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Misconceptions of Students in Learning Mathematics at Primary Level

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Abstract

The study was designed to identify the misconceptions of the students in learning mathematics at primary level. For this, curriculum of mathematics from class I to IV was used to develop a test inclusive of all the conceptual areas of mathematics from class I to Class IV. The curriculum of class V was left out because the sample students were planned to take into the study who were studying in class V. Twelve sample schools from Faisalabad district were selected randomly equal in number from all the three tehsils of Faisalabad. Test was conducted personally by the researchers. There were eight conceptual areas determined and included in the test viz. Numbers, Operations on Numbers, Fractions, Operations on Fractions, Decimals, Measurement, Information Handling and Geometry. The data collected from the all the students of class V of the selected sample schools, and 248 students took part in the exercise. Multi items were developed in each of the areas with different difficulties in order to have an idea of which of the selected eight areas was posing threat of misconceptions amongst the sample students. The data were analyzed using SPSS. Item-wise and gender-wise analyses was conducted to identify misconceptions and errors students committed and also to find out whether there was uniformity gender-wise or otherwise. The analyses disclosed that almost are the areas were having abandon of the errors and misconceptions and hence the achievement remained very low (mean score remained within 6.2 to 16.1 out of 51).

Keywords: Misconceptions, Mathematics, primary level

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Introduction

The researcher has worked as a mathematics teacher, teacher educator and trainer of mathematics teachers and head teacher of different government high schools. The experience and continuous interaction with students and teachers of mathematics, specifically at school level have resulted in his concern with the learning of mathematics. It has become increasingly important to know how students can be taught more effectively in the classroom. With this background, work experience and professional assignments, the researcher wished to identify the kinds of errors and misconceptions students make at Primary level and investigate the possible causes for errors and misconceptions to suggest remedial measures for the problems faced by the students.

Progress in the understanding of mathematical development of children has been quite considerable for the last few decades. The major contribution in this regard has come from psychological, educational and cross-cultural investigations. The knowledge about understanding of mathematical concepts has been enriched by the combination of experimental, survey research and observational studies and these have challenged the theories about how children think and learn in various mathematical domains (David Wood, 1998).

The errors and misconceptions that students develop during previous classes or bring with them to the school from the community can create hindrances in the on-going learning of mathematical conceptions, consequently producing poor achievement in mathematics. Even if, the errors and misconceptions are considered positive, there is certainly a need to rectify these and help students' progress in the development of mathematics knowledge during their education. It is also important to see whether the teachers' misconceptions have any effect on the students' misconceptions. There is, therefore, a need to identify not only the conceptual areas where most of the children make mistakes or construct wrong generalizations but also the reasons responsible for those and how to rectify or correct them. This need has been translated into this study.

Errors: Meaning and explanation

During the 39th meeting of International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM) in 1987 (in Sherbrooke, Canada) the key issue deliberated upon was the role played by errors in the learning of mathematics. According to Migoń, J. (2007), in the CIEAEM (1987) it was stated that *an error takes place when a person chooses the false as the truth.* When the actual result is different from the objective (erroneous result); when the procedure adopted is different from the accepted procedures (erroneous actions) erroneous conceptions might be hindering problem solving and producing irrational results. Errors are of various types and hence difficult to classify accurately.

Misconceptions: Meaning and explanation

It is important to establish the difference between an ‘error’ and a ‘misconception’ as both seem to be equivalent regarding the incorrect result they produce. An error might be caused due to a misconception. Other factors may include carelessness, problems in reading or interpreting a question and lack of numbers knowledge (Spooner, 2002). A misconception, on the other hand, is the result of a lack of understanding or in many cases misapplication of a ‘rule’ or mathematical generalization (Spooner, 2002).

The ideas about how students develop ‘misconceptions’ are emphasized by most of the empirical studies on learning mathematics during the last many decades. Piaget’s repeated demonstration in the late 1970s that children think about the world in very different ways than adults resulted in educational researches, and people began to listen carefully what students were saying and doing on a variety of subject matter tasks (Smith J. P., 1993). What researchers heard and subsequently reported was both surprising and disturbing: students had ideas that competed, often quite effectively, against the concepts presented in the classroom. Students did not come to school as blank slates. They had developed durable conceptions with explanatory power, but those conceptions were inconsistent with the accepted mathematical and scientific concepts presented during instructions.

Students do not come to the classroom as "blank slates" (Resnick, 1983) but with theories constructed from their everyday experiences --- an activity crucial to all successful learning. Some of the theories that students use to make sense of the world are, however, incomplete half-truths (Mestre, 1987) which may be called “misconceptions”. Misconceptions are a problem for two reasons. First, they interfere with learning when students use them to interpret new experiences. Second, students become emotionally and intellectually attached to their misconceptions, because they have actively constructed them. Hence, they find it difficult to accept new concepts which are unfamiliar and dissimilar to their misconceptions.

Mathematics is relatively more difficult subject to learn at school level. In Pakistan various researches have shown that mathematics is a difficult subject and students' achievement, both in terms of scoring well in it and a grasp of the content matter is not very good. This general assumption is based on some recently conducted studies including:

- Base –Line Survey of Class IV Students in Faisalabad by SAHE in Faisalabad Pakistan in 2006.
- EFA Assessment Country Report, Pakistan 2000 conducted by World Education Forum
- Provincial Educational Assessment System (PEAS) Report 2005
- Measuring Learning Achievement at Primary Level in Pakistan, by AEPAM/UNESCO Study No. 135, 2000

Methodology of the study

The study, entitled "Misconceptions of the Students in Learning of Mathematics at Primary Level" intends to identify some of the misconceptions students develop at primary level, to investigate the possible causes of the misconceptions and to design and conduct remedial measures for students and their respective teachers to rectify these misconceptions.

