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TARGETED SHOWERING OPTIMIZATION: TRAINING IRRIGATION TOOLS TO SOLVE CROP PLANNING PROBLEMS

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Optimization can play an important role in supporting agricultural community not only in designing and manufacturing mechanical equipment but also in optimal crop planning. The related optimization models are not necessarily linear due to varying resources and complex environmental processes. The traditional linear programming techniques may not be practical in such situations. Metaheuristics are powerful approaches to solve complex nonlinear models. Metaheuristics are developed by transforming dynamics of natural phenomena to artificial intelligence computational environment. Realizing the potential adaptability of working principles of irrigation tools, this paper develops a novel optimization algorithm called Targeted Showering Optimization (TSO) algorithm which aims to solve linear, nonlinear and multi-objective optimization problems arising in agriculture, engineering and other scientific areas. In the present work, the design of TSO algorithm has been elaborated in detail and is followed by the performance evaluation of TSO algorithm by applying it to six well-known benchmark functions. The obtained results reveal that the developed method finds the best quality solutions of at least four benchmark functions in just 100 iterations and in additional 100 iterations it supersedes other nature inspired algorithms. To show the applicability of the proposed method in agriculture, a case study regarding the model of optimal crop rotation in Slovenian organic farming has been solved by TSO. The results of optimization models of crop rotation produced by TSO are also promising and provide a clear trade-off between total income and the nitrogen off-take when the maximization of total income and minimization of nitrogen off-take are dealt simultaneously.
Keywords: Artificial showering, nature inspired algorithms, linear and nonlinear programming, optimal crop rotation, shadow price

INTRODUCTION

Decision making in crop planning is a crucial step in enhancing the profitability taking in to account multiple conflicting objectives under various unavoidable restrictions like input cost, water scarcity, water pollution, climate variation and the limited space and time. The design and employment of advanced and efficient decision making techniques to manage and benefit the entire agriculture system is one of the greatest challenges. The objectives of achieving highest possible profit in crop planning by consuming smallest possible resources and meeting all the restrictions can be modeled as nonlinear optimization problems. One of the greatest challenges in resulting nonlinear optimization models is the multimodality due to which classical methods get entrapped in local optimal solutions, and hence affect the reliability and worth of the obtained solution. Metaheuristic optimization algorithms are modern methods that can be used to overcome such disadvantages. Metaheuristics are mostly designed by imitating natural phenomena and are getting an increasing popularity.

Inspiration from natural phenomena has become an effective tool in designing so called meta-heuristic approaches to solve complex real world optimization problems. In search of the best optimizer, several nature inspired algorithms have been proposed. For example, invasive weed optimization (IWO) algorithm (Mehrabian and Lucas, 2006), water cycle algorithm (WCA) (Eskandar *et al.,* 2012), water wave optimization (WWO) algorithm (Zheng, 2015), differential evolution (DE) (Storn and Price, 1997), particle swarm optimization (PSO) algorithm (Eberhart and Kennedy, 1995), grey wolf optimizer (GWO) (Mirjalili *et al.,* 2014), artificial bee colony (ABC) algorithm (Karaboga and Aklay, 2011), firefly algorithm (Yang, 2010), teaching learning based optimization (TLBO) algorithm (Rao *et al.*, 2011), artificial showering algorithm (ASHA) (Ali *et al.*, 2015) and etc.

Over the last two decades, metaheuristics have been successfully applied to various real optimization problems, e.g., neural network training (Mirjalili *et al.*, 2012; Sheihan and Rad, 2013), mechanical engineering (Massinaei *et al.*, 2013; Li and Zhou, 2011), image processing (Rashedi and Nezamabadi, 2013), control engineering (Oliveira *et al.*, 2015; Precup *et al.,* 2012), civil and energy engineering

(Ganesan *et. al.*, 2013), telecommunication (Doraghinejad *et al.*, 2014), epidemiology (Ali *et al.,* 2018) and engineering design optimization problems (Luqman *et al.*, 2017; Yang and Deb, 2010; Rao and Patel, 2012; Tabassum *et al.*, 2015b, Sadollah *et al.,* 2015).

Unfortunately, there exist No Free Lunch (NFLs) theorems (Wolpert and Macready, 1997) which state that an algorithm best for one algorithm may not be better for the other problem. Consequently necessary modifications, enhancements, hybridization or adaptation of more effective natural phenomena are needed to evolve more robust method.

Human are the most intelligent creature of this planet and are trying to overcome the nature. The methods, tools and systems developed by them are more efficient and effective than those of their co-inhabitants. Therefore, it is expected that the balanced adaptation based on human intelligence might boost up the problem solving capabilities of the resulting optimizer.

The idea of artificial showering for the optimization task was introduced by Ali *et al.* (2015). Originally, very limited adaptation from the phenomena of flow and accumulation of multiple water units scattered by mechanical equipment in an irrigation field were incorporated in the design of artificial showering optimization approach. Many other intelligent and dynamical aspects of the mechanical tools like sprinklers, their moving platforms, automotive controllers and other related concepts offer considerable inspirations for research on more sophisticated adaptations to design a fully equipped optimizer. Keeping in view these facts, the present work imitates advanced characteristics of artificial showering phenomena to propose a novel optimization method.

Following are the main contributions of the present work.

1. Adaptation of phenomenal working of irrigation tools to model an equivalent optimization process.
2. Development of a novel metaheuristic (TSO) based on targeted showering through sprinklers in an irrigation field.
3. Validation of the proposed method through benchmark functions.
4. Efficient solution of optimal crop rotation in Slovenian organic farming by the proposed TSO.

Rest of the paper is organized as follows. Firstly, the analogies and the procedural steps of artificial showering to model the TSO algorithm are elaborated. Then numerical results on six standard benchmark functions are reported and discussed in detail. Thereafter the proposed TSO is applied to solve a case study of optimal crop rotation in Slovenian organic farming along with sensitivity analysis of the model. In the end, main achievements of the paper have been concluded and some future research directions have also been presented.

MATERIALS AND METHODS

***Assumptions and analogies in design of Targeted Showering Optimization algorithm*:** Targeted showering optimization algorithm aims to perform optimization process by generating and improving successive populations of solutions in the search space through iterative process. Following idealizations have been made getting inspired from the artificial showering phenomena (similar to those as in (Ali *et al.*, 2015).

1. The whole search space resembles to an imaginary field offering no resistance to water flow and the water infiltrates only at the lowest location.
2. There are no evaporation, raining and interflow of water.
3. The surmounted sprinklers cover every bit of the field.
4. Water is in abundance and remains constant throughout the iterations.
5. Each unit of the water has the probabilistic sense of moving downhill.
6. The objective function to be optimized is bounded below.

In addition to above assumptions, the adaptations from artificial showering by irrigation tools to design the optimization process are described point wise as follows.

