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Choice under Uncertainty; 'Allais Paradox' and its Paradoxical Implication

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Abstract

Human attitude towards risk is mixed. However, looking at the persuasive argument of diminishing marginal utility of wealth, academia has adopted risk aversion as the norm. Inculcating risk aversion, expected utility hypothesis (EUH) is used to rank risky options. 'Allais paradox' contradicted EUH but promoted the conviction of risk aversion intuitively on the basis of certainty effect. Most of the later studies also did not test risk aversion in strict statistical terms. Consequently the confusion initiated by 'Allais paradox' persists. This research has investigated three possible reasons for this paradoxical implication. One, Allais study put up such option-pairs which were not testable for risk aversion statistically. Two, by posing similar option-pairs, later researchers confirmed 'Allais paradox' and found even more systematic violations of EUH which further discredited EUH as an analytical tool. However, a few of these studies tested risk aversion directly. Three, analogy of the argument of diminishing utility of wealth and law of diminishing utility of a commodity holds if probability distributions in competing options are symmetric, otherwise it may or may not hold. Since all options in Allais study had asymmetric distributions, therefore violation of EUH should not have promoted the conviction of risk aversion intuitively. To clear this confusion, this paper recommends simultaneous testing of EUH and risk aversion and equal emphasis on the certainty and the big amount effects which tilt human attitude towards risk aversion and risk-seeking respectively.

Key Words: Choice under uncertainty; expected utility hypothesis; risk

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aversion; Allais paradox; certainty effect.

1. Introduction

In real life, human attitude towards risk is not consistent. On the one side, there are people who pay certain amount of money to assume risk while knowing the fact that the expected payoff of risky venture is, on the average, less than the money paid for assuming it. Such people buy lottery tickets, bet on various games and try their luck in gambling dens and casinos. In economic terminology, they show a risk-seeking attitude. On the other side, there are people who pay for getting rid of risk while knowing the fact that the expected value of uncertain loss is, on the average, less than the money paid for insuring it. Such people buy insurance policies of various kinds like car, health and life. They are therefore categorized as risk-averse. Furthermore, every individual is neither exclusively risk-seeking nor exclusively risk-averse; rather he/she is risk-seeking in some matters or at some times and risk-averse in other matters or at other times.[1]

In such an intermingled nature of human attitude towards risk, it is really difficult to prove whether humans are predominantly risk-seeking or risk-averse. However, looking at the persuasive argument of diminishing marginal utility of wealth, which is in line with the law of diminishing marginal utility that applies to every commodity in microeconomic analysis of consumer behavior, risk aversion has been taken as the norm and risk-seeking as an exception for choice under uncertainty in economic literature.[2] Diminishing marginal utility of wealth implies that disutility derived from a dollar's loss is always greater than the utility derived from a dollar's gain. Therefore, a utility maximizer must be risk-averse as he/she would not play a fair game for which expected payoff is equal to its price, not to speak of an unfair game. He/she

[1]In this regard, Domodaran (2008) writes in chapter 2, "The same person who puts his life at risk climbing mountains may refuse to drive a car without his seat belt on or to invest in stocks, because he considers them to be too risky."
[2]See, for example, Bernoulli (1738/1954) and Domodaran (2008) chapters 2-3.

would play only if expected payoff of the game is greater than its price.

Von Neumann and Morgenstern (1944) developed this concept to its complete axiomatic form known as Expected Utility Hypothesis (EUH). It provides a single-parameter criterion for evaluation of competing investments or bets. Each investment or bet has a unique expected utility calculated as $EU = \Sigma\ U(x_i)\ p_i$ where x_i denotes all possible payoffs $x_1, x_2, \ldots, x_n$; $U(x_i)$ denotes ordinal utility value attached subjectively to each payoff such that the total utility function against payoffs takes the shape of an increasing concave curve and p_i denotes probability of occurrence of each payoff such that $\Sigma p_i = 1$. The investment or bet, which has the highest value of expected utility, is ranked at the top and so on.

