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Novel computational study on MHD flow of nanofluid flow with gyrotactic microorganism due to porous stretching sheet

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Abstract.: The study of motile microorganism and the nano-particles of the MHD fluid flow over the porous stretching sheet with buoyancy forces are made. Boundaries conditions along non-zero mass flux are impose. We implement the dimensionless variable technique and the systems of coupled non-linear partial differential equations are transformed into ordinary differential equations by using the appropriate similarity transformation. Moreover, by using package ND-Solve on Mathematica problem is numerically integrated with the help of shooting technique. The ascendency of arising physical interpretation of thermo-physical parameter on temperature distribution, mass concentration and microorganism profile are obtained and observed. The evaluations are executed graphically and numerically for numerous prominent parameters Magnetic parameter, Porosity parameter, Deborah number, mixed convection parameter, radiation parameter, bio convection Rayeigh number, thermophoresis diffusion, Brownian motion, bio convection Peclet number, density of the motile microorganism and nano-particles versus velocity, temperature, concentration fields are also discussed numerically in detail. It is noted that the field of microorganisms decreases by raising the value of bio-convection and Peclet number.

AMS (MOS) Subject Classification Codes: 82D80; 92C70; 76Zxx; 65Nxx
Key Words: Nano fluid, Microorganism, Bio convection, shooting method.

1. INTRODUCTION

Now a day different industries use fluids to maximize the production by growing/reducing energy flow of the system. Various physical performances in production process like thermal conductivity, cooling and heat capacity are contributed by these fluids. Heat and thermal conductivity can produce different effect on final product as compare to the original one. Mega industries like polymer extrusion unit, power plant, paper production unit, glass fiber unit and chemical process unit mainly work on the role of heat transmission capability. The uniqueness flows properties of fluid are used in industries to making different products like paints, food stuff and printer etc. Flow properties like Newtonian and non-Newtonian are not the same. Newtonian fluids are those fluids that satisfied the Newton law of viscosity like ethyl alcohol, benzene and water etc. In our daily life different materials can act like non-Newtonian fluids. Such materials are syrup, ketchup, custard, shampoo, honey, paint, molten and blood. Newtonian fluids show linear relation among shear rate as well as shear stress while non-Newtonian fluids have vice versa relation. It may be time dependent or independent. Non-Newtonian fluids change their properties with respect to the amount of application of shear stress. Nanofluid is modern sorts of heat transfer source having nano-particles with shape less than one hundred (nm). Nanofluids cover two portions depending upon base fluids and nano-particles which are sturdily spread and homogenous lying in base fluid. Commonly these distributed nano-particles are metal or its oxides especially use to improve the nanofluids thermal conductivity. If we want to move heat transfer in fluid we should increase the conduction and convection coefficients which are examined by Choi [5]. Nanofluids have many advantages due to the heat transfer intensification like dispersion stability. Large area of surface leads to more heat transfer between fluids and particles. The absorption of heat energy is directly investigated by [28]. In 1999 Lee et al. [17] examined the increase in thermal conductivity which is main feature of nanofluids. The time independent boundary layer stream and two dimension flow of nanofluid passing through stretching surface is studied by [23]. By open working on nanofluids recommended the stretching sheet to explore their problem of flow [11]. The impact of particles in the form of dust suspended with in a nanofluid flow using a stretching medium is studies by [6]. Many researchers including [21] examined the impact of heat transfer and thermal diffusion of nanofluid passing through a stretching surface. On the other hand, the interaction between the electromagnetic fields and the conducting fluids is addressed by magnetohydrodynamics MHD. In science and engineering, it has countless applications primarily in designing heat exchanger equipment, MHD accelerators and generators, thermal insulation systems and plasma containment. There are plenty of research concerning Maxwell fluid and MHD flow. In the presence of heat source, [22] provided a heat transfer assessment of Maxwell fluid unstable MHD flow over a stretching surface. MHD two-dimensional nanofluid flow with diffusion of thermophoresis and Brownian motion was studied by [8]. Entropy generation through a rotating disk in slip and MHD flow problem with different characteristics was examined by [26]. Time dependent MHD fluid flow of nano-second grade with thermal radiation and mixed convention induced by permeable vertical sheet was analyzed by [25]. Microorganisms such as microalgae and bacteria are denser than water, allowing them to swing back to gravity. These random and voluntary movements of microorganisms are known as bio-convection. Microorganisms are denser than fluid, resulting a rise occur in fluids density due to their movement. Different stimulators such as chemo taxis, picture synthesis, gyrotactic etc. are simulating with this mobility of microorganisms. Many authors contributed on this topic, to cite few are including [15]-[3]. Thermal radiation as a heat allocation mode is an important and interesting phenomenon for researchers because of its significance in photochemical reactors, solar lakes, combustion, nuclear energy plants, turbines, solar collectors and numerous impulse devices used in engineering fields such as space vehicles, satellites, missiles and so on. Many authors contributed to this topic such as [4] & [30]. The sort of medium containing pores is known as porous media, where typically these pores are filled with fluid. Due to its various applications such as water flow in heat exchangers, reservoirs, catalytic reactors and petroleum manufacturing etc. The notion of fluid flow in porous media is of excellent concern to technicians, mathematicians, geologists and architectures. The heat transfer method in porous media has been used extensively in paper manufacturing, heat pipe technology, non-woven materials, power storage and electronic technology etc. [20] & [32]. The recent literature for further interest of readers and more understanding about the topic they may read [7]-[2]. The study of literature shows that such analysis has not yet been carried out. The consideration of non-Newtonian nanofluid, gyrotactic microorganism, thermal radiation and mixed convection has motivated us for this study. We also are going to solve this boundary value problem numerically. After converting it into first order initial value problem we apply shooting technique on it. Further, similarity variable and shooting scheme is used to solve the problem numerically. The physical characteristics of efficient parameters are underlined graphically and debated for the profiles concerned.