A sample of 12 primary schools of government of the Punjab was selected out of more than two thousand primary schools from district Faisalabad. Using a pre test consisting of 42 items including eight conceptual areas was used to collect data. After conducting test personally, the identification of errors and misconceptions was done. To understand the causes of these errors and misconceptions 4 students from each of 12 schools were interviewed. A training programme of the sample school teachers were designed on the basis of the knowledge thus gained.

Research Questions

In the light of these objectives following are the research questions of the study.

1. What are the students' misconceptions in mathematics at primary level?
2. What are the possible causes of these misconceptions?
3. How can these misconceptions be rectified / removed?
4. How can teachers help students to learn mathematics better and remove the targeted misconceptions?

Definitions of the Key Terms

Concept: The term is used for holistic understanding which results in generalization of an idea or some ideas.

Errors/misconceptions:

Errors and misconceptions occur when children make wrong or inappropriate generalization of an idea. Children construct erroneous rules without reference to the conceptual content or the meaning of arithmetic (Resnick, 1982; \Rensick&Omanson, 1987), systematic errors of strictly symbolic interpretation in decimal fractions (Hiebert& Wearne, 1985) and for elementary algebra (Matz, 1982; Sleeman, 1982)

Primary level: The level of school education including Nursery to grade V. It is six years education imparted in three types of government schools; Primary schools, Elementary/Middle schools and High/Higher Secondary schools.

Teachers of mathematics: In government primary schools, a single teacher usually teaches all subjects including mathematics. We consider primary school teachers who teach mathematics (along with other subjects) to students as teachers of mathematics.

Data Collection and Analysis

The following procedure was adopted:

Phase 1

In this article we will report phase-1 during which a pre-test was administered to the sample students and then the test was marked. The score of the students provided initial information about the areas of mathematics where the students committed errors or in other words identification of errors was done through the pre-test.

Overall analysis

The overall information regarding mean scores of the students, school-wise and concept-wise, before item-wise analysis is given in Table 4.1.

Table 1

Concept-wise mean score of students of sample schools

	C1 09 marks	C2 05 marks	C3 06 marks	C4 07 marks	C5 07 marks	C6 05 marks	C7 04 marks	C8 08 marks	Total 51 marks
Overall mean score	2.9435	1.2419	1.0323	1.0685	1.3105	1.1734	0.8992	0.9153	10.5847
School A	3.1600	1.7600	0.7600	0.8800	1.1200	0.9600	1.4800	0.5600	10.6800
School B	5.0435	1.0435	1.4348	1.3478	1.7391	1.3478	0.7391	1.0435	13.7391
School C	1.7647	0.9412	0.6471	0.9412	0.6471	0.4118	0.1176	0.7647	6.2353
School D	1.9375	1.4375	1.3125	0.9375	1.5000	0.7500	0.7500	2.1250	10.7500
School E	2.8750	1.6250	0.7500	1.3750	0.5000	0.5000	0.7500	0.8750	9.2500
School F	1.5789	0.7368	0.6842	1.1579	0.7895	0.6842	0.8421	0.1053	6.5789
School G	1.4500	1.0000	0.7500	1.2000	1.2000	1.3500	0.3500	0.5500	7.8500
School H	1.8421	0.8947	0.7895	0.7895	0.8947	0.3158	1.1053	0.8421	7.4737
School I	1.8421	0.8947	0.7895	0.7895	0.8947	0.3158	1.1053	0.8421	7.4737
School J	3.0000	1.3333	1.5238	1.1429	1.3810	1.5238	1.2381	1.6667	12.8095
School K	4.0400	1.6400	1.6000	0.8400	2.0400	1.4800	0.5200	1.2000	13.3600
School L	5.1333	1.3667	1.0333	1.5000	1.9667	2.4667	1.7000	0.9667	16.1333

Mean scores of the schools compared with the 8 sub concepts as shown in the table above are further explained using bar graph in figure 4.1. The mean score is derived by adding the concept-wise marks of the students of the particular school and dividing it by the number of the students in the pre test.