It is noteworthy that popularity of EUH as an analytical tool is knotted with the convincing argument of diminishing utility of wealth reflected through an increasing utility-of-wealth function. EUH can, however, be used equally well to reflect risk-seeking and risk-neutral attitudes through increasing convex and increasing straight line utility functions respectively.[3] Risk aversion is though rooted in an increasing concave utility function, yet its degree cannot be assessed explicitly from its curvature. On the contrary, mean-variance criterion reflects degree of risk aversion explicitly but it requires comparison of two parameters, mean and variance, of competing investments or bets. It is also not helpful if both parameters of one investment or bet are greater than those of the other. EUH is free from these shortcomings. It is probably for this reason that as an analytical tool for choice under uncertainty, EUH is generally preferred over mean-variance criterion in the literature.

Allais (1953) discovered a systematic violation of EUH known as 'Allais Paradox.' Like other paradoxes, it should have discredited both EUH as a tool and risk aversion as its underlying principle. However, it created doubts about EUH and not about risk aversion; rather it strengthened the conviction of risk aversion. The reason could be that Allais himself and most of the

3 See Machina (2008) for details.

studies which followed tested EUH without testing risk aversion in strict statistical sense that is possible through mean-variance criterion as explained in the next section. As a result, later authors who found other problems with EUH offered various psychological reasons, some of which have been discussed below, to discredit EUH but they also did not challenge risk aversion explicitly. One of the most publicized of these reasons is the certainty effect that reinforces risk aversion. Few researches though hinted upon the big amount effect that reinforces risk seeking attitude but they did not emphasize it. Consequently, choice under uncertainty, which had been progressing satisfactorily as a distinct branch of knowledge for many years, turned into a field in flux.[4] Therefore, the objective of this paper is to explore those reasons due to which 'Allais paradox' generated such a paradoxical implication. That is, it created doubts about EUH as an analytical tool but it accredited risk aversion as the norm for choice under uncertainty.

This research propounds three hypotheses to explain paradoxical implication of 'Allais paradox.' One is that 'Allais paradox' has been unduly posed as a paradox for EUH based on risk-aversion. The argument explained in the next section is that both option pairs in Allais choice-set neither confirm nor contradict risk aversion in pure statistical sense. The other is that later researchers who identified many other systematic violations of EUH interpreted them as in conflict with some specific axioms of EUH without directly calling risk aversion into question as the norm. The last is that risk aversion, which is the theoretical backbone of EUH, stems from the argument of diminishing marginal utility of wealth that is proclaimed to be a replica of the law of diminishing marginal utility of a commodity. It is, however, contended in this research that the analogy of two concepts is debatable. In the latter concept, the count of commodity for which utility is defined, is a non-random variable and in the former concept, the count of wealth for which utility is defined, is a random variable. Therefore, it matters for the proclaimed analogy to hold whether probability distribution of wealth or payoffs arranged in ascending order is symmetric or

[4] See Machina (1987).

asymmetric. It is argued that analogy of two concepts hold if probability distribution of payoffs is symmetric; otherwise it may or may not hold. In case of asymmetric distributions, both the certainty and the big amount effects are present in some degree and they work in opposite directions. If the big amount effect dominates, then the proclaimed analogy breaks down.

The contribution of this research is that first of all it criticizes 'Allais paradox' for supporting the conviction of risk aversion intuitively. Naturally, the criticism extends to all subsequent researches which tested EUH without testing human attitude towards risk in pure statistical sense. Then picking up the idea of big amount effect from Friedman and Savage (1948) and Tversky and Kahneman (1986), that talked about it explicitly as discussed below under the certainty effect, and from Lichtenstein and Slovic (1971 and 1973) and Blavatskyy (2010), which hinted upon it as discussed below under the response mode effect and the reverse common ratio effect respectively; its stance as a counter-balancing force to the certainty effect has been highlighted by taking examples of two hypothetical distributions, one symmetric and the other asymmetric. It has been explained that in case of asymmetric distributions, both the certainty and the big amount effects are always present in some degree and it can hardly be predicted as to which of them will dominate the other for a given person at a given time. A clear implication of this research is that in future, researchers should carry out testing of EUH or 'Allais paradox' and risk aversion through mean-variance criterion simultaneously.