2. Mathematical Formulation

Consider steady MHD flow of nanofluid embedded in porous stretching surface with gyrotactic microorganism. We assume that the surface is stretching in x-direction with stretching rate further the fluid is subjected to applied magnetic field of strength. In the presence of mixed convection, chemical reaction and thermal radiation, the related governing equations of nanofluid becomes.

The equation of mass conservation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2. 1}$$

Equation of Momentum

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_e B_0^2 u}{\rho_f}-\frac{\nu u}{K}+$$
$$\frac{1}{\rho_f}\left[(1-C_\infty)\,g\,(T-T_\infty)\,\beta\rho_f-(\rho_p-\rho_f)\,g\,(C-C_\infty)-\gamma\,(n-n_\infty)\,g\,(\rho_m-\rho_r)\right], \tag{2. 2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)\tau\left\{\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2+D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right\}-\frac{1}{(\rho c)_f}\frac{\partial \hat{q}_r}{\partial y}+\frac{Q_0}{(\rho c)_f}\left(T-T_\infty\right), \tag{2. 3}$$

Concentration Equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K_0\left(C-C_\infty\right), \tag{2. 4}$$

Density of Microorganism

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\left[\frac{bW_c}{C_W-C_\infty}\right]\left(\frac{\partial N}{\partial y}\frac{\partial C}{\partial y}+N\frac{\partial^2 C}{\partial y^2}\right)=D_n\frac{\partial^2 N}{\partial y^2}, \tag{2. 5}$$

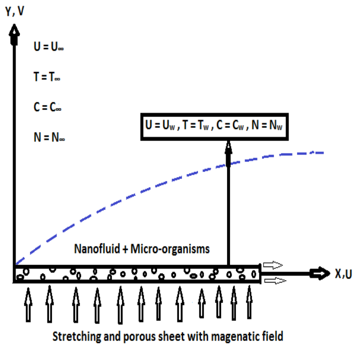

FIGURE 1. Geometry of the problem

Equations (2. 2) to (2. 5) are subjected to the following boundary conditions:

$$v = v_0, u = \lambda U_w, T = T_w, C = C_w, N = N_w \quad at \quad y = 0, \tag{2. 6}$$

$$u \to 0, T \to T_\infty, C \to C_\infty, N \to N_\infty \quad as \quad y \to \infty. \tag{2. 7}$$

3. SIMILARITY TRANSFORMATION

Let us adopt the following similarity variables, to convert PDEs to ODEs [31] & [1]:

$$\psi = (av)^{\frac{1}{2}} x f(\eta), \eta = \left(\frac{a}{v}\right)^{\frac{1}{2}} y, \tag{3. 8}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \chi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}, \tag{3. 9}$$

where η is the similarity variable and ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, which identically satisfies the equation of continuity. Now substituting (3. 8) and (3. 9) into the Eqs. (2. 2)(2. 5), we get

$$f''' + f f'' - f'^2 - (M + K_1) f' + \lambda (\theta - N_r \phi - N_c \chi) = 0, \tag{3. 10}$$

$$\frac{1}{p_r}\left(1 + \frac{4Rd}{3}\right)\theta'' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 + \Delta\theta = 0, \tag{3. 11}$$

$$\phi'' + Lef\phi' + \frac{N_t}{N_b}\theta'' - Le\gamma\phi = 0, \tag{3. 12}$$

$$\chi''(\eta) + S_c f(\eta)\chi'(\eta) - P_e\big[\chi(\eta)\phi'(\eta) + \sigma\phi''(\eta) + \chi'(\eta)\phi''(\eta)\big] = 0. \tag{3. 13}$$

