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Permuting Tri (α, β)-Derivation on Almost Distributive Lattices

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Abstract.: In this paper, we introduce the idea of permuting tri-(α, β)-derivation on ADL's and proved some results by using this notion. Let h be the trace of permuting tri (α, β)-derivation H on ADL G, if $\alpha \geq H$ and $\beta \geq H$, then $h(x \wedge y) = (\alpha(y) \wedge h(x)) \vee H(x, x, z) \vee H(y, y, z) \vee (\beta(x) \wedge h(y))$.

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1. INTRODUCTION

In 1975, Szasz initiated the idea of derivation in Lattices [11]. Xin studied this concept and proved some important results on derivations of lattices [12]. After these studies the idea of f-derivation and symmetric f bi-derivation and permuting f-tri-derivation of lattices were introduced in [1, 2, 4, 9]. In 2012, Khan et. al. generalized this idea by introducing new concept of (α, β)-generalized derivation of lattices [3].

The concept of ADL's was initiated by Rao and Swamy in 1981 [10]. They gave the new idea of Almost Distributive Lattices by relaxing some conditions of lattices. In 2016, Rao and Babu introduced the concept of symmetric bi-derivation on ADL's and in 2017, Rao et. al. introduced the idea of permuting tri-derivation in ADL's [6, 8]. They generalized this concept by using the notion of permuting tri f- derivation on ADL's [7]. In this paper, by generalizing the derivation defined in [5] and [7], we initiate the concept of permuting tri-(α, β)-derivation on ADL's.

2. PRELIMINARIES

Definition 2.1. [8] *An algebra* $(G, \vee, \wedge)$ *of type* $(2, 2)$ *is called an Almost Distributive Lattice if it satisfy the following axioms:*
$G_1 : (x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
$G_2 : x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
$G_3 : (x \vee y) \wedge y = y$
$G_4 : (x \vee y) \wedge x = x$
$G_5 : x \vee (x \wedge y) = x$, *for all* $x, y, z \in G$.

Lemma 2.2. [8] *For any* $x, y \in G$, *we have:*
(1) $x \wedge x = x$
(2) $x \vee x = x$
(3) $(x \wedge y) \vee y = y$
(4) $x \wedge (x \vee y) = x$
(5) $x \vee (y \wedge x) = x$
(6) $x \vee y = x$ *if and only if* $x \wedge y = y$
(7) $x \vee y = y$ *if and only if* $x \wedge y = x$.

Definition 2.3. [8] *An element* $0 \in G$ *is called zero element of* G, *if* $0 \wedge x = 0$, *for all* $x \in G$.

Lemma 2.4. [8] *If* G *has* 0, *then for any* $x, y \in G$, *we have:*
(1) $x \vee 0 = x$
(2) $0 \vee x = x$
(3) $x \wedge 0 = 0$
(4) $x \wedge y = 0$ *if and only if* $y \wedge x = 0$.

Definition 2.5. [8] *For any* $x, y \in G$, *we say that* x *is less than or equal to* y *and write* $x \leq y$, *if* $x \wedge y = x$ *or, equivalently,* $x \vee y = y$.

Definition 2.6. [8] *An element* $x \in G$ *is called maximal if, for any* $y \in G$, $x \leq y$ *implies* $x = y$.

Lemma 2.7. [8] *For any* $u \in G$ *the following are equivalent:*
(1) u *is maximal*
(2) $u \vee x = u$
(3) $u \wedge x = x$, *for all* $x \in G$.

Definition 2.8. [10] *For any* $x, y \in G$, x *is said to be compatible with* y *(written* $x \sim y$ *) if* $x \wedge y = y \wedge x$ *or equivalently,* $x \vee y = y \vee x$. *A subset* S *of* G *is said to b compatible if* $x \sim y$ *for all* $x, y \in S$.

Definition 2.9. [8] *A function* $\alpha : G \longrightarrow G$ *is said to be an ADL homomorphism if it satisfy the following:*
(i) $\alpha(x \wedge y) = \alpha(x) \wedge \alpha(y)$
(ii) $\alpha(x \vee y) = \alpha(x) \vee \alpha(y)$ *for all* $x, y \in G$.

Definition 2.10. [8] *A function* $h : G \longrightarrow G$ *is called an isotone, if* $h(x) \leq h(y)$ *for any* $x, y \in G$ *with* $x \leq y$.