The scheme of this paper is that introduction is followed by a critique of 'Allais paradox.' This section points out a statistical flaw considering which 'Allais paradox' should not have been considered a paradox for EUH in the literature. Section three discusses selected psychological factors like the certainty effect, the big amount effect, the common consequence effect, the common ratio or isolation effect, the reverse common ratio effect, the response mode effect, the framing effect and the hypothetical vs. cash payoffs effect which have further discredited EUH as an analytical tool but have not directly questioned risk aversion as the rational behavior. In section

four, analogy of the argument of diminishing utility of wealth and the law of diminishing utility of a commodity is reexamined. The last section is reserved for conclusion.

2. Critique of 'Allais Paradox'

In his widely cited experimental study, Allais (1953) asked his respondents first to choose from a certain option A and a probable option A^*, and then from two probable options, B and B^*, as given in table 1.

Table 1
Original Options in 'Allais Paradox'

Option A		Option A^*		Option B		Option B^*	
Payoff (million$)	Probability	Payoff (million$)	Probability	Payoff (million$)	Probability	Payoff (million$)	Probability
1	1.00	0	0.01	0	0.89	0	0.90
		1	0.89	1	0.11	5	0.10
		5	0.10				
E(A) = 1		$E(A^*) = 1.39$		E(B) = 0.11		$E(B^*) = 0.50$	
var(A) = 0		$var(A^*) = 1.46$		var(B) = 0.10		$var(B^*) = 2.25$	

Source: Allais (1953)

Majority of the respondents chose option A from option-pair AA^* and option B^* from option-pair BB^*. Therefore, expected utilities of options A and A^* are expressed as inequality 1^* below: -

$U(1) \geq 0.01\ U(0) + 0.89\ U(1) + 0.10\ U(5)$ inequality 1 or

$0.11\ U(1) \geq 0.01\ U(0) + 0.10\ U(5)$ inequality 1^*

Similarly, expected utilities of options B and B^* are expressed as inequality 2^* below: -

$0.89\ U(0) + 0.11\ U(1) \leq 0.90\ U(0) + 0.10\ U(5)$ inequality 2 or

$0.11\ U(1) \leq 0.01\ U(0) + 0.10\ U(5)$ inequality 2^*

Since the expressions on both sides of inequalities 1^* and 2^* are same but the inequality sign is reversed, therefore it shows a contradiction of EUH irrespective of the fact whether utility function inculcates a risk-averse, risk-neutral or risk-seeking attitude.[5]

To investigate the first hypothesis of this research, both option-pairs are evaluated by mean-variance criterion as well. According to this criterion, a risk-averse person should prefer option A^*(or B^*) over option A (or B), if following inequalities i) and ii) hold and weak inequality sign in at least one of them is replaced with the strict inequality sign:-

i) $E(A) \leq E(A^*)$ ii) $var(A) \geq var(A^*)$

However, if these inequalities take the form of following strict inequalities iii) and iv), then this criterion fails to rank competing option-pairs in strict statistical terms.

iii) $E(A) < E(A^*)$ iv) $var\,(A) < var\,(A^*)$

In the latter case, the choice of option A may be argued as reflecting risk-averse attitude if one compares only risks of both options, but as soon as one compares their expected values as well, then the choice of option A seems irrational. In other words, option A reflects risk-averse attitude only intuitively, not statistically.

Looking at the expected value and variance of each option given in the last row of table 1, it is evident that both option-pairs in Allais' study cannot be tested for risk aversion by mean-variance criterion as they lead to inequalities iii) and iv) above. It means that even if majority of Allais' respondents had chosen options AB, which are in line with EUH, they would

[5] The sign of the second derivative of a given utility function, which could be positive, zero or negative, shows whether the person who has conceived it is risk-averse, risk-neutral or risk-seeking respectively. Since the expression for expected utility of each option in these inequalities does not involve second derivative of utility function, therefore it is concluded that this violation of EUH applies to all types of utility functions.

not have confirmed risk aversion in strict statistical terms. Therefore, the actual option-choice AB* should not have been interpreted as a paradox for EUH based on risk averse attitude.