***Search space and the solution*:** Mathematically, the search space for optimization of a real valued function $f: \mathbb{R}^n \to \mathbb{R}$ is defined as the set:

$$\boldsymbol{\mathcal{S}} = \{\boldsymbol{x}: \boldsymbol{x} \in \Omega \subseteq \mathbb{R}^n \wedge l_i \leq x_i \leq u_i\} \qquad (1)$$

Here Ω is a subset of Euclidean field $\mathbb{R}^n$ due to some problem constraints and is omitted in the absence of constraints.

TSO imitates the search space, a solution and the quality of a solution according as follows.

$\boldsymbol{\mathcal{S}}: Search\ space$ ← an irrigation field,

$\boldsymbol{x}: solution$ ← a location in irrigation field,

$f(\boldsymbol{x})$ ← landscape level of the location.

For illustration, consider the following two-dimensional problem.

$$\left.\begin{array}{c} \text{Minimize } f(\boldsymbol{x}) = x_1^2 + x_2^2 \\ x_i \in [-10, 10] \end{array}\right\} \qquad (2)$$

The search space reduces to the following set.

$$\boldsymbol{\mathcal{S}} = \{\boldsymbol{x}: -10 \leq x_i \leq 10, 1 \leq i \leq 2\}$$

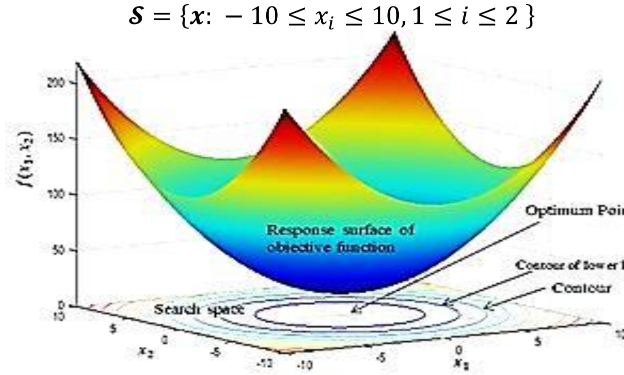

Figure 1. Geometry of problem given by relation (2).

Figure 1 shows the corresponding surface of the objective function and the search space as a rectangular field. Every location in the associated abstract irrigation field is identified by an ordered pair in R^2 and belonging to the set $\boldsymbol{S}$.

Supposing the following four members of initial population $\boldsymbol{P}^{(0)}$:

$\boldsymbol{x}^{(1)} = (2.5, -4)$, $\boldsymbol{x}^{(2)} = (-5.1, -6.091)$,
$\boldsymbol{x}^{(3)} = (8.51, 4)$, $\boldsymbol{x}^{(4)} = (-1.1, 5.091)$.

Then their corresponding landscape levels are given as under.

$h^{(1)} = f(\boldsymbol{x}^{(1)}) = 22.25 \leftarrow$ *The Lowest Landscape level*,
$h^{(2)} = f(\boldsymbol{x}^{(2)}) = 63.11 \leftarrow$ *Second Highest Landscape level*,
$h^{(3)} = f(\boldsymbol{x}^{(3)}) = 88.42 \leftarrow$ *The Highest Landscape level*,
$h^{(4)} = f(\boldsymbol{x}^{(4)} = 27.128 \leftarrow$ *Second Lowest Landscape level*.

Generating initial population of locations: For the generation of initial population of trial solutions, it is supposed that the entire search space is covered by overhead sprinklers. The initial population $\boldsymbol{P}^{(0)}$ of M locations is obtained by operating M, randomly selected, overhead sprinklers to release water units at the corresponding locations in the search space $\boldsymbol{S}$. The mathematical expression to be satisfied by each initial solution is given below.

$$\boldsymbol{x}^{(j)} = \left(x_1^{(j)}, x_2^{(j)}, x_3^{(j)}, \dots, x_n^{(j)}\right): 1 \le j \le M$$
$$x_i^{(j)} = l_i + rand(1) \times (u_i - l_i): 1 \le i \le n \qquad (3)$$

The symbol $rand(1)$ denotes a random number drawn from a normal distribution over the interval [0, 1]. Initialization process also involves the evaluation of landscape levels of the locations where the water units have been launched. An equivalent initial set $\boldsymbol{H}^{(0)}$ of objective function values is prepared.

$$\boldsymbol{H}^{(0)} = \left\{h^{(1)}, h^{(2)}, h^{(4)}, \dots, h^{(M)}\right\} \qquad (4)$$

Optimization process by targeted artificial showering: To perform optimization process the artificial showering defines a task to shower water units towards the most desired location of the search space which is analogous to search for the best possible solution denoted by $\boldsymbol{x}^{(b)}$. For a minimization problem the best solution is the one with the smallest objective function value whereas for the maximization problem it is the location with -1 time the largest objective function value. For the population

$$\boldsymbol{P}^{(0)} = \left\{\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}, \dots, \boldsymbol{x}^{(M)}\right\}$$

the best solution is calculated as:

$$\boldsymbol{x}^{(b)} = \arg\left(\min_{1 \le i \le M} \begin{cases} h^{(i)}\ for\ minization \\ -h^{(i)}\ for\ maxization \end{cases}\right) \qquad (5)$$

After identifying the best location gun-type sprinklers are designated to each location. Each sprinkler aims to project water unit by following the natural phenomenon of water flow along the steepest path determined by the landscape topology. The best natural steepest downhill direction $\boldsymbol{d}$ can be defined as:

$$\boldsymbol{d} = -\nabla f(\boldsymbol{x}) \qquad (6)$$

However, from algorithmic point of view, TSO needs only a descent direction. For this purpose, a target location $\boldsymbol{x}^{(j)}$, different from the current location $\boldsymbol{x}^{(i)}$ of the sprinkler, is selected randomly. The descent direction, say $\boldsymbol{d_1}$, is constructed according as under.

$$\boldsymbol{d_1} = \begin{cases} \boldsymbol{x}^{(j)} - \boldsymbol{x}^{(i)}\ if\ f_j - f_i < 0 \\ \boldsymbol{x}^{(i)} - \boldsymbol{x}^{(j)}\ otherwise \end{cases} \qquad (7)$$

Another better descent direction is found by following relation.

$$\boldsymbol{d_2} = \boldsymbol{x}^{(best)} - \boldsymbol{x}^{(i)} \qquad (8)$$

Figure 2 exhibits the above defined two directions.

With the help of path selection probability, say ρ, the gun-type sprinkler at location $\boldsymbol{x}^{(i)}$ selects one of the descent directions defined above and projects water unit by using one of the following governing relations.

$$\boldsymbol{x}^{(i)new} = \boldsymbol{x}^{(i)} + F_i \times (\boldsymbol{s} \otimes \boldsymbol{d_1}/\|\boldsymbol{d_1}\|) \qquad (9)$$
$$\boldsymbol{x}^{(i)\,new} = \boldsymbol{x}^{(i)} + F_i \times r \times \boldsymbol{d_2}/\|\boldsymbol{d_2}\| \qquad (10)$$

In above two equations r is a real number randomly chosen from $(0, 1)$, $\boldsymbol{s}$ is an n-dimensional vector of random numbers drawn from $(0, 1)$ and $\otimes$ is the Minkowski product of two vectors. The parameter F_i is called the projection speed of the sprinkler at location $\boldsymbol{x}^{(i)}$ and is calculated by using following Euclidean norm.

$$F_i = \left\|\boldsymbol{x}^{(i)} - \boldsymbol{x}^{(l)}\right\| \qquad (11)$$

The location $\boldsymbol{x}^{(l)}$ is randomly selected from the set $\boldsymbol{P}^{(0)} - \{\boldsymbol{x}^{(i)}, \boldsymbol{x}^{(j)}\}$.