Boundary condition (2. 6) and (2. 7) becomes:

$$f(0) = S, f'(0) = \lambda, \chi(0) = 1, \theta(0) = 1, \phi(0) = 1, \tag{3. 14}$$

$$f'(0) = 0, \theta = 0, \phi = 0, \chi = 0 \quad as \quad \eta \to \infty. \tag{3. 15}$$

Where Pr represent the Prandtl number, Nb for Brownian motion parameter, Sc is the Schmidt number, Nt is the thermophoresis parameter, Pe is the bio convection Peclet number, ∇ is heat generation/absorption coefficients, M is the magnetic parameter number, and σ is a dimensionless constant, Le is the Lewis number, λ is mixed convection parameter, Nr buoyancy ratio parameter, Nc bio convection Rayleigh number, γ is chemical reaction parameter, K for Porous medium parameter, and Rd is the thermal radiation parameter are defined as

$$R_d = \frac{4\sigma^* T_\infty^3}{k^* k}, N_b = \frac{\tau D_B(C_w - C_\infty)}{\nu}, Nt = \frac{\tau D_T(T_w - T_\infty)}{\nu T_\infty}, K_1 = \frac{\nu}{aK}, S_c = \frac{v}{D_n}, \Delta = \frac{Q_0}{a(\rho c)_f},$$
$$M = \frac{\sigma_e B_0^2}{\alpha \rho_f}, P_e = \frac{bWc}{\nu}, \sigma = \frac{N_\infty}{(N_w - N_\infty)}, \sigma = \frac{N_\infty}{(N_w - N_\infty)}, \lambda = \frac{\beta g\,(1 - C_\infty)\,(T_w - T_\infty)}{ax^2},$$
$$N_r = \frac{(\rho_p - \rho_f)\,(C_w - C_\infty)}{\rho_f\,(1 - C_\infty)\,(T_w - T_\infty)\,\beta}, N_c = \frac{\gamma\,(\rho_m - \rho_r)\,(n_w - n_\infty)}{\rho_f\,(1 - C_\infty)\,(T_w - T_\infty)\,\beta}, P_r = \frac{\nu}{\alpha} = \frac{k}{\nu(\rho c)_f},$$
$$\alpha = \frac{k}{\rho c}_f, \gamma = \frac{K_0}{a}\phi(\eta), Le = \frac{\nu}{D_B}.$$

The physical quantities of interest are define as

$$C_f = \frac{\tau_w}{\rho U_w^2}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, Nn_x = \frac{xq_m}{D_n(N_w - N_\infty)}, \tag{3. 21}$$

$$C_f Re_x^{(\frac{1}{2})} = -f^{''}(0), Nu_x Re_x^{(\frac{1}{2})} = -\theta^{'}(0), Sh_x Re_x^{(\frac{1}{2})} = -\phi^{'}(0), Nn_x Re_x^{(\frac{1}{2})} = -\chi^{'}(0), \tag{3. 22}$$

where $Re_x^{(\frac{1}{2})} = U_w(\frac{x}{v})$ is the local Reynolds number.

4. Numerical Techniques

Many physical systems produce extremely nonlinear differential equations for which exact solutions are usually not feasible. The shooting scheme is one of the popular and very useful numerical methods among all methods to fix any kind of boundary value problem. This technique is sensitive straightforward and free from complex discretization. One of the distinctive characteristics of this technique is that by using smart original guesses, missing boundary condition can be created. Using this method a good precision of solution convergence is noted. The resulting first order scheme of differential equations is to apply the shooting method, let us denote f by z_1, θ by z_4 , ϕ by z_6 and χ by z_8 . The subsequent equations are

$$\begin{aligned}
z_1^{'} &= z_2, \\
z_2^{'} &= z_3, \\
z_3^{'} &= z_2 z_2 - z_1 z_3 + (M + K) z_2 - \lambda(z_4 - Nr z_6 - Nc z_8), \\
z_4^{'} &= z_5, \\
z_5^{'} &= -\left(\frac{Pr}{(1 + \frac{4Rd}{3})}\right)\big(z_1 z_5 + Nt(z_5 z_5) + Nb(z_5 z_6) + \Delta z_4\big), \\
z_6^{'} &= z_7, \\
z_7^{'} &= -\big(Le z_1 z_7 + (Nt/Nb) z_5^{'}\big) - Le\gamma z_6, \\
z_8^{'} &= z_9, \\
z_9^{'} &= Pe\big(z_8 z_7 + z_7^{'}(z_9 + \sigma)\big) - Sc(z_1 z_9).
\end{aligned} \tag{4. 23}$$