Blavatskyy (2010) has indirectly pointed out this statistical flaw. He remarked that the sure monetary payoff in one option-pair in Allais' study and in many subsequent studies like Khaneman and Tversky (1979) was deliberately selected not too far below the expected value of the risky option so that the majority of people were likely to choose the sure payoff over the risky option. To put it differently, risk aversion was pre-programmed in that setup of payoffs. As a result, majority of the respondents chose sure payoff and thus the conviction that human attitude is risk-averse was strengthened. Contrary to this practice, he selected the sure payoffs that were far below the expected values of corresponding risky options. In other words, he escalated the big amount effect to the extent that it dominated the certainty effect. Consequently majority of the people chose risky options over the sure payoffs. In other words, the author pre-programmed a risk-seeking attitude but again only by intuitive reasoning. Had all those authors used sure payoffs exactly equal to expected value of competing risky options, they would have escaped controversial results and obtained statistically sound results as to whether people are risk-averse or risk-seeking

3. Various Psychological Factors Influencing Choice Under Uncertainty

So far, too many systematic violations of EUH based on risk aversion have been noted. It is hard to cover all of them in a single paper, therefore few of them that are quoted most commonly in the literature like the certainty effect, the big amount effect, the common consequence effect, the common ratio or isolation effect, the reverse common ratio effect, the response mode effect, the framing effect and the hypothetical vs. cash payoffs effect are discussed below.

The Certainty Effect: Kahneman and Tversky (1979) conducted a

comprehensive experimental research containing some questions having non-negative monetary payoffs like those used by Allais but being much less in amount, some others having negative monetary payoffs and yet some others having payoffs in kind. In addition, some options contained almost certain instead of fully certain payoffs. Overall, they verified 'Allais paradox' and concluded that people underweight probable payoffs while comparing them with certain or almost certain payoffs. They called this tendency as the certainty effect. It reinforces risk aversion when competing options contain only non-negative payoffs and reinforces risk seeking attitude when competing options contain only non-positive payoffs. Following table 2 reproduces a couple of their survey questions which highlight the certainty effect.

Table 2
Illustration of the Certainty Effect

Option A			Option A^*			Option B			Option B^*		
Payoffs	Prob.	E(P)	Payoffs	Prob.	E(P)	Payoffs	Prob.	E(P)	Payoffs	Prob.	E(P)
3000	1.00	3000	0	0.20	0	0	0.75	0	0	0.80	0
			4000	0.80	3200	3000	0.25	750	4000	0.20	800
E(A) = 3000			$E(A^*) = 3200$			E(B) = 750			$E(B^*) = 800$		
var(A) = 0			$var(A^*) = 2,560,000$			var(B) = 1,687,500			$var(B^*) = 2,560,000$		

Source: Kahneman and Tversky (1979) problems 3 and 4.

Respondents were first asked to choose from option-pair AA^* and then from option-pair BB^*. The majority chose options A and B^*. Their choice-pattern violated EUH because of the certainty effect as payoff in option A was certain. Subsequently Conlisk (1989) confirmed the certainty effect.[6] Moreover, the certainty effect is not confined to cases in which the payoff of a competing option is absolutely certain but it also extends to cases in which payoffs of a competing option are almost certain. Table 3 reproduces two survey problems in Kahneman and Tversky (1979) which highlight the almost certainty effect.

In this case, majority of the respondents also chose options A and B^*. Their choice-pattern violated EUH because of almost certainty effect. In

[6] See particularly the results of third variant of Allais questions in his paper.