The landscape level of the new location generated by relation (9) or (10) is calculated and is assigned to $\boldsymbol{x}^{(i)}$ if $f\left(\boldsymbol{x}^{(i)new}\right) < f\left(\boldsymbol{x}^{(i)}\right)$, otherwise it is discarded. This process is carried out for each member of the current population. For a successful up gradation of a location the corresponding descent direction is memorized and the projection speed is increased for next iteration whereas for a failed attempt new descent directions and the projection speed are defined. The iterative process continues until prescribed termination criteria are met.

***Re-installation*:** The A serious challenge faced by nature inspired algorithms is premature convergence or stagnation at a local optimum point. To equip TSO with strength to jump out of stagnation state, the phenomena of clustering of sprinklers and maximum number of failed attempts are incorporated. For this purpose, a small positive real number δ is used as clustering tolerance and a positive integer $T_i^{(allowed)}$ defines an upper bound for maximum allowed number of failed attempts made by the sprinkler at location $\boldsymbol{x}^{(i)}$. A re-installation at a new random location, defined by equation (3), of the search space takes place if one of the following conditions is satisfied.

$$\left\|\boldsymbol{x}^{(i)} - \boldsymbol{x}^{(best)}\right\| < \delta \qquad (12)$$
$$T_i^{(current)} > T_i^{(allowed)} \qquad (13)$$

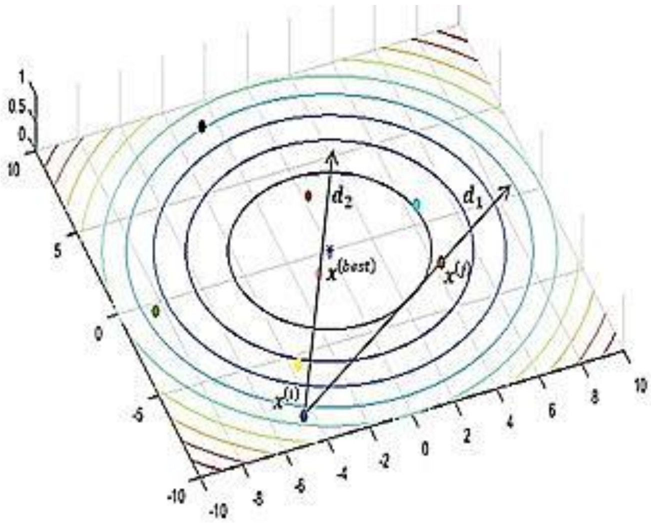

Figure 2. Graphical views of eight locations and search directions for i^{th} location.

Combining the above four phases, the complete TSO algorithm is described as under.

Main TSO algorithm

1. *Initialize* $M, T_i^{(allowed)}, \delta, \rho$ *and set* $T_i^{(current)} = 0$.
2. *Drive the overhead sprinklers to shower M units of water and evaluate their landscape levels.*
3. *For each unit* $i : 1, 2, 3, \ldots, M$

Choose a random number $r_i \in (0, 1)$.

$\boldsymbol{if}\ r_i \geq \rho$ *, find* $\boldsymbol{x}^{(j)new}$ *by using equation (9)*

$\boldsymbol{else}$ *Use equation (10) to find* $\boldsymbol{x}^{(i)new}$

$\boldsymbol{endif}$.

$\boldsymbol{if}\ f(\boldsymbol{x}^{(i)new}) < f(\boldsymbol{x}^{(i)})$

$then\ \boldsymbol{x}^{(i)} \leftarrow \boldsymbol{x}^{(i)new}\ and\ set\ T_i^{(current)} = 0$

$\boldsymbol{else}\ discard\ \boldsymbol{x}^{(i)new}\ and\ set$

$T_i^{(current)} = T_i^{(current)} + 1$

$\boldsymbol{endif}$.

$Update\ the\ best\ location.$

4. *For each unit* $\boldsymbol{x}^{(i)}$: $i = 1, 2, 3, \ldots, M$*; if condition (12) or (13) is satisfied then assign a new random location to* $\boldsymbol{x}^{(i)}$ *by using relation (3) and set* $T_i^{(current)} = 0$.
5. *If termination criteria are met then STOP otherwise go to step 3.*

ILLUSTRATIVE EXAMPLE

For explanation of the iterative process of TSO consider the well-known Rastrigin's function defined by following formula.

$$f(\boldsymbol{x}) = 10n + \sum_{i=1}^{2} [x_i^2 - 10\cos(2\pi x_i)]$$

The search space is taken as $[-2, 2] \times [-2, 2]$. Consider that the parameters of TSO are as under:

$$M = 10, \rho = 0.1, \delta = 10^{-6},$$
$$T^{(allowed)} = \{200, 180, \ldots, 20\}.$$

Following matrices correspond to the initial population and the landscape levels returned by computer simulation denoted by $P^{(0)}$ and $H^{(0)}$ respectively.

$$P^{(0)} = \begin{bmatrix} -1.86 & -0.47 \\ -1.08 & 1.54 \\ 0.85 & 0.57 \\ \mathbf{-0.87} & \mathbf{1.12} \\ -0.67 & -1.19 \\ -1.98 & 0.76 \\ -0.39 & 0.51 \\ 0.51 & -0.04 \\ 1.20 & 0.83 \\ 0.82 & -0.82 \end{bmatrix}, H^{(0)} = \begin{bmatrix} 27.08 \\ 24.34 \\ 24.21 \\ \mathbf{7.53} \\ 22.49 \\ 13.72 \\ 37.97 \\ 20.60 \\ 14.54 \\ 13.05 \end{bmatrix}$$

In $P^{(0)}$ The best solution is $\boldsymbol{x}^{(4)}$ with fitness value $h^{(4)}$. Execution of first iteration improves the current best solution in its vicinity. Next few populations of trial solutions and their fitness values are given as under.