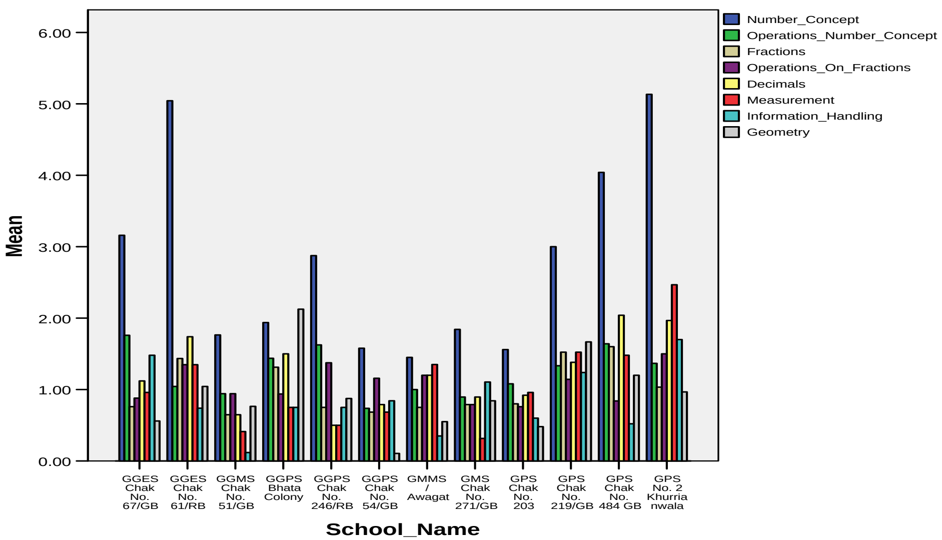


Figure 1: Mean scores of students of grade 5

Figure1. shows the mean scores of the students of grade V in the 12 sample schools. It is evident from the graph that students obtained relatively higher mean score in ‘numbers concept’. Whereas, in other tested domains/concepts students’ performance remained considerably low, particularly in ‘information handling’ and ‘geometry’.

Question-wise analysis of the pre-test

Section 1 of the Pre-test consisted of 33 items each having four alternatives. The frequency of students’ responses on these alternatives informed about the alternative concepts of the students. In the following lines item-wise analysis of students’ responses is discussed.

Question 1

Identify the biggest four digit number using the given four numerals which are; 4, 6, 3 and 7.

The right answer 7643 was correctly responded by a majority (68%) of the students, the least response turned out to be 6743 (12 responses; only 4.8%). The details are given in Table 4.2.

The response frequency shows a reasonably high frequency of students who did not respond to the item perhaps due to the reason that they did not understand the item. During the interview, the students performed the task correctly when they were explained the question in Punjabi. It seems that the students have the required skill to arrange the given digits to get the biggest possible number.

Question 2

Write these numbers in words 3, 59, 45, 007.

In Urdu language there is a difficulty in reading and writing numbers like 29, 39 etc. up to 99 due to their phonetic resemblance. Usually the pronunciation of number 29 resembles that of 39 and same is the case for almost all the double figure numbers with 9 at the end. The largest group of students marked option 1 as the right answer (37.9%) probably due to their mistake of reading 59 as *“Unchas” (49)* instead of correctly reading it as *“Unsath”.* This difficulty is usually overcome by providing a lot of practice in reading and writing of counting in Urdu during grade 1 through 3. In interviews with the sample students, it was revealed that they got very little practice of writing numbers in Urdu. And it was surprising to note that their teachers also had a lot of confusion themselves in reading the double figure difficult numbers with 9 as their 2nd number up to 100. Also, quite a considerable number of students made mistake in reading 45007 as forty five hundred and 7 instead of forty five thousand and 7, as there were 2 zeros in places of hundreds and tens. It seems that students have difficulties in reading numbers correctly as well as place value understanding of numerals in a figure.

Question 3

Which of the given numbers represents "Aath Crores, Nau Lac, Sattatar Hazar Chay Sau Untees"? (Eight Crore Nine Lac Seventy Seven Thousands Six Hundred Twenty Nine)

The same problem was felt in the response to this question also; as "Sattatar (77)" is very close to "Sattar (70)" , a large number of students (71out of 248; 28.9%) confused 'seventy-seven' with seventy due to this phonic resemblance of 'Satattar' with 'Sattar'. The interviews with the students and their teachers further added to the language issue both written and spoken. The students were mostly taught in provincial language which was their first language too i.e. Punjabi and the text books were in Urdu or English. Besides, they also confronted much difficulty in reading Urdu text. Some students showed that they read 09 as ninety instead of nine.

Question 4

Which of the following set of numbers is in ascending order?

78757, 78759, 77762, 78765, *900970, 900990, 901010, 901030,* 70048, 700500, 701510, 700470, 230003, 230010, 230017, 229024

About 20% of the students responded correctly, which of course was not encouraging. On the other hand, 32% of the students did not respond to the question provides evidence that the question was difficult to understand by the sample students. The concept of place value as well as skill to read and write a number seems also responsible for the overall difficulty students faced during attempting the questions regarding the number concept. It seems that the sample students did not see the common difference between each of the numbers in the sequence must be same. It was further revealed after the interviews that they felt difficulty in comprehending the question as their reading skills seemed very weak.

Question 5

Which Roman number represents 44?

A large number of students answered, this question correctly (117 out of 248; 47.2%). A considerably large number of students (25%) misunderstood (4th alternative of the table 5) the rule of writing X before L for representing 40 as XL. Responses to this question revealed that a large number of the sample students had remembered the ‘rote-learnt’ conversion table of Hindi Arabic numbers to Roman numbers. 62 students, however, committed an error of placing X to the right of L as LX to represent 40. Rote-learning was a habit of the students and their teachers tried their utmost for that considering it the main objective of their teaching. In their interviews most of the teachers complained that their students could not memorize the solutions of the questions.

Question 6

Find out all the factors of 28?

The highest number of the sample students responded correctly (42%), but a considerably large number of students thought 1 is not a factor of 28. And 54 out of 248 students not attempted the question showed their difficulty in reading or understanding the stem of the question. However, quite a considerable number of the students were confused factors with multiples. An analysis of this question tells that a considerable number of students lack clarity that 1 is a factor of 28 (1 is factor of all numbers) and they mixed factors with multiples.

Question 7

What is the least common multiple of 12 and 18?