Table 3
Illustration of the Almost Certainty Effect

Option A			Option A*			Option B			Option B*		
Payoffs	Prob.	E(P)	Payoffs	Prob.	E(P)	Payoffs	Prob.	E(P)	Payoffs	Prob.	E(P)
0	0.10	0	0	0.55	0	0	0.998	0	0	0.999	0
3000	0.90	2700	6000	0.45	2700	3000	0.002	6	6000	0.001	6
E(A) = 2700			E(A*) = 2700			E(B) = 6			E(B*) = 6		
var(A) = 810,000			var(A*) = 8,910,000			var(B) = 17,364			var(B*) = 35,964		

Source: Kahneman and Tversky (1979) problems 7 and 8.

option A, the only positive payoff was almost certain with probability 0.90 as compared to the only positive payoff of option A* with probability 0.45, whereas positive payoffs in option-pair BB* were merely probable. Probabilities of positive payoffs in option-pair BB* were simply scaled down figures (1/450) of probabilities of positive payoffs in option-pair AA* that should not have affected their choice according to EUH. It is also noteworthy that in these options, EUH and risk aversion were testable simultaneously. The choice-pattern AB* violated not only EUH but also mean-variance criterion in the option-pair BB*.

The Big Amount Effect: The big amount effect better explains gambling as well as insurance. It was stated more explicitly by Tversky and Kahneman (1986) and Friedman and Savage (1948). According to Tversky and Khaneman, people often attach a higher expected value than the actual one to a small probability of large prize while assessing expected value of a lottery for decision making. It reinforces risk-seeking attitude and it explains why many people willingly pay a price greater than the actual expected value of a lottery. According to Friedman and Savage, an individual who buys lottery ticket subjects himself to a large chance of losing a small amount (the price of lottery) plus a small chance of winning a big amount (the prize of lottery) in preference to avoiding both risks (keeping the price of lottery). That is, he chooses uncertainty in preference to certainty in hope of big amount of prize knowing the fact that the expected value of prize is generally less than the price of lottery. Similarly, an individual who buys fire insurance on his house accepts to pay a fixed sum (the insurance premium) in preference to a small chance of a big loss (the value of the house) and a large chance of no

loss. That is, he chooses certainty in preference to uncertainty in order to avoid a big loss knowing the fact that insurance premium is generally greater than the actual value of loss.

The Common Consequence Effect: 'Allais paradox' is not an isolated example of violation of EUH; rather it is a special case of a general empirical pattern termed as the common consequence effect. In questions like the ones in Allais study, one can find an implicit payoff with same probability on both sides of each option-pair that allegedly causes 'Allais paradox' to happen. Such a payoff is called the common consequence. Considering Allais questions given in table 1 above, expected utilities of options A and A^* are expressed as inequality 1^*after subtracting 0.89 U(1), the common consequence, from both sides of inequality 1. Similarly, expected utilities of options B and B^* are expressed as inequality 2^* after subtracting 0.89 U(0), the common consequence, from both sides of inequality 2. It means that the common consequence with higher expected value, 0.89 U(1), in option-pair AA^* makes respondents risk-averse while the common consequence with lower expected value, 0.89 U(0), in option-pair BB^* makes them risk-loving.[7]

The Common Ratio or Isolation Effect: If an option-pair having a certain option with outcome X or an option-pair having an almost certain option with 2 outcomes, zero and X and a probable option with 2 outcomes, zero and Y such that $Y > X > 0$ and $1 \geq p_X$ (probability of X) $> ½ > p_Y > 0$ is transformed to another option-pair by scaling down the probabilities of X and Y by the same proportion such that the ratio $(p_X\ X) / (p_Y\ Y)$ remains same (common) in both option-pairs, then according to EUH choice of respondents should not change in both option-pairs. However, empirically it has been noted by many researchers for example, Tversky (1975), Kahneman and Tversky (1979 and 1984) that majority of the respondents chose certain or almost certain option in the original option-pair and risky option in the option-pair with scaled down probabilities. The reason is said to be the certainty effect that is found in the original option-pair. Its numerical examples are the same as those given in the context of certainty effect in

[7] See Machina (1987) and Kahnemen and Tversky (1979) for more details.

table 2 and 3 above.