5. RESULTS

In this section, we discussed the graphical result of different parameters $M, K, Nc, Nr,$ $S, \lambda, Nt, \Delta, Nb, Pr, Rd, Le, Pe, Sc$ and γ on non-dimensional velocity, non-dimensional fluid temperature, dimensionless fluid concentration and non-dimensional motile micro-organism profiles. The first order system of equations (4. 23) together with boundary and initial conditions (3. 14) and (3. 15) is solved by using the command ND solve on Mathematica. For this the fixed values of some parameters are chosen arbitrarily as given in the following: $M = 1; S = 2; \lambda = 1; Sc = 1; \sigma = 0.1; Nb = 0.5; \gamma = 0.1; Nt = 0.5; Pe = 1; K = 1; Pr = 1; Le = 1; Nr = 0.5; Nc = 0.5; \lambda^{'} = 1; R = 0.1; \Delta = 0.5.$

5.1. **Velocity Profile.** In this subsection, we discuss some parameters appeared in velocity profile in the form of plots. The magnetic parameter M, porosity parameter K, Mixed convection parameter λ , bio-convection Rayleigh number Nc and buoyancy number Nr are plotted against function of velocity $f^{'}$ in Figs. (2)-(6). The impact of parameter M on fluid velocity function $f^{'}$ is presented in Fig. (2). The curve of flow speed is reducing with raise in the values of M, it is due to the interaction of opposing force namely Loretnz force is increasing. Fig. (3) showing the impact of porosity parameter K on fluid velocity profile $f^{'}$. The speed of flow shows down with increment in the values of permeable parameter. Similarly, the increase in the values of λ in stretching case is shown in Fig. (4). The impact of Nc bio-convection Rayleigh number on $f^{'}$ is shown in Fig. (5). It is clear that when variation in the values of Nc raises leads to reduction in the values of $f^{'}$. Actually, due to bio-convection, the higher values of Nc are ascribed to the buoyancy force as a conclusion to the raise in the profile of velocity. The impact of Nr on the profile of $f^{'}$ presented in Fig. (6) is showing vice versa impact of Nc.

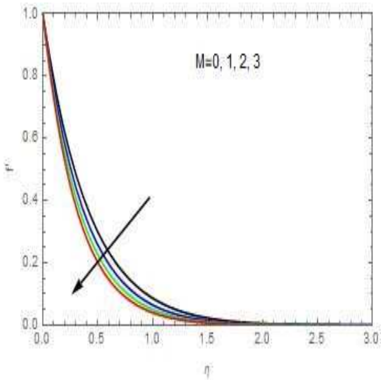

FIGURE 2. Graph pattern of f' for various values of M.

5.2. **Temperature profile.** In this subsection the impacts of porosity parameter K, generation coefficients Δ, Mixed convection parameter λ, Nb Brownian motion, thermophoresis parameter Nt, Prandtl number Pr and radiation parameter Rd on θ are depicted in Figs. (7)-(13). Fig. (7) shows the increment in K with respect to increase in temperature. The impact of heat Δ generation coefficients can be seen in Fig. (8). It can be seen that the Δ and the temperature profile has direct and increasing behavior. Similarly, the increases in the values of λ in stretching case rise in the temperature profile θ as given in Fig. (9).

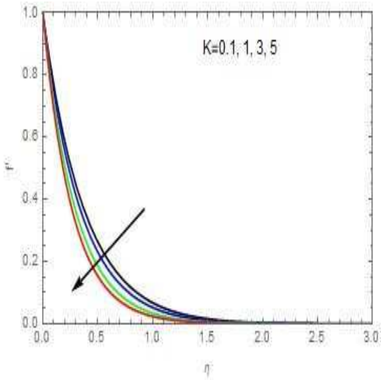

FIGURE 3. Graph pattern of f' for various values of K.