About one third of the students (77 out of 248) marked the right alternative i.e. 36, but a considerably large number of students selected the first alternative which was the Highest Common Factor (HCF) showing that perhaps they confused LCM with HCF. There seems difficulty in differentiating LCM and HCF as well as understanding the question and this is also supported by the findings of the previous question. However, 42 students thought 18, which was the biggest number of the given pair of numbers, as LCM and 46 students not responding to the question shows that their concept has not yet well developed.

Question 8

A class was divided into two groups of children. In one group there were 16 children whereas 12 were in the other group. Minimum how many apples are needed so that if divided in the first or the second group, the apples are divided completely?

The frequency of the students' responses on this item gives further evidence of confusion about LCM and HCF the students have. The word problem asked the students to calculate the LCM of the numbers 16 and 12 and had the right option at serial number 1, but the alternative 4 is the HCF of these numbers. About 33% of the students selected the correct response but about 32% of them marked the alternative 4 which is HCF. Similarly, quite a large number of students did not respond to this and the previous question showing their difficulty in understanding the questions. This is clear evidence that word problems are relatively more difficult for students to solve.

Question 9

Which of the following figures shows $\frac{2}{3}$ part of the figures shaded?

The majority (58%) of the students thought that $\frac{2}{3}$means two parts are un-shaded and three are shaded showing their wrong concept of fractions. Only 5% of the students responded correctly as shown in the table 4.10. It seems that fractions are not taken by the students as 'part of a whole' rather they thought a fraction as two numbers written up and down of a bar. The alternate concept that they have $\frac{2}{3}$ is two parts and three parts where two parts are not shaded but three are shaded. Why they thought like this instead of 2 parts shaded but 3 are not? It is probably due to the reason that there was not an alternate answer given in the test. However, it is evident that they do not have idea that there should be 3 parts out of which 2 are shaded as they perhaps were not taught fractions using common fractions like half, one third etc. Interviews with groups of students and their teachers had been revealing to some degree. There were very few among the sample students who understood fractions as part of whole even very simple 'half', 'one-third' and 'one-fourth' were, though understood in every day experience, not known to them that these are fractions and written mathematically as '$\frac{1}{2}$', '$\frac{1}{3}$' and '$\frac{1}{4}$'. The previous concepts possessed by the students were not related to the 'mathematical representation' of these concepts.

Question 10

Which of the following is an equivalent fraction to$5/7$*?*

The responses of the students on this question show that quite a large number of them have the concept of equivalent fraction but, about 25% of them have incorrect understanding of equivalent fraction. The alternate concept they have is that equal fractions are equivalent. So, they thought $7/9$ is equivalent to the fraction $5/(\ 7)$.

Question 11

The simplest form of $12/16$ *is*

Very few (only 8.5%) of the sample students responded correctly to this question. Large number of the students selected the simpler fraction rather the simplest form. The reason disclosed in the interviews seems quite interesting; they were taught the concept in ‘Punjabi’ and when told the meaning of the question that was asked in ‘Urdu’, they understood easily and responded correctly. They, however, easily converted a fraction into a simpler fraction but not to the simplest one. The procedural skill of cancelling the common factors in numerator and denominator is not well placed.

Question 12

What fraction do we get when we convert $18/7$ *into mixed fraction?*

About 56% of the sample students (25% + 31.5%) tried to convert the given fraction into an equivalent mixed fraction correctly out of which only 25% had converted the answer to the simplest desired fraction. A considerable number of students did not attempt the question points towards not understanding the question. Real understanding of similarity between a mixed fraction and improper fraction is ambiguous in this and the next question.

Question 13

Which of the following fractions is equivalent to $3\,\frac{4}{5}$*?*

$7/15, 23/5, 19/5, 12/5$

This question is the converse to the previous one i.e. it demanded the students to convert the mixed fraction into an equivalent improper fraction. Only 18.5% responded correctly. This question and the previous one needed to be understood as improper fractions can be written as mixed fraction having an integral part and fractional part (proper fraction) or in improper fractional form (having numerator larger than the denominator). Most of the students thought that integral part is to be multiplied by the numerator leaving the denominator unchanged. Why they thought, seems they have not developed the concept in appropriate way. Moreover, a large number did not respond to this question too showing their inability in understanding what was needed to be done to reach correct improper fraction.

Question 14

Which fraction is bigger in $1/4$ *and* $2/9$*?*

In order to decide between two fractions as which one is bigger than the other, a student need to have understanding that each fraction has a unique value (as numbers). The frequency of the responses on correct answer (only 6.5%) shows that most of the students understand incorrectly that the numerals involved in fractions determine the weight of the fraction. That is why most of the students responded that 2/9 is bigger than 1/4 proves that they consider the numbers to decide which one is bigger.

Question 15

$7/11+2/3=$*?*

Addition of fractions is very much different from the addition of simple numbers as fractions are added by converting them into equivalent fractions with same denominator. The addition of numerators and denominators, as has been done by majority (39.1%) of the students, show that most of the students have misconception regarding the value of a fraction. They treated addition of fractions as they are adding whole numbers, adding numerators and denominators separately.

Question 16

$5/12 – 4/15$=?

Similarly, the responses on the subtraction of two fractions show that most of the students do not have concept of fractions. Most of the students tried to subtract numerators and denominators instead of taking LCM of the denominators of the fractions or converting these fractions into equivalent fractions with same denominators. Only 10.1% students responded correctly. Most of the respondents posses misconception regarding subtraction of two fractions as they subtracted numerators and denominators thinking probably they are to subtract whole numbers. It is further odd to observe that quite a considerable number of students (63) added the numerators and denominators and a big number of them did not respond to the question. Perhaps they are careless in reading the question and are used to doing mathematics problems after getting help from their teachers as what to do to solve the question.