Kahneman and Tversky (1979) interpreted the same phenomenon as a two stage game.[8] For the first stage, the respondents were told that there was a probability of 0.75 to end the game without winning anything and a probability of 0.25 to move to the second stage of the game. For the second stage, the respondents were asked to choose from a sure payoff of 3000 Israeli pounds and a lottery of 4000 Israeli pounds with probability of 0.8 and nothing otherwise. The respondents were, however, required to reveal their choice for the second stage before they could try their luck at the first stage. The majority chose sure payoff for the second stage. The second stage game was nothing more than the scaled down option-pair BB^* in table 3 above. However, in that case majority of the respondents chose the risky option. The authors concluded that people often ignore or isolate the first stage or that information which is common for each option of the second stage. That was why they chose the sure option which is virtually not certain.

The Reverse Common Ratio Effect: If an option-pair having a certain option with outcome X and a probable option with 2 outcomes, zero and Y such that $Y > X > 0$ and $½ > p_X > p_Y > 0$ is transformed to another option-pair by scaling down the probabilities of X and Y by the same proportion such that the ratio $(p_X\ X) / (p_Y\ Y)$ remains same (common) in both option-pairs, then, according to Blavatskyy (2010) and few previous researches quoted by him, people showed exactly opposite behavior which they showed in the common ratio effect explained above. That is, majority chose risky option in the original option-pair and safer option in the option-pair with scaled down probabilities. He also found evidence for the reverse common ratio effect when in choice-sets described in context of common ratio or isolation effect above, the certain option in the original option-pair is set far below the expected value of probable option.

His conclusion is that in experimental testing, the choice of majority of the the respondents is pretty much pre-programmed because it depends upon

[8] See problem 10 in the study.

the amount of certain option as compared to the expected value of probable option. If the amount of certain option is set slightly below the expected value of probable option, the majority choice is certain option as in the first option-pair of Allais' and Kahneman and Tversky's studies shown in tables 1 to 3 above. On the other hand, if the amount of certain option is set far below the expected value of risky option as in option-pairs 1 to 3 and 7 to 11 of his study, the majority choice is risky option.

The Response Mode Effect: After numerous hypothetical experiments and actual gaming in a casino, Lichtenstein and Slovic (1971 and 1973) noted another systemic contradiction of EUH that is called the response mode effect. If respondents are asked to rank or choose from two competing gambling options, then they pay more attention to probabilities of individual payoffs or to the certainty effect. So they choose the option that contains the most probable payoff even if overall expected value of the chosen option is somewhat less than that of the competing option. On the other hand, if people are asked to bid or ask money for the same competing options, then they pay more attention to the amount of payoffs or to the big amount affect. So they bid or ask a bigger amount of money for the option with highest payoff even if its overall expected value is somewhat less than that of the competing option. According to the authors, the reason is that information processing system of human mind focuses more on probability while ranking or choosing from competing probable options focuses more on payoffs while assessing monetary value of competing options.

Tversky, Slovic and Kahneman (1990) confirmed preference reversal with a simple example. Respondents were asked to choose from option A that was 28/36 chance to win $10 and nothing otherwise (called P-bet) and from option A^* that was 3/36 chance to win $100 and nothing otherwise (called $-bet). Majority of the respondents chose option A even though its expected value (7.78) was less than that of option A^* (8.33). Later on, respondents were asked to state their lowest selling price of same options A and A^*. In response, the majority stated a higher price for option A^* even though variance of option A^* (763.89) was many fold of that of option A

(17.8).

The Framing Effect: Many authors including Slovic (1969), Payne and Braunstein (1971), Khaneman and Tversky (1979), Hershy and Schoemaker (1980) and Tversky and Kahneman (1986) have verified another source of contradiction of EUH called the framing effect. Alternative ways of stating or framing probabilistically identical options lead to systematic differences in choice. For illustration, the following example from Khaneman and Tversky (1979) is reported. In one experiment, investigators told their respondents to suppose that they were given $1,000 in addition to whatever wealth they had, and then they were asked to choose from a sure payoff of $500 and from a lottery of winning $1,000 with a probability of 0.5 and getting nothing otherwise. Majority of them chose the sure payoff. Later on, investigators told their respondents to suppose that they were given $2,000 in addition to whatever wealth they had, and then they were asked to choose from a sure loss of $500 and from a chance of losing $1,000 with a probability of 0.5 and losing nothing otherwise. Majority of them preferred to avail the chance. The second option-pair is, in fact, not much different from the first one with respect to financial position of respondents, but choice of respondents changed. The authors concluded that people change their choice-pattern due to framing effect and people are risk-averse in case of positive payoffs and risk-seeking in case of negative payoffs.

