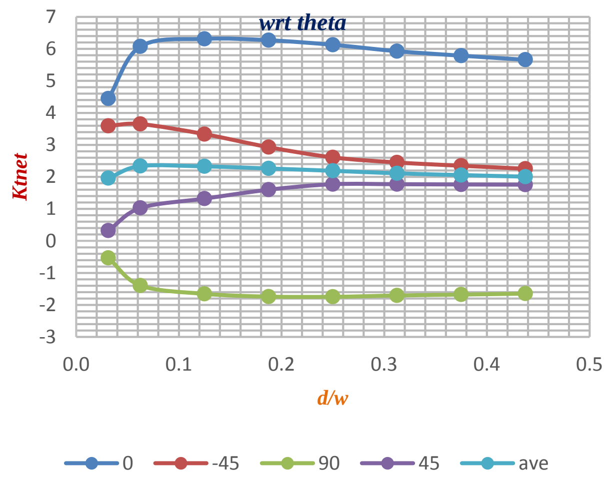


Figure-16: *d/w ratio effect on SCF for quasi-isotropic CFRP against material orientation*

4.2.4 Quasi-isotropic Configuration

The SCFs for a hole size of 4 mm diameter of quasi-isotropic stacking configuration of CFRP composite material obtained from the numerical models are given in table-6. The layup sequence of quasi-isotropic configuration is [45/90/-45/0]4s. The overall effect on the SCF against varying d/w ratios is shown in figure-16 and 17 for material orientation and global orientaion respectively.

layup	σ_{max}	*RF*	A_{net}	σ_{net}	*Kt*
0	1.15E+08	5.11E+02	2.80E-05	1.83E+07	6.312
-45	3.67E+07	5.11E+02	2.80E-05	1.83E+07	2.009
90	7.47E+06	5.11E+02	2.80E-05	1.83E+07	0.409
45	1.92E+07	5.11E+02	2.80E-05	1.83E+07	1.052
avg	4.47E+07	5.11E+02	2.80E-05	1.83E+07	2.445

Table-6: *hole size effect on SCF on each lamina of 4 mm diameter hole for quasi-isotropic CFRP material*

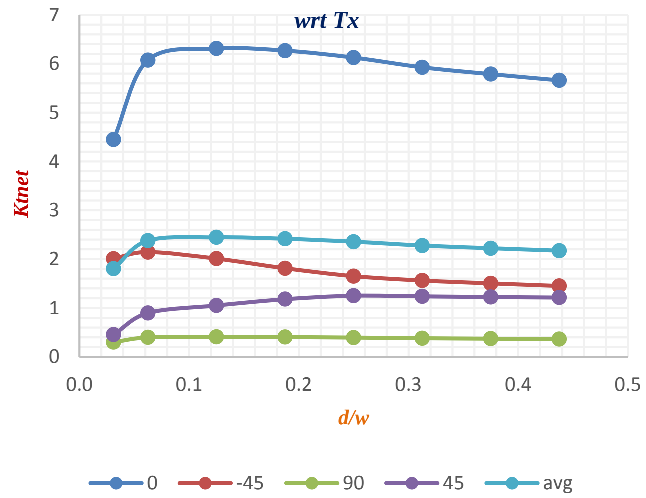


Figure-17: *d/w ration effect on SCF for quasi-isotropic CFRP against global orientation*

5. Discussion:

The combined representation for the effect of tensile stresses with varying hole sizes of all the stacking configurations of CFRP composite material and steel material is shown in figure-18. The highest value of maximum tensile stress is found for UD configuration and lowest for the case of angle-ply configuration. This indicates that the stress values mainly depends on the number of 0^0 laminas orientation. The more number laminas in 0^0 sequence the more will be the tensile stress value, however the transverse strength would be compromized in this case. Same result may be viewed on the effect of tensile and transverse stresses obtained against material orientation for quasi-isotropic configuration shown in figure-19 and 20 repectively.

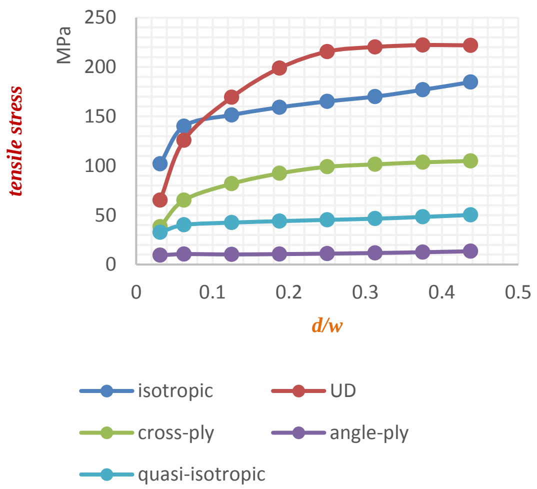


Figure-18: *d/w ratio effect on tensile stresses for different layup configurations of CFRP composite and steel materials*

