Journal of Business & Economics
Vol. 4 No.2 (July-December 2012) pp.200-215

Optimal Monetary Noise in an Economy with Bayesian Consumers and Risk-Averse Investors

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Abstract

An autonomous demand shock affects consumption spending. Variations in consumption spending contribute to the volatility in aggregate demand. As the investor is risk averse, volatility of aggregate demand reduces investment. Government injects monetary noise to reduce the volatility in aggregate demand and induce higher investment. Monetary noise clouds the observation of autonomous aggregate demand by the consumer who forms Bayesian beliefs that are consistent with the equilibrium they supported for forecasting autonomous aggregate demand and monetary noise. With a greater monetary noise, the consumer relies less on the inaccurate observation of autonomous aggregate demand and more on the prior distribution functions of autonomous aggregate demand and monetary noise to decide upon the level of consumption spending. Consumption spending therefore reflects less of the volatility in autonomous aggregate demand. Faced with a less volatile consumption spending, the investor increases investment.

Key Words: Monetary noise, aggregate demand, rational expectations, risk averse, Bayesian Decision Theory.

1. Introduction

In the current monetary economics and macroeconomics literature, it is invariably argued that monetary noise causes volatility in the economy and

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should be minimized if not eradicated.[1] This article argues to the contrary by using a Bayesian statistical decision theoretic model embedded in game theoretic context. In this model, the government injects monetary noise to cloud the observation of autonomous aggregate demand by the consumer. As a consequence of the monetary noise, consumption spending reflects less of the volatility of autonomous aggregate demand. The more stable consumption spending induces the risk averse investor to increase his investment.

There has been a lot of work discussing the relationship between volatility of output and aggregate demand and the long term growth rate of the economy. One of the views is that volatility of output and aggregate demand dampens investment (including human capital investment) and lowers the long term growth potential of the economy. This negative relationship arises due to several reasons, including the imperfections in credit market (especially regarding human capital), irreversibility of investment and risk averseness on the part of the investors.[2] So far, however, there is no research paper with a formal model that analyzes how government could act to improve the investment environment and growth potential by inducing a more stable aggregate demand with economic agents forming rational expectations or beliefs. This paper fills in this gap.

In this model, the players form Bayesian beliefs that are consistent with the equilibrium that the beliefs supported.[3] Players make full use of all the available information they possessed (including the structure of strategic interactions) to form these beliefs. Bayesian equilibrium consistent beliefs therefore agree with the basic tenets of rational expectations and are a variant of rational expectations.[4] The Bayesian modeling approach of this paper however allows the whole predictive distribution function, including the variance and mean to be mapped out. The current rational expectations

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[1] Refer to Friedman (1960, 1968) and Nelson (2007).
[2] Refer to Ramey and Ramey (1995), Martin and Rogers (2000), Kneller and Young (2001) and Dopke (2004) for theoretical arguments and empirical studies.
[3] Refer to Teng (2013).
[4] Refer to Muth (1961), Lucas (1972, 1975) and Sargent (1986, 1987).

approach, in contrast, typically focuses only on the expectation (or mean).[5]

By focusing only on the mean, the rational expectations theory has only point prediction and it has not touched upon other parameters of the distribution function other than the mean. The other parameters of the distribution function that might be of interest include the variance. The variance is a measure of the uncertainty involved. Uncertainty is a key factor deciding the level of economic undertakings including especially investment. Rational economic agents should attempt to form forecasts on the variance as well as the mean of economic variables that are of interest to them. This paper fills in this gap by modeling the whole predictive distribution function. The approach adopted in this paper could help to analyze macroeconomic phenomena such as business cycles by allowing the modeling of beliefs to include the variance (or higher order moments) and thereby taking into account the role of uncertainty.

In the model presented below, economic agents make full use of all the available information, including common knowledge about the structure of strategic interaction, to form their beliefs. They go through the process of decision and game theoretic analysis to arrive at the prior distribution function before observing the data. After observing the data, the prior distribution function is updated using the Bayes theorem. The paper models the prior distribution functions as well as posterior distribution functions. The model allows more complicated decision theoretic analysis to be done, including how risk aversion of the investor and variance of autonomous aggregate demand and monetary supply affect consumption spending and investment.

Section 2 presents the model. Section 3 has the comparative static exercises. Section 4 concludes the paper.