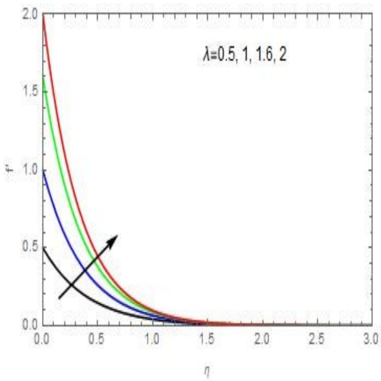

FIGURE 4. Graph pattern of f' for various values of $\lambda \geq 0$.

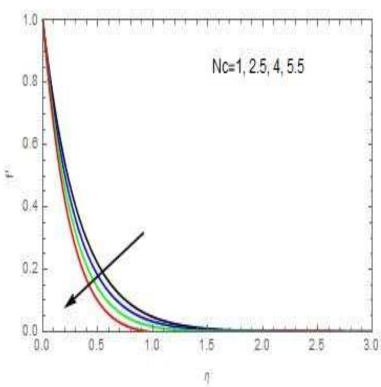

FIGURE 5. Graph pattern of f' for various values of Nc.

Fig. (10) indicates that the increasing Nc definitely leads to increase in temperature θ. The parameter Nt showing increasing relation with the increase in temperature profile as shown in Fig. (11). Fig. (12) shows that the increasing Prandtl Number Pr markedly

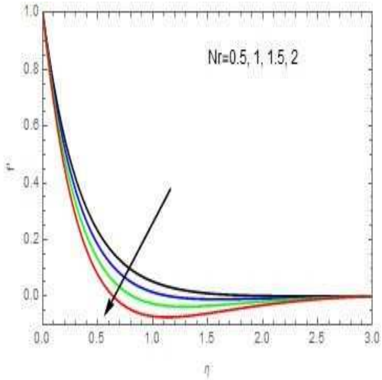

FIGURE 6. Graph pattern of f' for various values of Nr.

decreases the temperature profile. Radiation parameter Rd showing a significant rise in the temperature profile as plotted in Fig. (13).

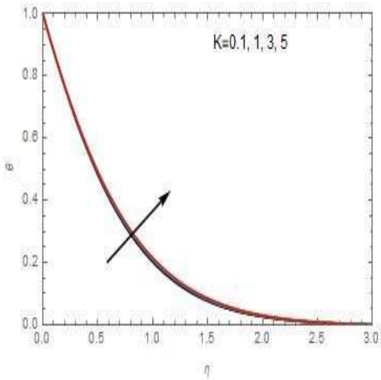

FIGURE 7. Graph pattern of θ for various values of K.

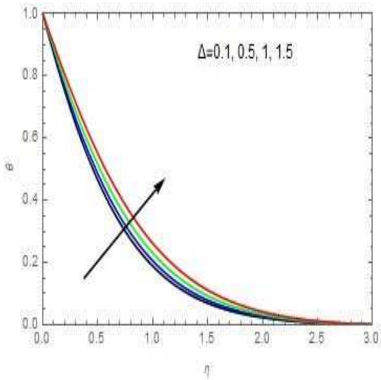

FIGURE 8. Graph pattern of θ for various values of $\Delta > 0$.

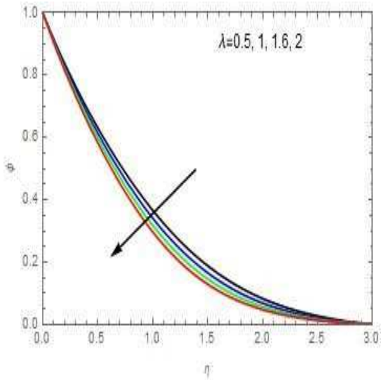

FIGURE 9. Graph pattern of θ for various values of $\lambda \geq 0$.

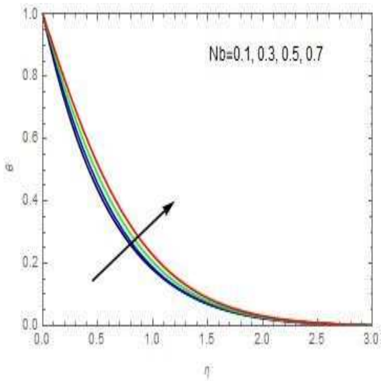

FIGURE 10. Graph pattern of θ for various values of Nb.