Definition 2.11. [8] *A map* $H : G \times G \times G \longrightarrow G$ *is called permuting map, if* $H(x, y, z) = H(x, z, y) = H(y, z, x) = H(y, x, z) = H(z, x, y) = H(z, y, x)$*, for all* $x, y, z, \in G$.

Definition 2.12. [8] *A map* $H : G \times G \times G \longrightarrow G$ *is called an isotone map if, for any* $x, y, z, w \in G$ *with* $x \leq w$, $H(x, y, z) \leq H(w, y, z)$.

Definition 2.13. [8] *A mapping* $h : G \longrightarrow G$ *defined by* $h(x) = H(x, x, x)$, *for all* $x \in G$, *is called trace of* H.

Definition 2.14. [8] *A permuting map* $H : G \times G \times G \longrightarrow G$ *is called a permuting tri-derivation on G, if*
$H(x \wedge w, y, z) = [w \wedge H(x, y, z)] \vee [x \wedge H(w, y, z)]$, *for all* $x, y, w, z \in G$.

3. Permuting Tri (α, β)-Derivation on Almost Distributive Lattices

In this section, we proved some results by using the notion of permuting tri (α, β)-derivation on Almost Distributive Lattices.

Definition 3.1. *A permuting map* $H : G \times G \times G \longrightarrow G$ *is called a permuting tri* (α, β)*-derivation, if there exist functions* $\alpha, \beta : G \longrightarrow G$ *such that*
$H(x \wedge w, y, z) = [\alpha(w) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(w, y, z)]$, *for all* $x, y, z, w \in G$.
A permuting tri (α, β)*-derivation* H *on* G *also satisfies*
$H(x, y \wedge w, z) = [\alpha(w) \wedge H(x, y, z)] \vee [\beta(y) \wedge H(x, w, z)]$ *and*
$H(x, y, z \wedge w) = [\alpha(w) \wedge H(x, y, z)] \vee [\beta(z) \wedge H(x, y, w)]$, *for all* $x, y, z, w \in G$.

Example 3.2. *Let* G *be an ADL with* 0 *and* $a \neq 0 \in G$. *If we define a map* $H : G \times G \times G \longrightarrow G$ *by* $H(x, y, z) = a$, *for all* $x, y, z \in G$ *and* $\alpha : G \longrightarrow G$ *by* $\alpha(w) = a$ *and* $\beta : G \longrightarrow G$ *by* $\beta(x) = x \vee a$, *for all* $x \in G$, *then* H *is a permuting tri* (α, β)*-derivation on* G *but* H *is not permuting tri-derivation on* G.

Proposition 3.3. *Let* G *be an Almost Distributive Lattice and* H *be a Permuting tri* (α, β)*-derivation on* G, *then*
$H(x, y, z) \leq \alpha(x) \vee \beta(x)$, $\forall\, x, y \in G$.

Proof. Since $H(x, y, z) \wedge \alpha(x) \leq \alpha(x)$ and $\beta(x) \wedge H(x, y, z) \leq \beta(x)$
$H(x, y, z) = H(x \wedge x, y, z) = [\alpha(x) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(x, y, z)] = (\alpha(x) \vee \beta(x)) \wedge H(x, y, z)$. Hence $H(x, y, z) \leq \alpha(x) \vee \beta(x)$. □

Proposition 3.4. *Let* H *be a permuting tri-*(α, β)*-derivation on an ADL* G *and* x *is compatible to* y, *for* $x, y \in G$, *then* $H(x, y, z) \wedge H(y, y, z) \leq H(x \wedge y, y, z)$.