After the first iteration following $P^{(1)}$ and $H^{(1)}$ were obtained.

$$\begin{bmatrix} -1.22 & 1.73 \\ -0.96 & 1.15 \\ 0.85 & 0.57 \\ -0.97 & 1.14 \\ -0.67 & -1.19 \\ -1.98 & 0.76 \\ -0.78 & 1.13 \\ 0.51 & -0.04 \\ 1.20 & 0.83 \\ \mathbf{0.10} & \mathbf{0.90} \end{bmatrix}, \begin{bmatrix} 24.02 \\ 6.43 \\ 24.21 \\ 5.99 \\ 22.49 \\ 13.72 \\ 13.40 \\ 20.60 \\ 14.54 \\ \mathbf{4.86} \end{bmatrix}$$

The 20th iteration produced $P^{(20)}$ and $H^{(20)}$ as under.

$$\begin{bmatrix} -0.9849 & 0.9310 \\ -0.0428 & 0.9335 \\ -0.0012 & 0.0073 \\ 0.0026 & 0.0064 \\ -0.0029 & 0.0059 \\ -0.0004 & -0.0095 \\ -0.0021 & 0.0057 \\ -0.0014 & 0.0059 \\ 1.0186 & 0.9220 \\ \mathbf{-0.00041} & \mathbf{0.0059} \end{bmatrix}, \begin{bmatrix} 2.8063 \\ 2.0935 \\ 0.0108 \\ 0.0094 \\ 0.0085 \\ 0.0181 \\ 0.0074 \\ 0.0073 \\ 3.1335 \\ \mathbf{0.0070} \end{bmatrix}$$

The 30th iteration was completed to generate following $P^{(30)}$ and $H^{(30)}$.

$$\begin{bmatrix} 5.26e-7 & 9.08e-6 \\ 8.96e-6 & 7.98e-6 \\ 6.54e-7 & 9.49e-6 \\ -4.56e-6 & 3.97e-5 \\ 3.53e-7 & 9.61e-6 \\ \mathbf{-3.14e-7} & \mathbf{9.08e-6} \\ -2.01e-5 & 9.11e-6 \\ -1.20e-5 & 9.02e-6 \\ -9.95e-5 & -4.11e-5 \\ 1.4179 & 0.7084 \end{bmatrix}, \begin{bmatrix} 1.64e-8 \\ 2.86e-8 \\ 1.80e-8 \\ 3.17e-7 \\ 1.83e-8 \\ \mathbf{1.643-8} \\ 9.62e-8 \\ 4.47e-8 \\ 2.30e-6 \\ 33.7975 \end{bmatrix}$$

The population $P^{(40)}$ and the landscape levels $H^{(40)}$ matrices are given as under respectively.

$$\begin{bmatrix} -1.89e-7 & -2.07e-8 \\ 1.07e-7 & -3.55e-8 \\ -3.50e-8 & -1.26e-7 \\ 5.55e-9 & -4.55e-8 \\ -1.15e-8 & 9.51e-8 \\ \mathbf{-8.13e-9} & \mathbf{-4.57e-9} \\ 9.55e-9 & 3.38e-8 \\ -1.67e-8 & -4.69e-9 \\ 6.19e-8 & -2.11e-7 \\ -0.0003 & -0.0235 \end{bmatrix}, \begin{bmatrix} 7.17e-12 \\ 2.52e-12 \\ 3.41e-12 \\ 4.16e-13 \\ 1.82e-12 \\ \mathbf{1.78e-14} \\ 2.45e-13 \\ 5.68e-14 \\ 9.56e-12 \\ 0.1097 \end{bmatrix}$$

At the end of 44th iteration the global optimum solution $f^*(0,0) = 0$ is achieved.

RESULTS AND DISCUSSION

The results of TSO are compared with ASHA, Water Cycle Algorithm (WCA) (Eskandar *et al.*, 2012), Water Wave Optimization (WWO) algorithm (Zheng, 2015), Differential Evolution (DE) (Storn and Price, 1997), Particle Swarm Algorithm (PSO) (Eberhart and Kennedy, 1995) and Artificial Bee Colony (ABC) algorithm (Karaboga and Akay, 2011). Each algorithm is implemented to optimize the given functions under the following limitations:

1. Number of independent runs = 30
2. Total number of iterations = 500
3. Number of function evaluations = 25000
4. The objective function values in the best run are observed at five different levels and the k^{th} level is defined as the end of $\left(\frac{k}{5} \times 500\right)^{th}$ iteration.

The objective is to evaluate efficiency, speed of convergence and accuracy of each of the algorithm at various levels of the best run of an algorithm under a limited budget. The relevant parameters of all the competing algorithms have been listed in Table 2. All the algorithms are programmed and implemented in MATLAB environment.

***Details of benchmark functions*:** Table 1 exhibits the mathematical form, bounds on decision variables and the global minimum value f^* of each benchmark. The selected benchmarks have been rigorously used in literature (Jamil and Yang, 2013; Hansen, 2006; Tanabe and Fukunaga, 2013; Chen *et al.*, 2014) for evaluating performances of algorithms. The considered benchmarks involve separable, non-separable, convex, non-convex, differentiable, non-differentiable, unimodal, multi-modal, convex and non-convex functions.

The values of u_j and v_j in f_6 are given as components of vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively as under:

$$\boldsymbol{u} = [0.1957, 0.1957, 0.1735, 0.16, 0.0844, 0.0627, 0.0456, 0.342, 0.0323, 0.0235, 0.0246]$$

$$\boldsymbol{v} = \left[4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}\right].$$

The algorithmic parameters of all of the competing algorithms have been presented in Table 2. Obtained results are presented

Table 1. Details of benchmark functions.

Name and Mathematical forms and Domains	$\boldsymbol{f^*}$
$f_1(\underline{\boldsymbol{x}}) = \sum_{i=1}^{n} x_i^2$; $-100 \le x_i \le 100$	0
$f_2(\underline{\boldsymbol{x}}) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$; $-10 \le x_i \le 10$	0
$f_3(\underline{\boldsymbol{x}}) = \sum_{j=1}^{n} \left(\sum_{i=1}^{j} x_i\right)^2$; $-100 \le x_i \le 100$	0
$f_4(\underline{\boldsymbol{x}}) = max\{\lvert x_i \rvert, 1 \le i \le n\}$; $-100 \le x_i \le 100$	0
$f_5(\underline{\boldsymbol{x}}) = \sum_{i=1}^{n} 100(x_i^2 - x_{i+1})^2 + (1 - x_i^2)^2$; $-30 \le x_i \le 30$	0
$f_6(\underline{\boldsymbol{x}}) = \sum_{j=1}^{11} \left(u_j - \frac{x_1\left(v_j^2 + v_j x_2\right)}{\left(v_j^2 + v_j x_3 + x_4\right)}\right)^2$; $-5 \le x_i \le 5$	0.00030749

Table 2. Parameters of the algorithms.

Algorithms	**Parameters**
TSO:	$M = 50, \rho = 0.1, \delta = 10^{-16}, T^{(allowed)} = \{200, 180, \ldots, 20\}$
$ASHA$:	$M = 50, \rho_0 = 0.1, \delta = 10^{-16}$
WCA:	$Npop = 50, Nsr = 4, dmax = 1e-32$
WWO:	$Npop = 50, h_{max} = 12, \alpha = 1.0026, \beta_{min} = 0.001, \beta_{max} = 0.25, \beta = \beta_{max}, k_{max} = min(12, n/2)$
DE:	$NP = 50, F = 0.5, CR = 0.9$
PSO:	$c_1 = 2, c_2 = 2, V_{max} = 6, W_{max} = 0.9, W_{min} = 0.2$
ABC:	$Food\ sources = Colonysize/2 = 25, Limit = 200$

Table 3. Comparison of median run results on 10-dimensional benchmark functions.