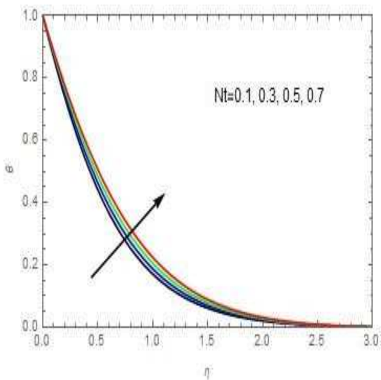

FIGURE 11. Graph pattern of θ for various values of Nt.

5.3. **Concentration profile.** In this subsection the effects of chemical reaction γ, λ, Le, Nb and Nt on concentration profile ϕ are depicted in Figs. (14)-(18). Fig. (14) indicates the impact of chemical reaction γ on space concentration function. It can be seen that the

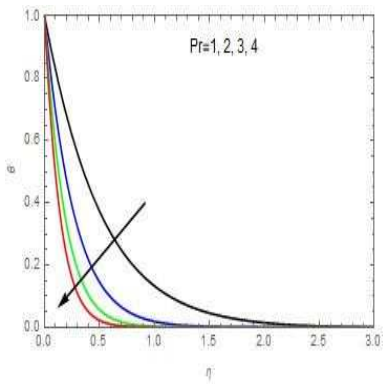

FIGURE 12. Graph pattern of θ for various values of Pr.

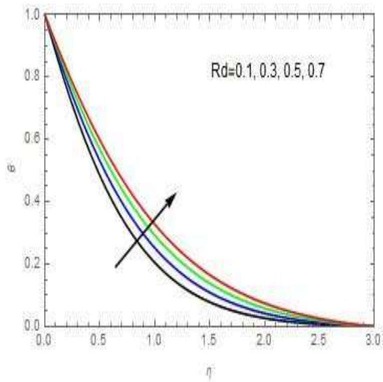

FIGURE 13. Graph pattern of θ for various values of Rd.

fluid concentration profile increase with raise in value of γ parameter. Figs. (15)-(17) respectively presents the decreasing pattern of fluid concentration ϕ for increasing values of λ, Le and Nb. The curve of concentration profile increase with raise in thermophoresis parameter Nt as shown in Fig. (18).

5.4. **Motile microorganisms density profile.** In this subsection, the effects of chemical reaction $\gamma, \lambda, Le, Nb, Nt, Pe, Sc$ and σ on density of motile micro-organism profile χ. Fig. (19) shows the impact of chemical reaction γ on density motile density profile ϕ. It can be seen that the density profile increases with increase in the values of γ. Similarly, the increase in the values of λ decreases the density profile χ as displayed in Fig. (20). The impact of Le the Lewis number and Nb on density of motile profile χ represents in Figs. (21)-(22). The curve of motile density decreases with increase in the values of Le and Nb. The profile of motile density increase with raise in thermophoresis parameter Nt as shown in Fig. (23). The graph of bio-convection Peclet number Pe, Schmidt number Sc and dimensionless parameter σ on motile density profile is given in Figs. (24)-(26). It can be seen that the density function decrease with increase in the values of σ, Sc and Pe.

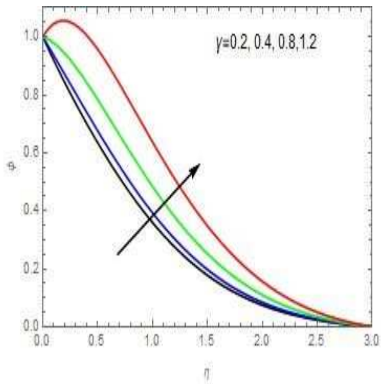

FIGURE 14. Graph pattern of ϕ for γ chemical reaction.

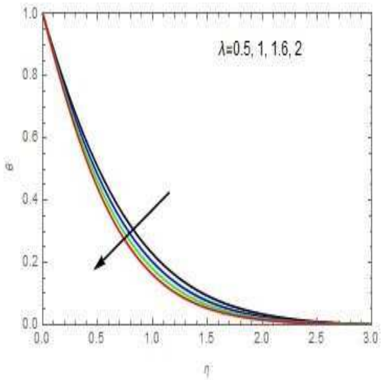

FIGURE 15. Graph pattern of ϕ for $\lambda \geq 0$.

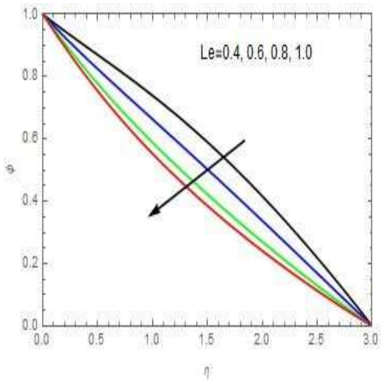

FIGURE 16. Graph pattern of ϕ for Le.