Question 17

If $2/3 + 1/4 =$ ------ $+ 2/3$, then what fraction should be in blank?

This question intended to know the understanding of the students regarding commutative property of addition in fractions. The responses of the students on different alternative responses tell nothing about the reason why about 90% of the students did not understand what has been asked to them. It seems that the properties of 'commutation', 'association' etc. are not taught to them. Moreover, teachers disclosed in their interviews with the researcher their lack of understanding these properties. Divergent distribution of the students' responses in different alternatives proved this.

Question 18

$1/4 \times 2/3 =$?

Most of the students among their responses on different options to the question responded incorrectly (25%) showing that they thought product of fraction similar to that of product of numerators and denominators as has been shown by them in previous questions regarding operations of addition and subtraction on fractions. Product of fractions, of course, consists of two steps; first, multiplication of the numerators and denominators and the second, the fraction must be converted into its simplest form. Quite a large number of the students added the numerators and multiplied the denominators (54 out of 248) and similarly other big group added both the numerators and denominators. It probably points towards their carelessness or inability in reading and understanding question. Those who did not respond confirmed this idea.

Question 19

If $(3/12 \times 2\ /\ 3)$ *x* $1/4 = \square$ *x* $(2/3 \times 1/4)$*, then which of the given fraction would be in the square?*

This question was intended to investigate students' understanding of associative property of fractions with respect to multiplication. These responses showing most of the students responding to correct option seems to be a chance. It shows they are not properly told about the properties of numbers and fractions as has already been explained earlier in question 17.

Question 20

$2/3 \div 1/(`3)$=?

It gives no clue as to why most of the students multiplied the numerators and denominators to choose $2/9$ as the correct answer. Only 12.5% students answered correctly. Quite a large number of the students abstained responding to the question also shows that they did not understand the question. Some have read the sign of division as 'minus' whereas a large number (93) of students multiplied numerators and denominators and quite a big group did not respond. They displayed lack of understanding about the question asked?

Question 21

A cake was eaten by Aslam, Sughra and their mother. $1/3$ *was eaten by the mother, whereas Aslam and his sister Sughra eaten* $1/4^{th}$ *of the cake. What part of the cake remained?*

The word problem regarding operations of fractions proved difficult to be understood by the sample students. Only about 11% of the students responded correctly. Quite a large number of them either left the question or answered there is nothing left without calculating the remaining part of the cake after being eaten by the three persons. It seems that the students marked the responses without calculating to reach the answer, rather marked by chance as evident from distribution of their responses.

Question 22

What the fraction $7/10$ *equals to?*

The question intended to know the concept of the students about decimal and common fraction and their ability to convert a fraction into its equivalent decimal. The largest number of respondents answered 70 as decimal equivalent to$7/10$. The concept of the decimal seems to be very vague in their minds. Students have a little sense of converting a common fraction with simple denominator 10 showing their different points of view regarding answer to this question. According to curriculum demand at this stage of mathematics learning they should know place values up to three digits after decimal. Quite a large number of them did not respond to the question.

Question 23

Which one is smaller in the decimals 0.083 and 0.1?

To answer this question a student should know the concept of place value in decimals. Otherwise on the face value it seems 0.083 seems bigger than 0.1 which was the case depicted by majority of the students (62%). Students seems lacking the place value concept of decimals as it is evident from the table 4.24. This further strengthens the proposition that students have very weak understanding of place value in decimals as had been mentioned in the previous comment.

Question 24

What is the sum of 2.52 + 1.29?

Summing two decimals seem simple operation after writing terms to be summed in order of decimal point. So, the most of the students in spite of not very familiar with place values of a decimal responded to the correct alternative. However, considerable number of students responded to the incorrect options for perhaps not maintaining the order in addition. The skill of adding two decimals is less problematic than the concept of place value in decimals.

Question 25

Find the difference 4.03 – 1.15?

As in the previous question, students found it easier to find the correct response in case of difference. The skill of subtracting two decimals is also not very weak as compared to the overall understanding of the decimals, but still a large number of them added instead of subtracting the decimals due to their carelessness.

Question 26

What will be the product of 2.67 × 8?

Multiplication of two decimals is performed as the simple multiplication of numbers but finally placement of decimal point in the answer requires carefulness.

The sample students shown their carelessness and hence about 20% of them selected the correct response. The largest group of the students placed decimal after one digit from right whereas they should have placed the decimal after two digits starting from right. About half of the sample students did not posses correct concept of decimals and operations of multiplication of a decimal with a whole number.

Question 27

Divide 1.2 by 2.

Division of a decimal by a number provided very much difficulty to solve to the sample students. Only 8.5% of the students responded correctly. A large number of the students divided 12 by 2 without observing that it was 1.2 instead of 12. Still a larger group without understanding operation of division multiplied 1.2 by 2 and still the largest group added. Hence the students have very vague concept of division of a decimal with a whole number.

Question 28

How many centimetres are there in 1.05 meters?

The correct response seems quite good as compared to the previous lot of questions but, still it is very difficult to infer that students have correct concept of the units and subunits of length. Interviews with the sample students provided further insight that very few of them have got experience of measuring lengths in meters and centimetres. The table 4.29 further elaborates their lack of understanding the conversion of units of length.