Function		Objective function value at iteration number					*w/d/ℓ*
		100	**200**	**300**	**400**	**500**	
	TSO	7.66e-14	5.05e-29	0	0	0	5/1/0
	ASHA	6.89e-02	9.90e-07	2.58e-11	2.02e-15	6.76e-21	
	WCA	4.31e-11	3.73e-19	1.36e-22	2.02e-28	0	
f_1	WWO	2.26e02	8.01e-02	1.96e-04	6.02e-07	3.91e-10	
	DE	2.87e-02	4.83e-08	1.51e-13	3.90e-19	3.36e-24	
	PSO	5633.58	798.4096	1.14e-02	4.85e-10	1.84e-23	
	ABC	8.06-04	4.23e-07	4.07e-11	1.56e-15	2.92e-17	
	TSO	8.97e-06	1.06e-12	2.75e-14	0	0	5/1/0
	ASHA	2.96e-02	3.06e-05	1.90e-08	4.94e-11	1.66e-13	
	WCA	1.69e-06	5.05e-10	9.20e-13	1.78e-15	0	
f_2	WWO	1.54745	1.82e-01	8.08e-03	7.23e-04	3.39e-05	
	DE	4.65e-02	5.32e-05	1.80e-07	2.76e-10	4.06e-13	
	PSO	26.6593	8.706	2.75e-02	1.06e-06	2.55e-13	
	ABC	4.36e-02	4.78e-04	1.49e-06	5.23e-09	7.53e-12	
	TSO	5.34e-05	4.29e-10	3.69e-16	2.90e-24	3.16e-29	6/0/0
	ASHA	18.8964	2.96e-01	5.83e-03	5.09e-05	4.35e-06	
	WCA	4.36e-04	3.99e-07	1.75e-11	5.24e-17	7.10e-19	
f_3	WWO	5.6828	2.42e-01	2.09e-02	8.70e-04	2.23e-05	
	DE	2.30e-01	8.57e-05	6.31e-08	1.10e-11	4.04e-15	
	PSO	328.7168	265.3724	4.5167	9.79e-03	1.64e-06	
	ABC	42.57807	4.94037	3.1782	1.4799	9.81e-01	
	TSO	2.548e-03	4.68e-06	9.45e-10	7.11e-15	7.11e-15	6/0/0
	ASHA	1.78e-01	6.29e-03	1.82e-04	1.37e-06	2.02e-08	
	WCA	9.16e-03	3.96e-04	1.09e-05	1.31e-06	3.16e-08	
f_4	WWO	1.3743	1.73e-01	1.61e-02	2.38e-03	1.53e-04	
	DE	1.53e-01	1.51e-03	1.96e-05	1.66e-07	1.66e-09	
	PSO	8.6092	5.3837	4.40e-02	3.69e-03	1.53e-06	
	ABC	1.8765	5.98e-01	1.28e-02	9.15e-02	3.04e-02	
	TSO	2.84e-01	3.83e-02	1.42e-03	4.40e-04	1.07e-04	6/0/0
	ASHA	96.2577	73.33551	1.8843	6.08e-01	3.72e-01	
	WCA	1.51	2.22e-01	7.09e-02	2.08e-02	4.94e-04	
f_5	WWO	380.998	18.5866	4.8820	4.4669	4.4218	
	DE	5.8203	3.0627	2.2073	1.1978	7.29e-01	
	PSO	384635.02	8537.766	59.5539	1.61e-01	1.22e-01	
	ABC	6.19649	1.2789	6.39e-01	6.05e-02	9.22e-03	
	TSO	3.08e-04	3.0749e-04	3.0749e-04	3.0749e-04	3.0749e-04	4/2/0
	ASHA	7.19 e-04	5.87e-04	0.00042836	3.74e-04	3.18e-04	
	WCA	3.0749e-04	3.0749e-04	3.0749e-04	3.0749e-04	3.0749e-04	
f_6	WWO	6.61e-04	5.1047e-04	0.00050313	4.76e-04	3.55e-04	
	DE	3.08e-04	3.0749e-04	3.0749e-04	3.0749e-04	3.0749e-04	
	PSO	2.29e-03	1.26e-03	8.58e-04	8.30e-04	8.17e-04	
	ABC	1.99e-03	1.54e-03	1.34e-03	7.66e-04	7.66e-04	

in Table 3 whose last column contains the symbols "w","d" and "ℓ" which represent the numbers of wins, draws and losses respectively. A win, draw or loss respectively means that TSO finds a better, equally good or an inferior solution as compared to some other algorithm. The bold face numbers in Table 3 represent the best approximates of optimal values produced by an algorithm among all the competing algorithms.

***Validation of TSO on benchmark functions*:** From the results presented in Table 3 it can be observed that TSO was able to find the smallest objective function value of 7.66e-14 for the benchmark function f_1 $(\boldsymbol{x})$ using only 100 iterations and converged to zero optimal value in just 300 iterations. On the other hand ASHA, WCA, WWO, DE, PSO and ABC were unable to find optimal solution up to 400 iterations. For benchmark function $f_2(\boldsymbol{x})$ WCA showed fast convergence for first 100 iterations but could reach an accuracy of 1.78e-15 at the level of 400 iterations. TSO proved to be superior to WCA and other algorithms at the levels of 200, 300, and 400 iterations. ASHA was able to find the third best approximate objective function value of 1.66e-13 for this function. TSO produced the approximate solutions of third benchmark function and secured 6 wins at each of observation levels. For the benchmark function $f_4(x)$ TSO stood the first by producing the smallest function values 2.548e-03, 4.68e-06, 9.45e-10, 7.11e-15, 7.11e-15 at five levels respectively. DE found the second best final value of 1.66e-09 for this function which was surpassed by TSO within 200 iterations. Rest of the algorithms showed much inferior solutions as compared to that of TSO algorithm. The performance of TSO on fifth benchmark function is quite similar to that on fourth benchmark function. On the sixth benchmark function TSO was better than ASHA, WWO, PSO and ABC but showed equal performance in comparison with WCA and DE. At the end of 500 iterations TSO was able to locate the optima of all the objective functions with the best accuracies. It is worth mentioning that among the other water based algorithms only WCA showed comparable performance on f_1, f_2 and f_6 functions but overall performance of TSO ranked the first.