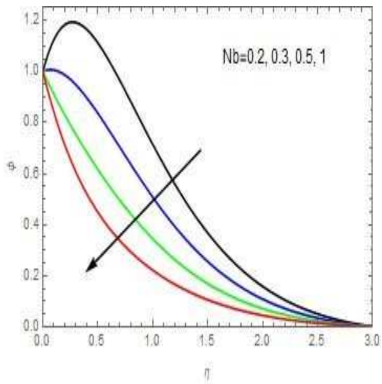

FIGURE 17. Graph pattern of ϕ for Nb.

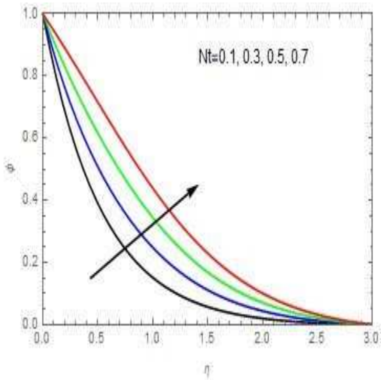

FIGURE 18. Graph pattern of ϕ for Nt.

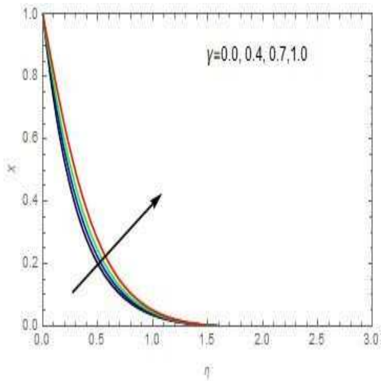

FIGURE 19. Graph pattern of χ for some values of γ chemical coefficient.

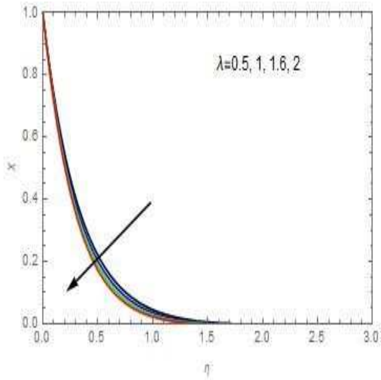

FIGURE 20. Graph pattern of χ for some values of $\lambda \geq 0$.

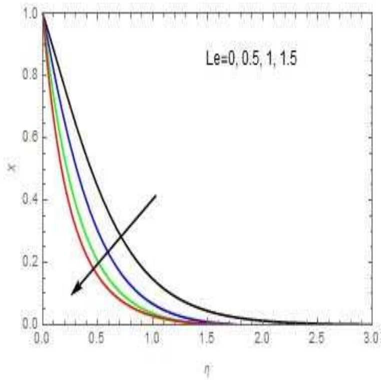

FIGURE 21. Graph pattern of χ for some values of Le.

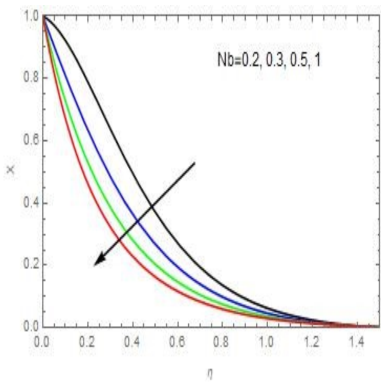

FIGURE 22. Graph pattern of χ for some values of Nb.

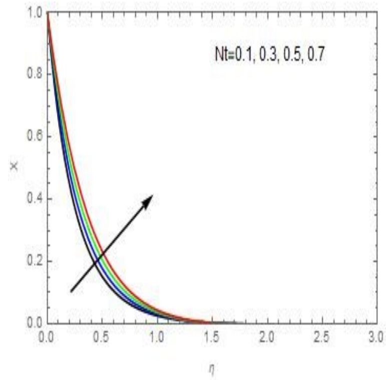

FIGURE 23. Graph pattern of χ for some values of Nt.

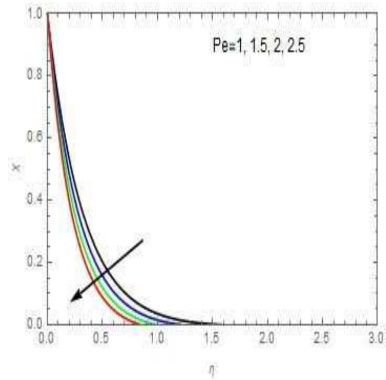

FIGURE 24. Graph pattern of χ for some values of Pe.