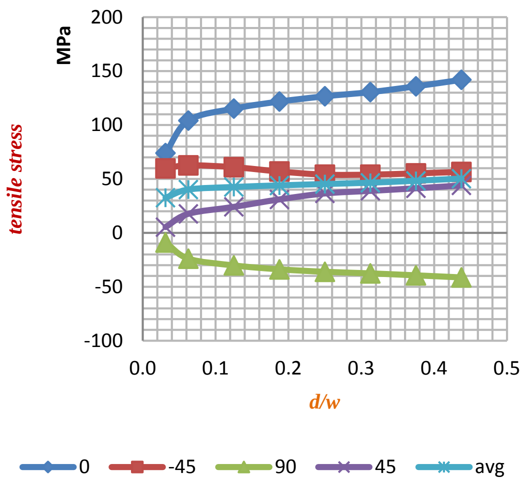


Figure-19: *d/w ratio effect on tensile stresses in material orientation for CFRP composite material of quasi-isotropic configuation*

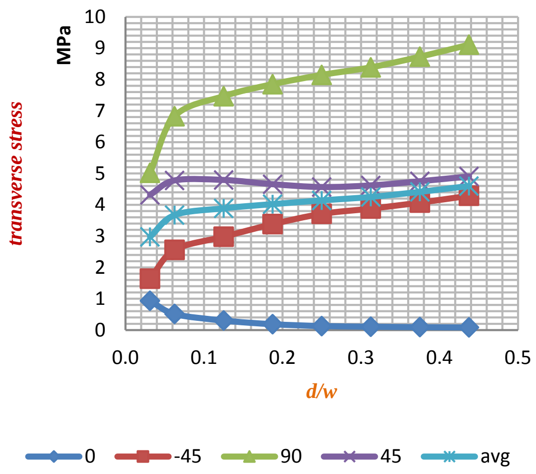
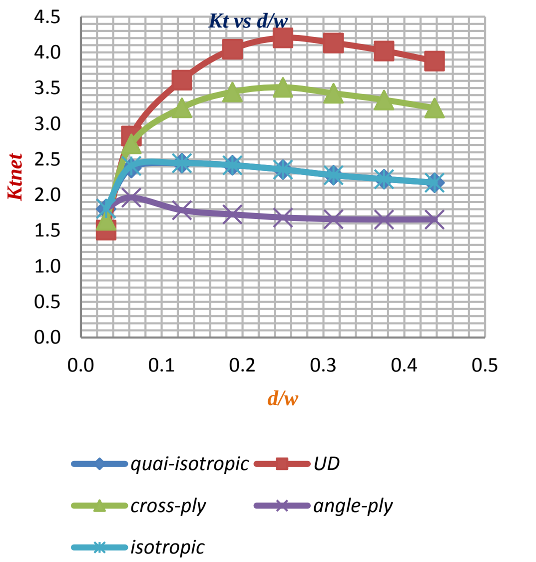


Figure-20: *d/w ratio effect on transverse stresses in material orientation for CFRP composite material of quasi-isotropic configuation*

Similarly the SCF found highest for the case of UD confoguration and lowest for the case of angle-ply configuration as shown in figure-21.



Graph-21: *d/w ratio effect on SCF for different layup configurations of CFRP composite and steel materials*

It is also observed that the SCF starts decreasing upon d/w ratio of 0.25 onwards for the case of UD and cross-ply configurations. It means that the size or width of the two tensile load bearing strips termed as “ligaments” decreased. In this case case bending toward’s the hole centre cause compresive flexural component that will reduce the maximum tensile stress, the larger the hole size the greater would be the influence of maximum tensile stress. Resultantly, the central circular hole would present the shape of an eliptical hole as shown in figure-22. This inward deflection of the ligament produces flexural stress which supress the value of tensile stress leads to the decrease in SCF.

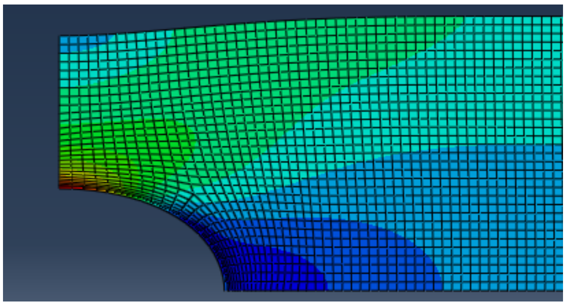


Figure-22: *deflection of the ligament towards centre of the hole*

Furthermore, laminate FE modelling is performed by using effective material properties of the laminate. For all the cases, the values of SCF obtained through effective laminate properties found same as obtained an average values of SCF through previous numerical models. It is important to note that the use of stress values or stress concentrations values obtained by effective laminate material properties or an average value of all the laminas for the design of composite structures would be misleading because the failure may start from the weakest link. Therefore designing of fibre reinforced polymer composite materials for engineering structures requires that the values of stresses and stress concentrations for all laminas should be estimated also keeping in view the first ply failure concept.

6. Conclusion

Fibre reinforced polymer composite materials are extensively used in various engineering structures comprising holes. These holes pose discontinuties due to which high stresses and stress concentrations produced near the hole boundary. The strenght of the engineering structure is the combined effect of material properties, stacking sequences, geometry of the structure and upon the applied load. For designing of engineering structures true estimation of stresses and SCFs are essential. The study shows that lamina orientations are extremely important to take into account for structural designes instead of using effective laminate material properties. Further studies are required for safe safety margins needed for the engineering structures based on stresses and stress concentrations. Also further work is needed for true estimation of the stress intensity factors due to abrupt geometric changes like holes and notches.

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