***Application of TSO to the case study regarding the model of optimal crop rotation in Slovenian organic farming*:**

***Model data*:** The proposed TSO algorithm is applied to the model of optimal crop rotation in Slovenian organic farming considered by Prisenk and Turk (2015). The model aims to determine optimum crop production combination to maximize the total income and minimization of nitrogen off-take in crops by satisfying the constraints of mechanical labour costs, manual labour costs, cropped areas and fertilizer costs. There are seven crops Maize, Rye, Barely, Oats, Wheat, Potato and Grass Silage with related incomes, mechanical labour costs, manual labour costs, fertilizers costs and nitrogen off-takes (kg N/ha) denoted by c_i, e_i, a_i, z_i and t_i respectively. Table 4 presents the relevant data of the model.

***Development of mathematical models*:**

Consider the decision variables, $x_i : 1 \leq i \leq 7$, represent the number of hectares allocated to the crops Maize, Rye, Barely, Oats, Wheat, Potato and Grass Silage respectively. Consequently a seven dimensional design vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_7)$is formed. The model is solved by considering three optimization problems. The first problem is the minimization of the objective function $\varphi_1(\boldsymbol{x})$ that maximizes the total income by obeying four inequality ($\leq$) constraints. The second problem is minimization of objective function $\varphi_2(\boldsymbol{x})$ denoting the nitrogen off-take with three inequality constraints of mechanical labour cost, manual labour cost, fertilizers cost and one equality constraint of cropping area. Third problem is the minimization of $\varphi_3(\boldsymbol{x})$ that is weighted aggregate of two objective functions $\varphi_1(\boldsymbol{x})$ and $\varphi_2(\boldsymbol{x})$ and is equivalent to simultaneous maximization of total income and minimization of nitrogen off-take. This problem considers fixed budget of 1734€ for mechanical labour cost and three other inequality constraints.

***Problem 1*:**

Minimize $\varphi_1(\boldsymbol{x}) = -\sum_{i=1}^{7} c_i x_i$

Subject to the constraints:

$\sum_{i=1}^{7} e_i x_i \leq 1734$; $\sum_{i=1}^{7} a_i x_i \leq 1854$; $\sum_{i=1}^{7} z_i x_i \leq 1880$, $\sum_{i=1}^{7} x_i \leq 7$.

***Problem 2*:**

Minimize $\varphi_2(\boldsymbol{x}) = \sum_{i=1}^{7} t_i x_i$

Subject to the constraints:

$\sum_{i=1}^{7} e_i x_i \leq 1734$; $\sum_{i=1}^{7} a_i x_i \leq 1854$, $\sum_{i=1}^{7} z_i x_i \leq 1880$, $\sum_{i=1}^{7} x_i = 7$.

***Problem 3*:**

Minimize $\varphi_3(\boldsymbol{x}) = w_1 \varphi_1(\boldsymbol{x}) + w_2 \varphi_2(\boldsymbol{x})$

Subject to the constraints:

$\sum_{i=1}^{7} e_i x_i = 1734$; $\sum_{i=1}^{7} a_i x_i \leq 1854$, $\sum_{i=1}^{7} z_i x_i \leq 1880$, $\sum_{i=1}^{7} x_i \leq 7$.

Each of the above problems is converted to an unconstrained optimization problem by means of penalty function approach. The weights $w_1$and $w_2$for third problem are set as unity. The proposed TSO algorithm is implemented by considering same parameters setting as shown in Table 2. For comparison of

Table 4. Input data for the model.

Organic Crop	**Cropped area (ha)**	**Costs (€/ha)**				**Nitrogen off-take: t_i**
		Total income:c_i	**Mechanical labour cost: e_i**	**Manual labour cost: a_i**	**Fertilizers cost: z_i**	
Maize	x_1	2430	274.4	192.5	260.8	78.0
Rye	x_2	1505	213.1	176.0	217.2	37.5
Barely	x_3	1470	217.2	176.0	215.8	56.0
Oats	x_4	1380	217.2	176.0	211.5	42.5
Wheat	x_5	1680	217.1	176.0	246.3	45.0
Potato	x_6	7350	501.7	786.5	381.3	112.5
Grass silage	x_7	1289	93.6	170.5	347.1	412.5
Restrictions	$\leq, = 7$	Maximize	$\leq, = 1734$	≤ 1854	≤ 1880	Minimize

Source: Cuttle *et al*. (2003), Jeric *et al*. (2011) and Prisenk and Turk (2015)

Table 5. Optimal results of TSO optimal crop rotation problem.

Objectives	Results of WGP (Prisenk and Turk 2015)				The Proposed TSO algorithm		
	Problem 1	Problem 2	SC1	SC4	Problem 1	Problem 2	Problem 3
Total Income (€)	19618.97	n/a	19227.45	17698.38	19620.964	n/a	18964.5329
Mechanical labour costs(€)	1734.32	2045.82	1675.74	1734.33	1734	1512	1734
Manual labour costs (€)	1853.54	2377.31	1853.50	1706.55	1854	1232	1854
Fertilizers costs (€)	1508.43	1879.91	1460.25	1646.07	1507.89336	1565.8	1531.91482
Cropped area (ha)	5.11	7.00	4.91	5.98	5.10391	7.00	5.42518
N off-take (kg N/ha)	n/a	423.83	423.81	423.81	n/a	306	423.81
Organic crop rotation							
Maize	3.64	0.00	3.00	2.64	3.63674271	0.00	2.8566
Rye	0.00	2.00	0.00	0.00	0.00	2.00	1.17298
Barley	0.00	0.00	0.00	0.00	0.00	1.00	0
Oats	0.00	1.12	0.00	0.00	0.00	2.00	0
Wheat	0.00	2.00	0.37	2.34	0.00	2.00	0
Potato	1.47	1.88	1.54	1.00	1.46716723	0.00	1.3956
Grass silage	0.00	0.00	0.00	0.00	0.00	0.00	0

results of TSO algorithm the results produced by those of Prisenk and Turk (2015) using Weighted Goal Programming (WGP) have been considered. Comparison of results has been presented in Tables 4 and 5.

From Table 5 it can be observed that the proposed TSO method was able to find the optimal total income of 19620.9639€ for problem 1 that is better than that of infeasible solution found by Prisenk and Turk (2015). The minimum nitrogen off take in problem 2 found by our proposed TSO algorithm is 306 kg N/ha which is 27.8% smaller than the reported solution of WGP. The problem 3 is equivalent to a multi-objective optimization problem and possesses many priority based optimal solutions. All such solutions have been obtained under two scenarios and are exhibited in Figure 3 and 4. Figure 3 shows the optimal solutions under the equality constraint on mechanical labour cost and Figure 4 shows the optimal curve under all inequality constraints.