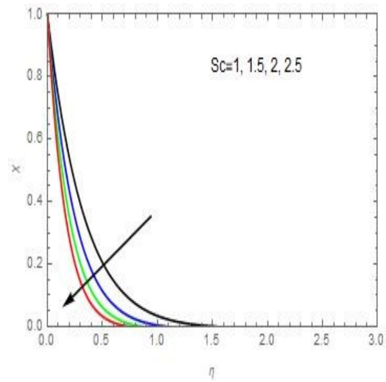

FIGURE 25. Graph pattern of χ for some values of Sc.

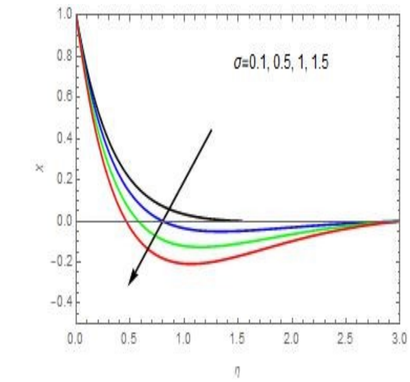

FIGURE 26. Graph pattern of χ for some values of σ.

6. Applications

Table I and **II** showing the present results for $-f^{''}(0), -\theta^{'}(0), -\phi^{'}(0)$ and $-\chi^{'}(0)$ are for magnetic parameters M and stretching parameter λ.

Table I

Values of $-f^{''}(0), -\theta^{'}(0), -\phi^{'}(0)$ and $-\chi^{'}(0)$ for various values of M when $\lambda^{'} = 0.1, R = 0.1, \lambda = 0.1, Sc = 1, Le = 1, \sigma = 0.2, Nt = 0.1, S = 1, Nb = 0.1,$ $\gamma = 0.1, Nc = 0.1, Pe = 0.3, K = 0.1, Nr = 0.1, \Delta = 0.5$ and $Pr = 6.2$.

M	$-f^{''}(0)$	$-\theta^{'}(0)$	$-\phi^{'}(0)$	$-\chi^{'}(0)$
0	1.56714	0.69753	0.770793	2.1117
1	1.94426	0.672445	0.762699	2.08181
1.5	2.10451	0.662717	0.760327	2.07094
2	2.25157	0.654285	0.75859	2.06180

Table II

Values of $-f^{''}(0), -\theta^{'}(0), -\phi^{'}(0)$ and $-\chi^{'}(0)$ for various values of λ when $Nr = 0.1; \Delta = 0.5; Pr = 6.2; Nt = 0.5; Sc = 1; \sigma = 1; Nb = 0.5; S = 2.5;$ $Pe = 1; K = 0.1; R = 0.1; Nc = 0.1; \lambda^{'} = 0.1; \gamma = 0.1; M = 0; Le = 2.$

λ	$-f^{''}(0)$	$-\theta^{'}(0)$	$-\phi^{'}(0)$	$-\chi^{'}(0)$
0	0.0030	1.47131	0.97732	3.57150
1	2.8508	1.59792	1.09056	3.84566
1.5	4.5192	1.65224	1.14145	3.96715
2	6.3063	1.70241	1.18923	4.08079

7. Conclusion

The main discoveries of this work are as follows

- The impact of magnetic field is reducing in all the physical quantities of interest.
- The velocity profile for nanofluids decreases by increasing suction parameter $S > 0$.
- The velocity function decreases by increasing in Nb and Nt.
- The velocity function nanofluids decreases by increasing buoyancy ratio parameter Nr, porosity parameter K and Nc.
- The temperature function for nanofluids decreases by increasing in Prandtl number Pr.
- The temperature nanofluids profile increases by increasing Brownian motion parameter Nb, thermal radiation Rd, heat generation parameter and thermophoresis parameter Nt.
- The density of the microorganism is enhanced by raising the thermophoresis parameter Nt, while convection Peclet number Pe, Lewis number Le, and Schmidt number Sc, the vice versa relation is noted.

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9. Contribution

This work is done by all authors equally. Authors Muhammad Jawad, Rabia Safder, Khurrem Shehzad & Sajjad Hussain managed the analysis of the study, achieved the statistical analysis and obtained the numerical solution of problem. Wrote the draft of the study, literature review and managed the technical aspects. All authors read and approved the final manuscript.

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