Tversky and Kahneman (1986) had another similar experiment. They asked two groups of respondents to imagine outbreak of an unusual disease which would kill 600 people. Then they segregated the two groups and told one group that two alternative medical programs A and B were in view to combat that disease. If program A were adopted, 200 people would be saved and if program B were adopted, there was 1/3 probability that 600 people would be saved and 2/3 probability that no people would be saved. Majority of respondents voted for program A. They told the other group that two alternative medical programs C and D were in view to combat that disease. If program C were adopted, 400 people would die and if program D were adopted, there was 1/3 probability that nobody would die and 2/3 probability

that 600 people would die. Majority of the respondents voted for program D. In fact, alternative medical programs A and B had exactly the same results as did programs C and D respectively but they were phrased differently. Acceding to the authors, that violation of EUH was simply due to the framing effect.

The Hypothetical vs. Cash Payoffs Effect: Conlisk (1989) empirically tested 'Allais paradox' both with cash and hypothetical payoffs. Cash payoffs in his experiment were \$0, \$5 and \$25 in lieu of hypothetical ones \$0, \$1 million and \$5 million with same probabilities as in the original Allais questions, reported in table 1 above. The results were contradictory. In case of hypothetical payoffs, 'Allais paradox' was confirmed but in case of cash payoffs, majority of the respondents chose risky options A^* and B^* in both option-pairs. The author, however, was not sure whether the switch over of respondents from risk-averse to risk-seeking attitude was due to replacement of hypothetical payoffs with cash ones or due to replacement of payoffs in millions of dollars with payoffs in numbers of dollars. In any case, his experiment with cash payoffs clearly demonstrated that cash payoffs might tilt human attitude towards risk-seeking.

4. Analogy of the Argument of Diminishing Utility of Wealth and the Law of Diminishing Utility of a Commodity Reexamined

The concept of diminishing marginal utility of wealth apparently seems and has actually been interpreted as a replica of the law of diminishing marginal utility of a commodity. This research, however, takes issue with this view. It is hypothesized that analogy of two concepts holds if probability distribution for payoffs is symmetric but it may or may not hold if it is asymmetric. The argument that disutility derived from a dollar's loss is always greater than utility derived from a dollar's gain surely applies if the amount to win and to lose and their probabilities are equal. That is, the probability distribution is symmetric. In case of asymmetric distribution, a dollar's expected loss may not necessarily be greater than a dollar's expected gain due to the big amount effect.

For illustration, two hypothetical bets A and B given in table 4 are considered. Each can be played by anyone without paying any price.

Table 4
Illustration of Certainty and Big Amount Effects by Hypothetical Bets

Bet A			Bet B		
Payoffs	Probability	E(P)	Payoffs	Probability	E(P)
-10	0.45	-4.50	-10	0.80	-8.00
0	0.10	0	0	0.10	0
10	0.45	4.50	80	0.10	8.00

Note: Negative payoffs have been included to keep the expected payoff of each bet equal to zero

Although the expected value of the first and the last payoff in each bet is same in absolute terms that is 4.50 in bet A and 8.00 in bet B, yet neither the certainty effect nor the big amount effect is present in bet A while both of them are present in bet B. That is, in bet A, the amount to lose and to win is same and their probability is also same. Hence, the analogy of diminishing marginal utility of wealth and that of a commodity should hold.