Proof. $H(x \wedge y, y, z) = [\alpha(y) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(y, y, z)]$.
It implies

$$\alpha(y) \wedge H(x, y, z) \leq H(x \wedge y, y, z) \;\; and \;\; \beta(x) \wedge H(y, y, z) \leq H(x \wedge y, y, z) \tag{3. 1}$$

By definition (2.8), we have
$H(x \wedge y, y, z) = H(y \wedge x, y, z) = [\alpha(x) \wedge H(y, y, z)] \vee [\beta(y) \wedge H(x, y, z)]$.
This gives

$$\alpha(x) \wedge H(y, y, z) \leq H(x \wedge y, y, z) \;\; and \;\; \beta(y) \wedge H(x, y, z) \leq H(x \wedge y, y, z) \tag{3. 2}$$

Combining equation (3. 1) and (3. 2), we obtain

$$[\alpha(x) \wedge H(y, y, z)] \vee [\beta(x) \wedge H(y, y, z)] \leq H(x \wedge y, y, z) \quad (3.\ 3)$$

Now by Proposition (3.3), we get
$H(x, y, z) \wedge H(y, y, z) \leq (\alpha(x) \vee \beta(x)) \wedge H(y, y, z) \leq [\alpha(x) \wedge H(y, y, z)] \vee [\beta(x) \wedge H(y, y, z)]$. Hence $H(x, y, z) \wedge H(y, y, z) \leq H(x \wedge y, y, z)$.

□

Proposition 3.5. *Let H be a permuting tri (α, β)-derivation on an ADL G, then $H(x \wedge y, y, z) \leq H(x, y, z) \vee H(y, y, z)$*

Proof. Let $x, y, z \in G$.

$$\alpha(y) \wedge H(x, y, z) \leq H(x, y, z) \quad (3.\ 4)$$

and

$$\beta(x) \wedge H(y, y, z) \leq H(y, y, z) \quad (3.\ 5)$$

$H(x \wedge y, y, z) = [\alpha(y) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(y, y, z)]$.
By equation (3. 4) and (3. 5), we have
$H(x \wedge y, y, z) \leq H(x, y, z) \vee H(y, y, z)$. □

Proposition 3.6. *Let h be a trace of permuting tri (α, β)-derivation on an ADL G. If G has least element 0 such that $\alpha(0) = 0$ and $\beta(0) = 0$, then $h(0) = 0$.*

Proof. By Proposition (3.3), we have
$H(x, y, z) \leq \alpha(x) \vee \beta(x)$. Since 0 is the least element therefore, $0 \leq H(0, 0, 0) \leq \alpha(0) \vee \beta(0) = 0 \vee 0 = 0$. Thus $0 \leq H(0, 0, 0) \leq 0$. From this we get $H(0, 0, 0) = 0$. Hence $h(0) = 0$. □

Proposition 3.7. *Let G be an ADL with greatest element 1 and 1 is compatible to x, H be a permuting tri (α, β)-derivation on G and $\alpha(1) = \beta(1) = 1$, then following hold:*
(1) *If $\alpha(x) \leq H(1, y, z)$ and $\beta(x) \leq H(1, y, z)$, then $H(x, y, z) = \alpha(x) \vee \beta(x)$*
(2) *If $\beta(x) \geq H(1, y, z)$, then $H(x, y, z) \geq H(1, y, z)$.*

Proof. Let $x, y, z \in G$.
(1) $H(x, y, z) = H(x \wedge 1, y, z) = [\alpha(1) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(1, y, z)] = [1 \wedge H(x, y, z)] \vee \beta(x) = H(x, y, z) \vee \beta(x)$. It implies

$$H(x, y, z) \geq \beta(x) \quad (3.\ 6)$$

Similarly $H(x, y, z) = H(1 \wedge x, y, z) = [\alpha(x) \wedge H(1, y, z)] \vee [\beta(1) \wedge H(x, y, z)] = [\alpha(x) \vee H(x, y, z)]$. This gives

$$H(x, y, z) \geq \alpha(x) \quad (3.\ 7)$$

By combining (3. 6) and (3. 7), we have

$$\alpha(x) \vee \beta(x) \leq H(x, y, z) \quad (3.\ 8)$$

By Proposition (3.3), we obtain

$$H(x, y, z) \leq \alpha(x) \vee \beta(x) \quad (3.\ 9)$$

Thus by equation (3. 8) and (3. 9), we have $H(x, y, z) = \alpha(x) \vee \beta(x)$.