Question 29

Add 1 kilogram 250 grams and 5 kilograms 800 grams.

The frequency of the students' correct responses in this question informs that in case of mass units students have more lack of understanding. Only 12.5% of them responded correctly. Interviews strengthened the idea that due to lack of experiences with measuring they are having slightest idea of how the weights are to be added if given in units and subunits. Without providing experience of measurement of lengths and weights (mass) to the students, they usually do not understand the units and subunits. The data reveals their inability to understand written quantities as meaningful; rather they tried to estimate wildly the correct answer.

Question 30

Subtract 4 kilometres 700 meters 9 decimetres from 7 kilometres 250 meters 7 decimetres.

Quite a large number of the students added the quantities which were to be subtracted (25.4%). They could not understand the question, it may be inferred. Most of the students abstained answering this question reflects that they have no understanding of subtracting two quantities given in km and m. Quite a large number of students subtracted quantities in kilometres prior to meters and hence subtracted 250 meters from 700 meters and 7 decimetres from 9 decimetres whereas they had to subtract 700 metres from 250 metres and 9 decimetres from 7 decimetres. The second largest group of students (63) added the quantities instead of subtracting showing 'weak literacy' regarding reading the question carefully.

Question 31

Add 5 grams 200 milligrams and 7 grams 900 milligrams.

The students in majority selected the correct response but, still it is evident from the distribution of students in all the four alternatives and the group who did not attempt this question is mostly uniform. This shows that they have very vague understanding of dealing with units and subunits of mass. Smaller units of mass like milligrams are not well understood by the students naturally as they had little experience of grams and kilograms which would become foundation for enabling a student to apply operations of addition and subtraction on quantities in units and subunits.

Question 32

Add 4 litres 500 millilitres and 3 litres 700 millilitres.

Considerably large frequency of the students' response to the first alternative, i.e. 7 litres 1200 millilitres, proves that they feel it difficult to understand 1200 millilitres is actually 1 litre and 200 millilitres. So, conversion can be blamed for not responding to the correct response. As the students have shown their relative inability in dealing with quantities of length and mass, they did not posses sound conception of quantities of volume.

Question 33

An iron girder is 5 meters 12 centimetres long. If 1 meter 21 centimetres long piece was cut from it, how long the girder will remain?

Quite a large number of students responded to first, second and fifth alternatives show that they did not understand the word problem asking for subtraction, subtracting in wrong order and not understanding the question so deciding not to answer. Only 16.5% responded correctly. Working with word problems posed enhanced difficulty of comprehending the quantities with which they had to apply operations of addition and subtraction, hence their performance proved weak.

Question 34

Sort out prime numbers from the given list of numbers.

16, 2,18,17,23,24,29,30

There were 9 numbers given to the students out of which 4 were primes. The students were required to identify those and write down those four prime numbers. Those who provided all primes correctly were called ‘correct’, while those who provided few correct primes but not all, were termed as ‘partially correct’ and those whose provided no prime are declared ‘incorrect’. Only 1.6% of the students responded correctly. More than 50% of the students responded incorrect regarding prime numbers. A considerable number (52) of students did not respond to the question. It seems that most of the students did not know the prime numbers. Those who were partially correct thought that 1 is also a prime number which shows that they did not have correct concept of a prime number which is that the number having two (exactly two) factors.

Question 35

Find prime factors of 16 and 24.

The performance on this question depends on the concept of prime numbers and process of applying small division to find the prime factors. The sample students performed at about similar level regarding their incorrect response. 127 responded incorrectly in the previous question whereas 125 responded incorrect in this question. Still a large number of students abstain responding to this question showing their difficulty in understanding of the question. The alternate concept of prime factorization possessed by the students (mostly those who thought the product of the factors must contain 1 as a factor which of course was a factor but not a prime factor).

Question 36

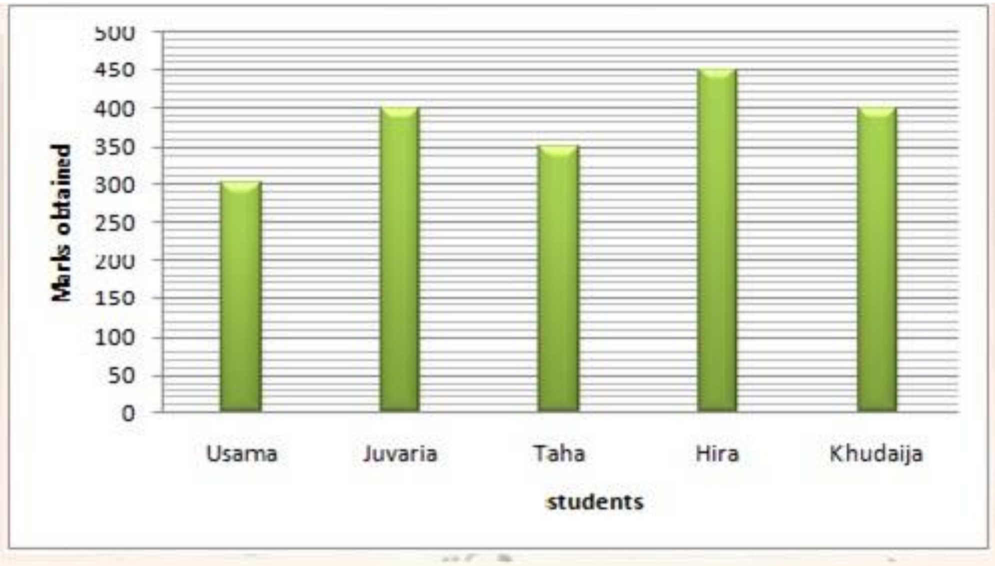
Find first ten multiples of 7 and 9.