In bet B, on one side of the distribution is the certainty effect as the probability to lose is much greater than that of to win and on the other side is the big amount effect as the amount to win is much greater than the amount to lose. Therefore, a person who is more concerned about certainty will surely not participate in this bet because the certainty effect reinforces risk aversion. However, another person who is more concerned about a big amount may participate in this bet if the lure of big amount tempts him/her to ignore risk aversion emanating from diminishing marginal utility of wealth when it is a non-probabilistic variable. In real life, it is the force of big amount which tempts people to participate in lotteries and gambling.

5. Conclusion

In everyday life, human attitude towards risk is mixed. Many people willingly assume risk as they take part in gambling and invest in stocks. Almost as many others want to get rid of risk as they buy car insurance and

keep their money in cash. Moreover, every individual is neither exclusively risk-averse, nor exclusively risk-seeking as he/she act in both ways. That is, he/she gambles as well as buys insurance and he/she invests in stocks as well as keeps some of his/her wealth in cash. In such a complicated and intermingled situation, it is awfully difficult to prove statistically whether human attitude is predominantly risk-averse or risk-seeking. However, lured with the persuasive argument of diminishing utility of wealth that is in line with the law of diminishing utility of a commodity, economists have adopted risk aversion as the norm and risk-seeking as an exception. Axiomatic presentation of utility theory further strengthened this belief by providing researchers with a powerful analytical tool.

Allais pointed out one of the earliest and most widely quoted systemic violation of EUH known as 'Allais paradox.' Later researchers, however, not only justified it on the basis of certainty effect but they also interpreted it as strong empirical evidence in support of risk-averse attitude. That is, on the one hand, 'Allais paradox' created doubts about EUH as an analytical tool but, on the other hand, it strengthened its underlying principle that is risk aversion. This research therefore intended to investigate as to why 'Allais paradox' generated such a paradoxical implication.

This research put up three hypotheses in this regard. One, 'Allais paradox' does not qualify on statistical grounds to be designated as a paradox for EUH reflecting risk-averse attitude. The other, later researchers who discovered other psychological issues with EUH interpreted them as violations of some axioms of EUH without directly investigating risk aversion. It cast more doubts about EUH as an analytical tool. As a result, the confusion that was created by 'Allais paradox' that EUH is not a robust analytical tool but risk aversion is a sound underlying assumption to study human attitude towards risk still persist. The last, treatment of risk aversion as the rational behavior mainly owe to the argument of diminishing utility of wealth. This argument apparently seems a replica of the law of diminishing utility of a commodity. This study has, however, contended that the analogy of the argument of diminishing utility of wealth and the law of diminishing

utility of a commodity holds if probability distributions of competing options are symmetric, otherwise it may or may not hold.

With regard to the first hypothesis of this research, it has been illustrated in section two of this research that none of the four possible choice-patterns for Allais questions confirmed EUH based on risk aversion in strict statistical terms. Therefore the actually chosen choice should not have been given the status of paradox for EUH. With regard to the second hypothesis, it has been documented that Allais himself and most of the researches who conducted experimental testing on the subject of decision making under certainty had been preoccupied with EUH while ignoring testing of its underlying principle, risk aversion. With regard to the last hypothesis, it has been highlighted that in case of asymmetric distributions of payoffs, both the certainty and the big amount effects, which work in opposite direction, are present. The certainty effect reinforces risk aversion while the big amount effect induces risk seeking attitude. Moreover, the amount at which risk seeking attitude that is provoked by the big amount affect dominates risk aversion is not same for all people and for all the times. Therefore, EUH, which ignores the big amount effect, may not bring consistent results.

The recommendation of this paper is that further research should concentrate more on risk aversion than on EUH.[9] It can better be achieved by posing such option-pairs which are useful to test EUH and risk aversion simultaneously. More specifically, further research should find empirical evidence on the big amount effect. If it is found significant, then the next task should be to investigate the amount at which this effect approximately offsets the certainty effect.

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[9] Levy and Levy (2001) tested risk aversion without testing EUH and found evidence against it. Evidence of risk-seeking attitude in case of cash payoffs in Conlisk (1989) also supports this recommendation.

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