(2) $H(x, y, z) = H(x \wedge 1, y, z) = [\alpha(1) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(1, y, z)] = H(x, y, z)$ $\vee\ H(1, y, z)$. We have $H(x, y, z) \geq$ H(1,y,z). □

Theorem 3.8. *Let G be an ADL with greatest element 1 and H be an isotone permuting tri (α, β)-derivation on G. Let $\alpha(1) = \beta(1) = 1$ and either $\alpha(x) \geq \beta(x)$ or $\alpha(x) \leq \beta(x)$, for all $x \in G$, then $H(x, y, z) = (\alpha(x) \vee \beta(x)) \wedge H(1, y, z)$.*

Proof. Since H is an isotone permuting tri (α, β)-derivation on G therefore, $H(x, y, z) \leq H(1, y, z)$.
Case 1: $\alpha(x) \geq \beta(x)$
$H(x, y, z) \leq \alpha(x) \vee \beta(x) = \alpha(x)$. It implies

$$H(x, y, z) \leq \alpha(x) \wedge H(1, y, z) \tag{3. 10}$$

$H(x, y, z) = H((x \vee 1) \wedge x, y, z) = [\alpha(x) \wedge H(x \vee 1, y, z)] \vee [\beta(x \vee 1) \wedge H(x, y, z)]$. Since 1 is greatest element of ADL therefore, $H(x, y, z) = [\alpha(x) \wedge H(1, y, z)] \vee [\beta(1) \wedge H(x, y, z)]$ $= [\alpha(x) \wedge H(1, y, z)] \vee [1 \wedge H(x, y, z)] = [\alpha(x) \wedge H(1, y, z)] \vee H(x, y, z)$. Thus by equation (3. 10), we have $H(x, y, z) = [\alpha(x) \wedge H(1, y, z)]$. Since $\alpha(x) \geq \beta(x)$, so we can write $H(x, y, z) = \alpha(x) \vee \beta(x) \wedge H(1, y, z)$.
Case 2: $\beta(x) \geq \alpha(x)$
$H(x, y, z) \leq \alpha(x) \vee \beta(x) = \beta(x)$ from this we get

$$H(x, y, z) \leq \beta(x) \wedge H(1, y, z) \tag{3. 11}$$

$H(x, y, z) = H(x \wedge (x \vee 1), y, z) = [\alpha(x \vee 1) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(x \vee 1, y, z)]$. Since 1 is greatest element of ADL therefore, $H(x, y, z) = [\alpha(1) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(x, y, z)]$ $= [1 \wedge H(x, y, z)] \vee [\beta(x) \wedge H(1, y, z)] = H(x, y, z) \vee [\beta(x) \wedge H(1, y, z)]$. Now by equation (3. 11), we have $H(x, y, z) = [\beta(x) \wedge H(1, y, z)]$. Since $\beta(x) \geq \alpha(x)$ therefore, $H(x, y, z) = (\alpha(x) \vee \beta(x)) \wedge H(1, y, z)$. □

Theorem 3.9. *Let h be the trace of permuting tri (α, β)-derivation on an ADL G, then $(\alpha(x) \vee \beta(x)) \wedge h(x) = h(x)$.*

Proof. By our hypothesis $h(x) = H(x, x, x) = H(x \wedge x, x, x) = [\alpha(x) \wedge H(x, x, x)] \vee [\beta(x) \wedge H(x, x, x)]$. By left distribution of meet over join, we have $h(x) = (\alpha(x) \vee \beta(x))$ $\wedge H(x, x, x) = (\alpha(x) \vee \beta(x)) \wedge h(x)$. □

Theorem 3.10. *Let H be a permuting tri (α, β)-derivation on G and $\alpha(x) \geq H(u, y, z)$, u and $\beta(u)$ are maximal in G, then $H(x, y, z) \geq H(u, y, z)$.*

Proof. $H(x, y, z) = H(u \wedge x, y, z) = [\alpha(x) \wedge H(u, y, z)] \vee [\beta(u) \wedge H(x, y, z)]$. Since $\beta(u)$ is maximal therefore, $H(x, y, z) = [\alpha(x) \wedge H(u, y, z)] \vee H(x, y, z)$. By our hypothesis, we obtain $H(x, y, z) = H(u, y, z) \vee H(x, y, z)$. Thus $H(x, y, z) \geq H(u, y, z)$.