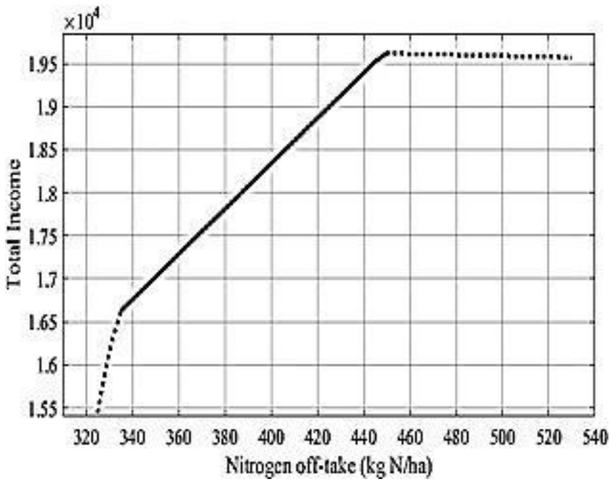

Figure 3. The optimal curve of first scenario of problem 3.

One of the optimal solutions under equality constraints on mechanical labour cost is presented in the last column of Table 5. The total income found by TSO algorithm is 18964.5329€ at nitrogen off-take level 423.83 kg N/ha. This solution provides a better optimal crop rotation option as compared to that of SC4 of WGP.

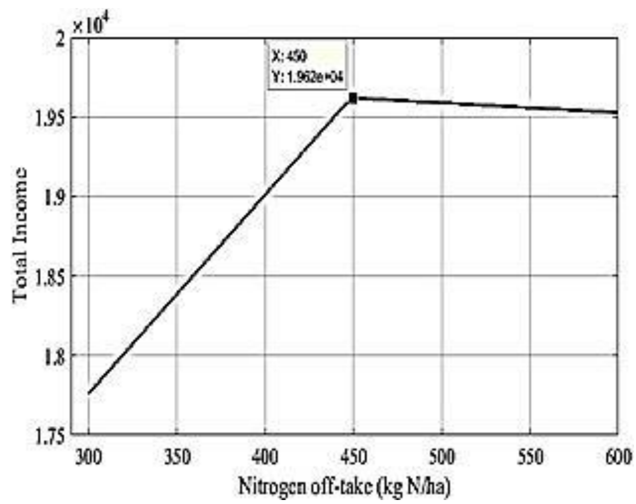

Figure 4. The optimal curve of second scenario of problem 3.

Figures 3 and 4 provide the extended insightful knowledge of the optimal crop rotation options. It can be observed from Figure 3 that the total income increases almost linearly from 16625.3266€ to 19620.1799€ when nitro off-take increases from 335 kg N/ha to 450 kg N/ha. It clearly describes that total income cannot be increased beyond 19620.1799€ whenever nitrogen off-take falls from 335 kg N/ha or exceeds the value 450 kg N/ha. Similarly Figure 4 shows that the total income is maximally achieved at nitrogen off-take level around 450 kg N/ha.

***Sensitivity analysis*:** Table 6 presents the sensitivity analysis of three optimization models. For problem 1, it can be observed that the fertilizer cost can be reduced to about 1508€ whereas the total cultivated area can be reduced to 5.2 ha. The farmer has an option to utilize rest of the area for some other usage. The shadow prices of mechanical and manual labour costs are 4.16242€ and 6.70€ respectively which indicate that an increase or decrease in the manual labour cost effects the total income more significantly.

Table 6. Sensitivity analysis of crop rotation models.

Restrictions	Mechanical labour cost	Manual labour cost	Fertilizers cost	Total cropped area
	Problem 1			
Final Value	1734	1854	1507.89336	5.10391
Shadow Price	4.162419222	6.690037224	0	0
Constraint R.H. Side	1734	1854	1880	7
Allowable Increase	336.8488836	864.3396452	1E+30	1E+30
Allowable Decrease	551.3530833	637.5459184	372.10664	1.89609
	Problem 2			
Final Value	1512	1232	1565.8	7
Shadow Price	0	0	0	56
Constraint R.H. Side	1734	1854	1880	7
Allowable Increase	1E+30	1E+30	1E+30	1
Allowable Decrease	222	622	314.2	1
	Problem 3			
Final Value	1734	1854	1531.91482	5.42518
Shadow Price	-1.50937311	10.16561029	0	0
Constraint R.H. Side	1734	1854	1880	7
Allowable Increase	0.767230706	1.615058423	1E+30	1E+30
Allowable Decrease	1.955505398	1.202764501	348.08518	1.57482

Moreover the allowable increase and decrease in the mechanical labour cost are 551.35€ and 336.85€ respectively whereas those of manual labour cost are 637.55€ and 864.34€ respectively. These variations do not disturb the feasibility of the obtained solution. For problem 2, the overall mechanical labour cost can be reduced by 222€, the overall manual labour cost can be reduced by 622€ and the overall fertilizers cost can be reduced by 314.2€ without effecting the feasibility of the solution. The increments in these quantities will be unnecessarily excessive. On the other hand a unit increase or decrease in the cropped area will increase or decrease 56 units in nitrogen off-take respectively.

For problem 3, the change in mechanical labour cost has opposite effect on total income as well as nitrogen off-take. The relevant allowable increase and decrease in mechanical labour cost are 0.767€ and 1.956€ which leave an opposite effect of 1.5094 units on the objective function value. The manual labour cost can be increased and decreased by 1.6151€ or 1.2028€ respectively by causing a variation of 10.1656 units in the objective function value. The solution remains feasible even the farmer reduces the allocated budget for fertilizers by 348.085€. The overall cropped area can be reduced by 1.575 ha and the excessive land can be utilized for some other needful crops.

***Conclusions*:** In this work, a new nature-inspired optimization algorithm, called Targeted Showering Optimization algorithm, based on artificial showering and moving of gun-type sprinklers has been presented. The developed TSO is a general purpose optimizer and has a great potential to solve linear as well as nonlinear optimization problems. TSO tackles the crop rotation optimization problems, especially involving conflicting objectives, in an effective and competent way.

To evaluate the potential of TSO, two experiments have been conducted. First, TSO has been compared with six state-of-the-art nature inspired algorithms namely ASHA, WCA, WWO, DE, PSO and ABC. The final values found by TSO for functions f_{1-6} by utilizing 500 iterations are 0, 0, 3.16e-29, 7.11e-15, 1.07e-04 and 3.0749e-04 which result in a total of 32 wins, 4 draws and no losses over the competing optimizers (please see last column of Table 3). Based on these facts and figures it can be concluded that TSO shows top ranked performance in solving the considered test suite.

The proposed TSO algorithm has been successfully applied to solve three models of optimal crop rotation problem in Slovenian organic farming. The solution of first problem demonstrates that the maximum income found by TSO is 19620.964€ that is better than the solution reported by Prisenk and Turk (2015) and is achievable at 19.79% reduction in the budget allocated for fertilizers along with 27.1% less utilization of cropped area. The optimal value of second problem found by TSO is 306 kg N/ha which is around 27.8% better than that reported by Prisenk and Turk (2015). The results of problem 3 found by TSO have been exhibited in Figures 3 and 4 which evidently reflect that up to what extent a compromise can be made over the nitrogen off-take or total income. It is worth noting that the solutions of all the considered crop rotation problems found by the proposed TSO are fully feasible whereas the solution obtained by Weighted Goal Programming (Prisenk and Turk 2015) possess large violations of the allowable resources.