About 35% of the students responded correctly to this question but it seems that they were probably good in rote learnt table of 7 and 9. They used their memory in reproducing numbers required to be written in the given space. Quite a large number of students made mistakes in writing tables resulting in 68 incorrect responses. The concept of multiples seems as learnt by association of the given number with the values of its table taken up to multiple of 10 : that is multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63 and 70 which consist of the table of 7 up to 10. Number of students "Not Attempted" depicts their inability to understand the true concept of the multiples.

Question 37

Read and answer questions regarding bar graph.

In the following Graph marks out of 500 obtained by Usama, Juvaria, Taha, Hira and Khudija are shown. Read the graph and answer questions given in the end.



Who got highest marks?

Who got lowest marks?

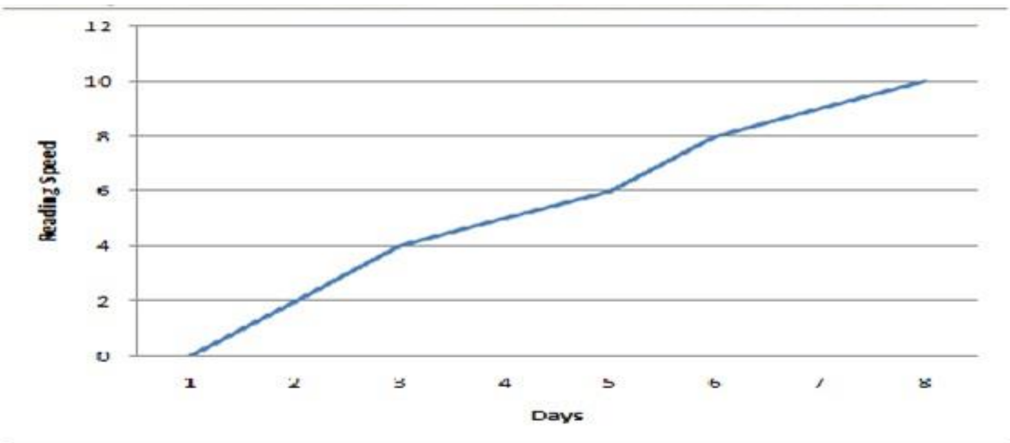
How many more marks were obtained by Taha than Usama?

The bar graph given in this question consisted of scores of four students. There were two questions on the graph, first was quite simple as they had to answer 'whose score was maximum?' and the second was a bit complex i.e. to identify difference between sores of two students. About 50% of the students responded the first bit easily (120 out of 248) but still it is not satisfactory. In responding second part of the three questions, they had to read the scale of the graph to see how many scores both of the students got and hence they could find the difference between their scores to which only 10.9% of them responded correctly. Hence, it seems that the students may easily understand geometrically which of the bars is the longest among the number of bars but what the length of a bar actually indicates is difficult as it requires a higher order skill of reading a scale.

Question 38

Read and answer questions on line graph.

The line graph below represents growth of mango plant during six days. Read the graph and give the answer to the questions at the end.



How many centimetres the plant grown between 3rd and 4th day?

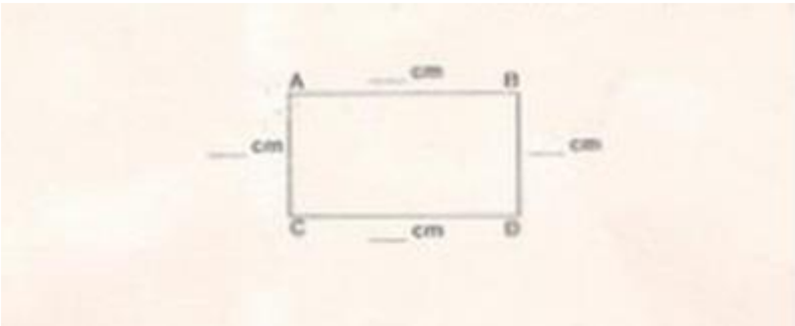
How high was the plant on 4th day?

How much high of the plant did increase between 2nd and 3rd day?

This question proved the most difficult of the questions of the test as only 2 students out of 248 could respond correctly to it. Line graph was a complex one posing a good challenge to the students to read the scale and answer the two questions given. It seems that reading a line and understanding variations in height of the line along the horizontal variable (here the days on which the height of the plant was measured) is more difficult than understanding variations in height of discrete columns (as the vertical bars of previous question). A considerable large number of the students (30.2%) did not attempt this question.

Question 39

A special figure of geometry is drawn below. Name this figure and with the measurement of the sides in the space given.



Name of Figure: ____________________

Measuring the sides of given rectangle and writing name of the figure.

Only 3.2% of the sample students could measure correctly the sides of given rectangle, and 23.4% had measured at least one of the four sides correctly. It reveals that they had not practiced using rulers to measure a line segment accurately up to millimetres. To measure accurately, one needs to know the measurement should start from '0' of the ruler instead of starting edge of it or from '1'. Only 3.2% of them measured correctly showing their inexperience using measuring scale.