□

Lemma 3.11. *Let H be a permuting tri (α, β)-derivation on G if $\alpha \leq \beta$, then $H(x, y, z)$ $= \beta(x) \wedge H(x, y, z)$.*

Proof. $H(x, y, z) = H(x \wedge x, y, z) = [\alpha(x) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(x, y, z)] = [\alpha(x) \vee \beta(x)] \wedge H(x, y, z)$. It implies $H(x, y, z) = \beta(x) \wedge H(x, y, z)$. □

Theorem 3.12. *Let H be a permuting tri (α, β)-derivation on G, then for any $x, y, z, w \in G$, following hold:*
(1) *If H is an isotone map on G with $\alpha \geq \beta$, then $H(x, y, z) = \alpha(x) \wedge H(x \vee w, y, z)$*
(2) *If β is an increasing function on G with $\alpha \leq \beta$, then $H(x, y, z) \geq \alpha(x) \wedge H(x \vee w, y, z)$.*

Proof. (1) Since H is an isotone map on G, therefore $H(x, y, z) \leq H(x \vee w, y, z)$. Also $H(x, y, z) = H((x \vee w) \wedge x, y, z) = [\alpha(x) \wedge H(x \vee w, y, z)] \vee [\beta(x \vee w) \wedge H(x, y, z)]$. By definition of isotone map, we have $H(x, y, z) = [\alpha(x) \wedge H(x \vee w, y, z)] \vee [\beta(x \vee w) \wedge H(x, y, z) \wedge H(x \vee w, y, z)] = [\alpha(x) \vee (\beta(x \vee w) \wedge H(x, y, z))] \wedge H(x \vee w, y, z)$ $= [(\alpha(x) \vee \beta(x \vee w)) \wedge (\alpha(x) \vee H(x, y, z))] \wedge H(x \vee w, y, z)$. Since $\alpha \geq \beta$, therefore $H(x, y, z) = [\alpha(x) \wedge (\alpha(x) \vee H(x, y, z))] \wedge H(x \vee w, y, z)$. Now by left absorption law, we get $H(x, y, z) = \alpha(x) \wedge H(x \vee w, y, z)$.

(2) Suppose β is an increasing function on G, then $\alpha(x) \leq \beta(x \vee w)$. Now $H(x, y, z)$ $= H((x \vee w) \wedge x, y, z) = [\alpha(x) \wedge H(x \vee w, y, z)] \vee [\beta(x \vee w) \wedge H(x, y, z)]$. By Lemma 3.11, we have $H(x, y, z) = [\alpha(x) \wedge H(x \vee w, y, z)] \vee [\beta(x \vee w) \wedge (\beta(x) \wedge H(x, y, z))] =$ $[\alpha(x) \wedge H(x \vee w, y, z)] \vee [(\beta(x \vee w) \wedge \beta(x)) \wedge H(x, y, z)]$. Since $\beta(x) \leq \beta(x \vee w)$, so $H(x, y, z) = [\alpha(x) \wedge H(x \vee w, y, z)] \vee [\beta(x) \wedge H(x, y, z)]$. By Lemma 3.11, we obtain $H(x, y, z) = [\alpha(x) \wedge H(x \vee w, y, z)] \vee H(x, y, z) = [\alpha(x) \wedge H(x \vee w, y, z)] \vee H(x, y, z)$. Thus $H(x, y, z) \geq \alpha(x) \wedge H(x \vee w, y, z)$. □

Theorem 3.13. *Let H be a permuting tri (α, β)-derivation on G with u and $\alpha(u)$ are maximal elements of G and α and β are homomorphisms on G, then the following are equivalent:*
(1) *H is an isotone map on G*
(2) *If $\alpha(x) \leq \beta(x)$, then $H(x, y, z) = \beta(x) \wedge H(u, y, z)$*
(3) *H is a join preserving map on G*
(4) *H is a meet preserving map on G.*

Proof. **(1)⇒ (2)**
Let H is an isotone map on G, $H(x, y, z) = H(x \wedge u, y, z) = [\alpha(u) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(u, y, z)]$.