2. The Model

There are three players in an economy: the government, the investor and

[5] Refer to Gertchev (2007) and Mlambo (2012) for critiques and defenses of rational expectations.

the consumer. In this model, the government moves first by setting the monetary policy which determines the level of monetary noise in the economy and chooses the variance of money supply while fixing the mean to zero. As the monetary policy is set, the investor sets his level of investment. This draws the level of autonomous aggregate demand from a predetermined prior distribution function. Finally, the consumer observes inaccurately the autonomous aggregate demand due to the confounding monetary noise term injected by the government and infers about the autonomous aggregate demand using the Bayesian decision theory framework to decide on his optimal consumption spending.

In the second stage of the game, the investor decides his level of investment before nature reveals the level of autonomous aggregate demand. He does so by anticipating the decision of the consumer. The government, being the first mover, takes into consideration the reaction functions of the investor and consumer, and optimally selects the variance of money supply. The structure of the game is common knowledge. The prior distribution function of the autonomous aggregate demand is also common knowledge.

The autonomous aggregate demand, *R*, is normally distributed and has mean $\bar{R}$ and variance σ_R^2. That is,

$$R \sim N\left(\bar{R}, \sigma_R^2\right) \tag{1}$$

M, the level of money supply, is the action of the government. *M* serves only to confound the observation of the consumer on the autonomous aggregate demand and does not change the level of aggregate demand by itself. In other words, this paper focuses its analysis on the long run effects of monetary policy where there is neutrality of money. The consumer observes not *R* but *R*+*M*. We denote the observation as *X*=*R*+*M*.[6] The consumer tries

[6] One could think of R as the changes in real prices that reflect changes in autonomous aggregate demand whereas M as changes in nominal prices due to inflation.

to make inference about the actual level of *R* given the observation, *X*.

Given the observed level of nominal autonomous aggregate demand, a consumer decides his consumption spending. Since the real autonomous aggregate demand is inaccurately observed by the consumer with a noise term caused by the monetary policy of the government, the sampling distribution on *X* is

$$X \mid M \sim N\left(R + M, \sigma_R^2\right) \tag{2}$$

For making Bayesian inference and decision, the consumer forms prior belief on the distribution of *M*:

$$M \sim N\left(0, \sigma_M^2\right) \tag{3}$$

and makes use of the prior distribution function of *R*.

Combining the prior distribution function and the likelihood function lead to the posterior distribution functions of *R* and *M*:

$$R \mid X \sim N\left(\hat{R}, \hat{\sigma}_R^2\right) \tag{4}$$

and

$$M \mid X \sim N\left(\hat{M}, \hat{\sigma}_M^2\right) \tag{5}$$

where

$$\hat{R} = \frac{\sigma_R^2}{\sigma_M^2 + \sigma_R^2}(X) + \frac{\sigma_M^2}{\sigma_M^2 + \sigma_R^2}\bar{R} = \alpha(X) + (1-\alpha)\bar{R} \tag{6}$$

$$\hat{M} = \frac{\sigma_M^2}{\sigma_M^2 + \sigma_R^2}\left(X - \bar{R}\right) + \frac{\sigma_R^2}{\sigma_M^2 + \sigma_R^2}(0) = (1-\alpha)\left(X - \bar{R}\right) \tag{7}$$

and

$$\alpha = \frac{\sigma_R^2}{\sigma_M^2 + \sigma_R^2} \tag{8}$$

and

$$\hat{\sigma}_R^2 = \hat{\sigma}_M^2 = \frac{\sigma_R^2 \sigma_M^2}{\sigma_M^2 + \sigma_R^2} \tag{9}$$

In determining the optimal response to the inaccurately observed level of autonomous aggregate demand, the consumer solves

$$\max_S E(U) = \int \left(A(RY - S) - \frac{B}{2}(RY - S)^2 \right) f(R|X)\, dR + GS \tag{10}$$

S is the amount of consumer spending. *Y* is wealth or income. *R* is the autonomous aggregate demand that positively affects the value of wealth or income and *f (R*|*X)* is the posterior distribution function of *R* given the observed *X*. *A, B* and *G* are the taste parameters.