From above facts and figures, it can be concluded that working principles of irrigation tools can be effectively imitated to evolve efficient optimizer for a wide range of practical optimization problems, especially in agriculture.

REFERENCES

Ali, J., M. Saeed, N.A. Chaudhry, M. Luqman and M.F. Tabassum. 2015. Artificial showering algorithm: a new meta-heuristic for unconstrained optimization. Sci. Int. 27:4939-4942.

Ali, J., first nameSaeed, M., Rafiq and I. Shaukat. 2018. Numerical treatment of nonlinear model of virus propagation in computer networks: an innovative evolutionary Padé approximation scheme. Adv. Differ. Equ. 2018:214. https://doi.org/10.1186/s13662-018-1672-1

Chen, L., Z. Zheng, H.L. Liu and S. Xie. 2014. An evolutionary algorithm based on covariance matrix leaning and searching preference for solving CEC 2014 benchmark problems. In: Evolutionary Computation (CEC), 2014 IEEE Congress; pp.2672-2677.

Cuttle, S., M. Shepherd and G. Goodlass. 2003. A review of leguminous fertility-building crops, with particular reference to nitrogen fixation and utilization. Written as a part of Defra Project OF0316 "The development of improved guidance on the use of fertility-building crops in organic farming.

De Kock, H.C. and S.E. Visagie. 1987. Multi-mix feedstock problems on microcomputers. J. Oper. Res. Soc. 38:585-590.

Doraghinejad, M., H. Nezamabadipour and A. Mahani. 2014. Channel assignment in multi-radio wireless mesh networks using an improved gravitational search algorithm. J. Netw. Comput. Appl. 38:163-171.

Eberhart, R. and J. Kennedy. 1995. A new optimizer using particle swarm theory. In: Micro Machine and Human Science. MHS'95, Proceedings of the Sixth International Symposium; pp.39-43.

Eskandar, H., A. Sadollah, A. Bahreininejad and M. Hamdi. 2012. Water cycle algorithm–A novel metaheuristic optimization method for solving constrained engineering optimization problems. Comput. Struct. 110:151-166.

Ganesan, T., I. Elamvazuthi, K.Z.K. Shaari and P. Vasant. 2013. Swarm intelligence and gravitational search algorithm for multi-objective optimization of synthesis gas production. Appl. Energy 103:368-374.

Hansen, N. 2006. Compilation of results on the 2005 CEC benchmark function set. Online May, 2006.

Jamil, M. and X.S. Yang. 2013. A literature survey of benchmark functions for global optimisation problems. IJMMNO 4:150-194.

Jeric, D., A. Caf, A. Demsar-Benedicic, S. Leskovar, O. Oblak, A. Soršak, M. Sotlar, S.D. Trpin, V. Velikonja, D. Vrtin and M. Zajc. 2011. Calculations catalogue for management planning on farms in Slovenia (CMPS). Chamber of Agriculture and Forestry of Slovenia, Ljubljana.

Karaboga, D. and B. Akay. 2011. A modified artificial bee colony (ABC) algorithm for constrained optimization problems. Appl. Soft Comput. 11:3021-3031.

Li, C. and J. Zhou. 2011. Parameters ident cation of hydraulic turbine governing system using improved gravitational search algorithm. Energy Convers. Manag. 52: 374-381.

Luqman, M., M. Saeed, J. Ali and M.F. Tabassum. 2017. Radial artificial bee colony algorithm for constraint engineering problems. Pak. J. Sci. 69:127-135.

Massinaei, H., M. Falaghi, and H. Lzadi. 2013. Optimisation of metallurgical performance of industrial oration column using neural network and gravitational search algorithm. Can. Metallurgical Quart. 52:115-122.

Mehrabian, A.R. and C. Lucas. 2006. A novel numerical optimization algorithm inspired from weed colonization. Ecol. Inform.1:355-366.

Mirjalili, S.A., S.Z.M. Hashim and H.M. Sardroudi. 2012. Training feed forward neural networks using hybrid particle swarm optimization and gravitational search algorithm. Appl. Math. Comput. 218:11125-11137.

Mirjalili, S., S.M. Mirjalili and A. Lewis. 2014. Grey wolf optimizer. Adv. Eng. Softw. 69:46-61.

Oliveira, P.B., E.J. S. Pires and P. Novais. 2015. Design of Posicast PID control systems using a gravitational search algorithm. Neurocomputing 167:18-23.

Precup, R.E., R.C. David, E.M. Petriu, S. Preitl and M.B. Radac. 2012. Novel adaptive gravitational search algorithm for fuzzy controlled servo systems. IEEE Trans. Ind. Informat. 8:791-800.

Rao, R.V. and V. Patel. 2012. An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems. IJIEC 3:535-560.

Rao, R.V., V.J. Savsani and D.P. Vakharia. 2011. Teaching-learning-based optimization: A novel method for constrained mechanical design optimization problems. Comput. Aided. Des. 43:303-315

Rashedi, E. and H. Nezamabadi. 2013. A stochastic gravitational approach to feature based color image segmentation. Eng. Appl. Artif. Intell. 26:1322-1332.

Reynolds, R.G. 1994. An introduction to cultural algorithms. In: Proceedings of the third annual conference on evolutionary programming. World Scientific; pp.131-139.

Sadollah, A., H. Eskandar, A. Bahreininejad, J.H. Kima. 2015. Water cycle algorithm with evaporation rate for solving constrained and unconstrained optimization problems. Appl. Soft Comput. 30:58-71.

Sheihan, M. and M.S. Rad. 2013. Gravitational search algorithm optimized neural misuse detector with selected features by fuzzy grids-based association rules mining. Neural Comput. Appl. 23:2451-2463.

Storn, R. and K. Price. 1997. Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. J. Global Optim. 11:341-359.

Tabassum, M. F., M. Saeed, N.A. Chaudary, J. Ali and A. Sana. Year. Integrated approach of set theory and pattern search methods for solving aircraft attacking problem. Pak. J. Sci. 69:136-143.

Tanabe, R. and A. Fukunaga. 2013. Evaluating the performance of SHADE on CEC 2013 benchmark problems. In: Evolutionary Computation (CEC), 2013 IEEE Congress; pp.1952-1959.

Wolpert, D.H. and W.G. Macready. 1997. No Free Lunch Theorem for Optimization. IEEE Trans. Evol. Comput. 1:67-82.

Yang, Y.S. 2010. Firefly algorithms for multimodal functions, stochastic algorithms: foundations and applications. SAGA 5792:169-178.

Yang, X.S. and S. Deb. 2010. Engineering optimization by cuckoo search. IJMMNO 1:30-43.

Zheng, Y.J. 2015. Water wave optimization: a new nature-inspired metaheuristic. Comput. Oper. Res. 55:1-11.

Prisenk, J. and J. Turk. 2015. A multi-goal mathematical approach for the optimization of crop planning on organic farms: A Slovienian case study. Pak. J. Agri. Sci. 52:971-979.