$$H(x, y, z) \geq \alpha(u) \wedge H(x, y, z) \tag{3. 12}$$

and

$$H(x, y, z) \geq \beta(x) \wedge H(u, y, z) \tag{3. 13}$$

On the other hand, $\beta(x) \wedge H(x \wedge u, y, z) = \beta(x) \wedge [[\alpha(u) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(u, y, z)]]$ $= [(\beta(x) \wedge \alpha(u)) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(u, y, z)]$. Since $\alpha(u)$ is maximal so, $\beta(x) \wedge H(x \wedge u, y, z) = [\beta(x) \wedge H(x, y, z)] \vee [\beta(x) \wedge H(u, y, z)]$. Since $\alpha(x) \leq \beta(x)$, therefore by Lemma 3.11, we have $\beta(x) \wedge H(x \wedge u, y, z) = H(x, y, z) \vee [\beta(x) \wedge H(u, y, z)]$. By equation (3. 13), we get

$$\beta(x) \wedge H(x \wedge u, y, z) = H(x, y, z) \tag{3. 14}$$

Since H is an isotone map on G, therefore

$$H(x \wedge u, y, z) \leq H(u, y, z) \tag{3. 15}$$

By equation (3. 14), we have $\beta(x) \wedge H(x \wedge u, y, z) = H(x, y, z)$. This alongwith equation (3. 15) gives

$$H(x, y, z) \leq \beta(x) \wedge H(u, y, z) \qquad (3.\ 16)$$

By equation (3. 13) and (3. 16), we get $H(x, y, z) = \beta(x) \wedge H(u, y, z)$.

(2)⇒ (3)

Let $H(x, y, z) = \beta(x) \wedge H(u, y, z)$. Then $H(x \vee w, y, z) = \beta(x \vee w) \wedge H(u, y, z)$. Since α and β are Homomorphism on G, therefore, $H(x \vee w, y, z) = (\beta(x) \vee \beta(w)) \wedge H(u, y, z)$. This implies

$H(x \vee w, y, z) = [\beta(x) \wedge H(u, y, z)] \vee [\beta(w) \wedge H(u, y, z)]$. By using (2) we have, $H(x \vee w, y, z) = H(x, y, z) \vee H(w, y, z)$. Hence H is a join preserving maps on G.

(2)⇒ (4)

$H(x \wedge w, y, z) = \beta(x \wedge w) \wedge H(u, y, z)$. Since α and β Homomorphism on G, we have $H(x \wedge w, y, z) = (\beta(x) \wedge \beta(w)) \wedge H(u, y, z) = [\beta(x) \wedge H(u, y, z)] \wedge [\beta(w) \wedge H(u, y, z)]$ $= H(x, y, z) \wedge H(w, y, z)$. Hence H is a meet preserving maps on G. □

Theorem 3.14. *Let h be the trace of permuting tri (α, β)-derivation H on ADL G if $\alpha \geq H$ and $\beta \geq H$, then $h(x \wedge y) = (\alpha(y) \wedge h(x)) \vee H(x, x, z) \vee H(y, y, z) \vee (\beta(x) \wedge h(y))$.*

Proof. Since $\alpha(y) \wedge H(x, x \wedge y, x \wedge y) = \alpha(y) \wedge [(\alpha(y) \wedge H(x, x, x \wedge y) \vee (\beta(x) \wedge H(x, y, x \wedge y)))]$. This gives $\alpha(y) \wedge H(x, x \wedge y, x \wedge y) = [(\alpha(y) \wedge H(x, x, x \wedge y)) \vee (\alpha(y) \wedge (\beta(x) \wedge H(x, y, x \wedge y)))]$. Since $\beta \geq H$, therefore $\alpha(y) \wedge H(x, x \wedge y, x \wedge y) = [(\alpha(y) \wedge H(x, x, x \wedge y)) \vee (\alpha(y) \wedge H(x, y, x \wedge y))]$. Also $\alpha \geq H$, so $\alpha(y) \wedge H(x, x \wedge y, x \wedge y) = [(\alpha(y) \wedge H(x, x, x \wedge y)) \vee H(x, y, x \wedge y)]$. Now by definition of permuting tri (α, β)-derivation, we have $\alpha(y) \wedge H(x, x \wedge y, x \wedge y) = \alpha(y) \wedge [(\alpha(y) \wedge H(x, x, x)) \vee (\beta(x) \wedge H(x, x, y))] \vee [\alpha(y) \wedge H(x, y, x) \vee (\beta(x) \wedge H(x, y, y))]$. This implies $\alpha(y) \wedge H(x, x \wedge y, x \wedge y) = (\alpha(y) \wedge h(x)) \vee H(x, x, y) \vee H(x, y, x) \vee H(x, y, y)$. From this we get

$$\alpha(y) \wedge H(x, x \wedge y, x \wedge y) = (\alpha(y) \wedge h(x)) \vee H(x, x, y) \vee H(x, y, y) \qquad (3.\ 17)$$