The first order condition is

$$\frac{\partial E(U)}{\partial S} = \int \left(-A + B\left(RY - S\right)\right) f\left(R \middle| X\right) dR + G = 0$$

(11)

The optimal solution is

$$S = \hat{R}Y - \frac{A}{B} + \frac{G}{B}$$

(12)

Therefore, the distribution function of *S* is

$$S \sim N\left(\left(\alpha(X) + (1-\alpha)\bar{R}\right)Y - \frac{A}{B} + \frac{G}{B}, (\alpha Y)^2 \left(\sigma_R^2 + \sigma_M^2\right)\right)$$

(13)

For simplicity, we assume that the autonomous aggregate demand affects the profit of the investor only indirectly through consumer spending. We assume that consumption spending positively affects the rate of returns of investments. We also assume that the investment returns function is linear in k, the level of investment. We assume that the expected utility of the investor is

$$E(V) = -\frac{qk^2}{2} + \int\int bk\left(\left(\alpha\left(X - \bar{M}\right) + (1-\alpha)\bar{R}\right)Y - \frac{A-G}{B}\right) - ak\left(S(R) - S\left(\bar{R}\right)\right)^2 f\left(R, M \middle| X\right) dR dM$$

(14)

where *b* measures the marginal profit of investment, a is a measure of risk aversion and *q* is the cost of capital formation.

Making use of the results previously derived, the above could be simplified:

$$E(V) = -\frac{qk^2}{2} + \int\int bk\left(\left(\alpha(X)\right)Y\right)f\left(R,M\middle|X\right)dRdM + bk(1-\alpha)\bar{R}Y - bk\frac{A-G}{B} - ak\left(\alpha Y\right)^2\left(\sigma_M^2 + \sigma_R^2\right)$$
$$= -\frac{qk^2}{2} + bk\bar{R}Y - bk\frac{A-G}{B} - akY^2\frac{\left(\sigma_R^2\right)^2}{\left(\sigma_M^2 + \sigma_R^2\right)} \tag{15}$$

The risk averse investor solves

$$\max_k E(V) = -\frac{qk^2}{2} + bk\bar{R}Y - bk\frac{A-G}{B} - akY^2\frac{\left(\sigma_R^2\right)^2}{\left(\sigma_M^2 + \sigma_R^2\right)} \tag{16}$$

The first order condition is

$$\frac{\partial E(V)}{\partial k} = -qk + b\bar{R}Y - b\frac{A-G}{B} - aY^2\frac{\left(\sigma_R^2\right)^2}{\left(\sigma_M^2 + \sigma_R^2\right)} = 0 \tag{17}$$

The optimal level of investment is

$$k = \frac{1}{q}\left(b\bar{R}Y - b\frac{A-G}{B} - aY^2\frac{\left(\sigma_R^2\right)^2}{\left(\sigma_M^2 + \sigma_R^2\right)}\right) \tag{18}$$

The government, being the first mover in this game, anticipates the reaction functions of the consumer and the investor. The government is concerned with the formation of capital in the economy. As the investor is risk averse, the government aims to minimize the volatility of consumption spending by the choice of variance of the monetary noise. In determining the optimal variance of the monetary policy, the government solves:

$$\max_{\sigma_M^2} E(W) = \frac{1}{q}\left(b\bar{R}Y - b\frac{A-G}{B} - aY^2 \frac{\left(\sigma_R^2\right)^2}{\left(\sigma_R^2 + \sigma_M^2\right)} \right) - h\sigma_M^2$$

(19)

h measures the costs of randomness in monetary policy, such as extra administrative costs involved due to volatility in money supply.

The first order condition is

$$\frac{\partial E(W)}{\partial \sigma_M^2} = \frac{1}{q}\left(aY^2 \frac{\left(\sigma_R^2\right)^2}{\left(\sigma_R^2 + \sigma_M^2\right)^2} \right) - h = 0$$

(20)

The optimal solution is

$$\left(\left(\frac{a}{hq}\right)^{\frac{1}{2}} Y - 1 \right) \sigma_R^2 = \sigma_M^2$$

(21)

Through substitution, we have

$$\alpha = \frac{\sigma_R^2}{\left(\left(\frac{a}{hq}\right)^{\frac{1}{2}} Y - 1 \right)\sigma_R^2 + \sigma_R^2} = \frac{1}{\left(\frac{a}{hq}\right)^{\frac{1}{2}} Y} = \frac{1}{Y}\left(\frac{hq}{a}\right)^{\frac{1}{2}}$$