This implies $\beta(x) \wedge H(y, x \wedge y, x \wedge y) = \beta(x) \wedge [(\alpha(y) \wedge H(y, x, x \wedge y)) \vee (\beta(x) \wedge H(y, y, x \wedge y))]$. Since $\alpha \geq H$ and $\beta \geq H$, therefore $\beta(x) \wedge H(y, x \wedge y, x \wedge y) = H(y, x, x \wedge y) \vee (\beta(x) \wedge H(y, y, x \wedge y))$. Now by definition of permuting tri (α, β)-derivation, we obtain $\beta(x) \wedge H(y, x \wedge y, x \wedge y) = [(\alpha(y) \wedge H(y, x, x)) \vee (\beta(x) \wedge H(y, x, y))]$ $\vee [\beta(x) \wedge [(\alpha(y) \wedge H(y, y, x)) \vee (\beta(x) \wedge H(y, y, y))]$. This implies $\beta(x) \wedge H(y, x \wedge y, x \wedge y)$ $= H(y, x, x) \vee H(y, x, y) \vee H(y, y, x) \vee \beta(x) \wedge H(y, y, y)$. From this we have

$$\beta(x) \wedge H(y, x \wedge y, x \wedge y) = H(y, p, p) \vee H(y, y, x) \vee (\beta(x) \wedge h(y)) \qquad (3.\ 18)$$

This gives $h(x \wedge y) = H(x \wedge y, x \wedge y, x \wedge y)$. This implies $h(x \wedge y) = (\alpha(y) \wedge H(x, x \wedge y, x \wedge y)) \vee (\beta(x) \wedge H(y, x \wedge y, x \wedge y))$. By equation (3. 17) and (3. 18), we have $h(x \wedge y) = (\alpha(y) \wedge h(x)) \vee H(x, x, y) \vee H(x, y, y) \vee (\beta(x) \wedge h(y))$. □

Conclusions: In this paper, we have initiated the idea of permuting tri (α, β)-derivation on Amost Distributive Lattices and by using this notion we have proved significant results in ADL's.

REFERENCES

1. Y. Ceven, *Symmetric bi-derivation of lattices*, Quaestiones Mathematcae, **32,**No. 2(2009) 241-245.
2. M. A. Chaudhry and A.R. Khan, *On symmetric f-bi-derivations of lattices*, *Quaestiones Mathematcae*, **35,** No.2(2012) 203-207.
3. A. R. Khan, M. A. Chaudhry and I. Javaid, *On (α,β)-generalized derivations on lattices*, Ars Combinatoria, **105**(2012)525-533.
4. A. R. Khan and M .A Chaudhry, *Permuting f-triderivation on lattices*, International Journal of Algebra, **5,**No. 10(2011) 471-481.
5. S. A. Ozbal and A. Firat, *Symmetric f-biderivation of lattices*, Ars Combinatoria **97**(2010) 471-477.
6. G. C. Rao and K. R. Babu,*Permuting tri-derivations and generalized d-derivations on almost distributive lattices*, Southeast Asian Bulletin of Mathematics **42,** No. 6(2018) 907-919.
7. G. C. Rao and K. R. Babu,, *The theory of derivations in almost distributive lattices*, Bulletin of International Mathematical Virtual Institute, **7,** No. 2(2017) 203-216.
8. G. C. Rao and K. R. Babu, *Permuting tri-f-derivations on almost distributive lattices*, Bulletin of International Mathematical Virtual Institute, **7,** No. 2(2017) 317-325.
9. G. C. Rao and K. R. Babu, *Symmetric Bi-derivation in Almost Distributive Lattices*, Discussion Mathematicae-General Algebra and Applications, **36**, No. 2(2016) 169-177.
10. U. M. Swamy and G.C. Rao, *Almost distributive lattices*, J. Austral. Math. Soc., **31**(1981) 77-91.
11. G. Szasz G., *Derivations of lattices*, Acta Sci. Math., **37**(1975) 149-154.
12. X. L. Xin, T.Y. Li, and J. H. Lu,*On derivations of lattices*, Information sciences, **178,** NO. 2 (2008) 307-316.