(22)

In the equilibrium where the Bayesian beliefs are consistent with the equilibrium they supported, the joint distribution function of the autonomous aggregate demand, monetary noise and consumption spending is

$$\begin{pmatrix} R \\ M \\ S \end{pmatrix} \sim N\left(\begin{pmatrix} \bar{R} \\ 0 \\ \bar{R}Y - \dfrac{A-G}{B} \end{pmatrix}, \begin{pmatrix} \sigma_R^2 & 0 & \left(\dfrac{hq}{a}\right)^{\frac{1}{2}} \sigma_R^2 \\ 0 & \left(\left(\dfrac{a}{hq}\right)^{\frac{1}{2}} Y - 1\right)\sigma_R^2 & \left(\dfrac{hq}{a}\right)\left(\left(\dfrac{a}{hq}\right)^{\frac{1}{2}} Y - 1\right)\sigma_R^2 \\ \left(\dfrac{hq}{a}\right)^{\frac{1}{2}} \sigma_R^2 & \left(\dfrac{hq}{a}\right)\left(\left(\dfrac{a}{hq}\right)^{\frac{1}{2}} Y - 1\right)\sigma_R^2 & \left(\dfrac{hq}{a}\right)^{\frac{1}{2}} Y \sigma_R^2 \end{pmatrix} \right)$$

(23)

We denote the consumption spending without monetary noise as S'. The distribution function of S' is

$$S' \sim N\left(\bar{R}Y - \frac{A}{B} + \frac{G}{B}, Y^2 \sigma_R^2 \right)$$

(24)

Proposition 1:

Consumption spending under the monetary regime with a noise term of optimal positive variance has a smaller variance than consumption spending under a monetary regime with a monetary noise of zero variance.

Proof:

$$\begin{aligned} &(\alpha Y)^2 (\sigma_R^2 + \sigma_M^2) - Y^2 \sigma_R^2 \\ &= Y^2 \left[\left(\frac{\sigma_R^2}{\sigma_R^2 + \sigma_M^2} \right) \left(\frac{\sigma_R^2}{\sigma_R^2 + \sigma_M^2} \right) (\sigma_R^2 + \sigma_M^2) - \sigma_R^2 \right] \\ &= -Y^2 \sigma_R^2 \left(\frac{\sigma_M^2}{\sigma_R^2 + \sigma_M^2} \right) < 0 \end{aligned}$$

(25)

Q. E. D.

Monetary noise term clouds the observation on the volatile autonomous aggregate demand. As the consumer reacts less to the inaccurately observed autonomous aggregate demand, consumption spending becomes less volatile and has a smaller variance.

We denote investment without monetary noise as *k'*:

$$k' = \frac{1}{q}\left(b\bar{R}Y - b\frac{A-G}{B} - aY^2\sigma_R^2 \right) \tag{26}$$

Proposition 2:

Investment under the monetary regime with a noise term of optimal positive variance is larger than investment with no monetary noise.

Proof:

$$\begin{aligned} & k - k' \\ &= \frac{1}{q}\left(b\bar{R}Y - b\frac{A-G}{B} - aY^2\frac{\left(\sigma_R^2\right)^2}{\left(\sigma_M^2 + \sigma_R^2\right)} \right) - \frac{1}{q}\left(b\bar{R}Y - b\frac{A-G}{B} - aY^2\sigma_R^2 \right) \\ &= \frac{1}{q}\left(-aY^2\frac{\left(\sigma_R^2\right)^2}{\left(\sigma_M^2 + \sigma_R^2\right)} + aY^2\sigma_R^2 \right) \\ &= \frac{1}{q}aY^2\sigma_R^2\left(\frac{\sigma_M^2}{\sigma_M^2 + \sigma_R^2} \right) > 0 \end{aligned} \tag{27}$$

Q. E. D.

The investor is risk averse. As consumption spending under optimal monetary noise is less volatile, the investor increases his investment.

3. Comparative Statistics

We present the comparative static analyses of the monetary noise variance and the relative weight that the consumer gives to the inaccurate observation when making Bayesian statistical inference on the level of autonomous aggregate demand.[7]

$$\frac{\partial \sigma_M^2}{\partial a} = \frac{1}{2}(ahq)^{-\frac{1}{2}} Y \sigma_R^2 > 0 \tag{28}$$

$$\frac{\partial \alpha}{\partial a} = -\frac{1}{2} a^{\frac{-3}{2}} Y^{-1} (hq)^{\frac{1}{2}} < 0 \tag{29}$$

A greater risk aversion on the part of the investor prompts the government to generate a larger monetary noise variance to stabilize consumption spending and stimulate investment. The consumer therefore relies less on the observation and more on the prior distribution functions for making inference.

$$\frac{\partial \sigma_M^2}{\partial h} = -\frac{1}{2} a^{\frac{1}{2}} h^{-\frac{3}{2}} q^{-\frac{1}{2}} Y \sigma_R^2 < 0 \tag{30}$$

$$\frac{\partial \alpha}{\partial h} = \frac{1}{2} Y^{-1} (ha)^{\frac{-1}{2}} (q)^{\frac{1}{2}} > 0 \tag{31}$$

[7] The comparative static analyses of consumption spending and investment are less central to the main arguments of this paper. Readers interested in those comparative static analyses could easily derive them themselves.

A larger administrative cost in generating randomness in monetary policy causes the government to generate a smaller monetary noise variance. The consumer therefore relies more on the observation and less on the prior distribution functions for making inference.

$$\frac{\partial \sigma_M^2}{\partial q} = -\frac{1}{2} a^{\frac{1}{2}} h^{-\frac{1}{2}} q^{-\frac{3}{2}} Y \sigma_R^2 < 0 \tag{32}$$

$$\frac{\partial \alpha}{\partial q} = \frac{1}{2} Y^{-1} \left(qa\right)^{\frac{-1}{2}} \left(h\right)^{\frac{1}{2}} > 0 \tag{33}$$

A larger cost of capital formation makes the investor less responsive to greater stability in nominal aggregate demand. Consequently, the government generates a smaller monetary noise variance. The consumer therefore relies more on the observation and less on the prior distribution functions for making inference.

$$\frac{\partial \sigma_M^2}{\partial Y} = \left(\frac{a}{hq}\right)^{\frac{1}{2}} \sigma_R^2 > 0 \tag{34}$$

$$\frac{\partial \alpha}{\partial Y} = -Y^{-2} \left(a\right)^{\frac{-1}{2}} \left(hq\right)^{\frac{1}{2}} < 0 \tag{35}$$

A larger wealth or income would translate a certain level of volatility in autonomous demand into a larger volatility in consumption spending. Consequently, the government generates a larger monetary noise variance to stabilize consumption spending. The consumer therefore relies less on the observation and more on the prior distribution functions for making inference.

$$\frac{\partial \sigma_M^2}{\partial \sigma_R^2} = \left(\frac{a}{hq} \right)^{\frac{1}{2}} Y - 1 > 0 \tag{36}$$

$$\frac{\partial \alpha}{\partial \sigma_M^2} = 0 \tag{37}$$

$$\frac{\partial \alpha}{\partial \sigma_R^2} = 0 \tag{38}$$

A more volatile autonomous demand prompts the government to generate a larger monetary noise variance to stabilize consumption spending. However, the size of the monetary noise variance is proportionate to the size of the variance of autonomous aggregate demand. Therefore, changes in the size of both variances have no effect on the relative weights that the consumer puts on the observation and the prior distribution functions for making inference.

4. Implications and Conclusions

The model shows that by clouding the observation of autonomous aggregate demand with monetary noise, the government induces the consumer to rely more on prior information and less on inaccurate direct observation to make inference on the actual level of autonomous aggregate demand. Consequently, consumption spending reacts less to changes in autonomous aggregate demand. The investor, who is risk averse, takes advantage of the more stable consumption spending and increases his investment. The model therefore argues for lowering the level of volatility in aggregate demand by increasing volatility in monetary supply. This is in sharp contrast to the argument by Milton Friedman (1960, 1968) for a constant growth rate of money supply, which is a received wisdom in current monetary economics.[8]

Future research could relax some of the restrictive assumptions in the model. For instance, future modeling could let the government choose both the mean and variance of the monetary policy. Future research could also relax the assumption that the consumer and investor know the type of the government. In sum, the modeling approach introduced in this paper allows beliefs to be modeled with firm statistical decision theoretic foundation and is very useful for analyzing economic topics where beliefs or forecasts and expectations play an important role.

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