Question 40

Drawing three line segments of given lengths.

Drawa the line segments of the measurements given in the space left below. Drwa neet and clean diagram.

Line segment AB:4cm

Line segment LM: 6.3cm

Line segment OD: 1.1cm

Similar is the situation in this question as we witnessed in the previous one, that is they have lack experience using rulers to draw the line segments of given length. Only 3.6% could draw correctly the line segments of 4, 6.3 and 1.1 centimetres respectively. This provides further evidence that most of the students have very little experience of measuring length of a given line segment and draw the line segments of given length. Following table shows it.

Question 41

Draw a circle of radius 3.7 cm.

Practical geometry needs the learners to be able to draw line segments, circles or triangles using simple instruments like rulers, a pair of compasses and protractors etc. Our sample students seem having very little experience of drawing themselves.

Question 42

Construct a triangle with sides 4.5cm, 3.5 cm and 2.5 cm.

This question needed use of pair of compasses and ruler to draw the required triangle. Only 4% of the students could draw the triangle of given measurements. It confirms that the sample students have too little experience of drawing geometrical figures using simple measuring instruments. The four questions of geometry were answered by the students committing similar mistakes shows that they have not been through the required hand-on experience of measuring and drawing using the ruler, a pair of compasses and protractor.

Discussion and Conclusion

In this section students' ideas are consolidated concept-wise.

Number Concept and Operations on Numbers

Students come to schools with basic knowledge of counting. The counting concept consists of only discrete whole numbers. Through grade 1 to IV, when their average age falls in the cohort 5 to 10, the curriculum and implementation processes by the schools have to provide integration of sequentially complex understanding, skills and application abilities on the foundation they had brought with them through social interaction and informal learning at home. The curriculum they had been through demanded consolidation of their concepts and skills that had not yet been developed even after four years of studying in school. The course of mathematics they followed was in 'Urdu' which was their second language. Their teachers usually communicated with them in 'Punjabi'. Further, they had shown unease in reading question statements particularly the specific terms such as 'Tarteeb-e-Saudi' (ascending order), 'Aads' (factors), 'ZuazaafAqal' (Least Common Multiple) etc. There was another problem worth noting faced by the students commonly; the number names in Urdu for the numbers like 29, 39, 49, 59, 69, 79, 89 and 99 are confusing due to the phonetic representation of their names of each with the next ones that is 29 is confused with 39 etc. All these problems noted by the researcher during interviews with the students and teachers. However, they were skilful in addition, subtraction, multiplication and division of simple numbers.

Fractions

Fractions are numbers which are not whole but part of whole. The whole numbers and integers can also be written in the form of fraction. The concept of fractions as the numbers having a unique value was not possessed by the students. Majority of the students and teachers thought that fractions are group of numbers written in the form of P/Q (P over Q). Most of them, therefore, added numerators and denominators to add two fractions and thought 2/9 is bigger than$\frac{1}{4}$. The real problem faced by the sample students lied in lack of concept development on their previously held concepts of fractions like 'half', 'one-third' or 'one-fourth' etc.

Decimals

Decimals are another way of writing fractions. In fact, fraction may be represented in terms of tenths, hundredths, and thousandths etc instead of writing in the form of common fraction. It was found that one cannot understand decimals without having sense of fraction. The concept of decimals includes understanding of a decimal as a number having unique value, place values of the digits in the decimal, conversion of a common fraction into its decimal representation and conversion of a decimal into its equivalent common fraction. Students could understand the fraction only if it is embedded in the constituents, which was not in place among the students as emerged during the interviews. However, those who marked at correct response in the pre-test also lack true understanding of this. They performed addition, subtraction, multiplication and division of the decimals taking the decimals as two groups of numbers each on either side of the 'decimal'. The teachers themselves were confused about the place value of digits in decimals, for example, some thought 0.01 is smaller than 0.008 and few did not know the place of a decimal on a number line.

Measurement

Measurement of quantities of length, mass (weight) and volume is included in the domain of learning mathematics at this stage. True learning of this area of mathematics is founded in conceptual development of students about understanding of 'units' and 'subunits' of all the three quantities (length, weight and volume). Mostly the sample students had learnt tables of conversion from units to subunits and vice versa, but didn't remember the table as they had no real understanding of how much a meter was long or how much a kilogram was heavy. Some of the students who help their parents in measuring grains or vegetables had correct sense, but still lack in mathematical representation of the problems correctly. Provision of practical situations to the students where they had opportunity of measuring themselves and then adding, subtracting and multiplying the quantities in units and sub units, like 6 kilograms 390 grams and 8 kilograms 850 grams, might have resulted in true conceptual development – was simply not there in sample schools. Some teachers themselves were not competent in handling specifically 'volume' given in litres and millilitres or area of floor or a field.

Information handling

The students were expected to read bar and line graph at this level of education by the curriculum of mathematics. Both the teachers and students had no experience in drawing line graph in their classrooms. However, bar graph reading was not very difficult for them except to compare two bars. They could easily identify which variable is the highest and which is the lowest, but they were unable to find out which variable is bigger than the other, perhaps it requires understanding of scale of the graph and it was not understood by the students and teachers, generally. Very few of the teachers had shown students graphs on writing board; instead mostly they had provided the experience of bar graphs by pointing on the pictures of the graphs (in textbooks) to the students. Line graphs were not clear to most of the teachers themselves. The teachers did not understand variation between two variables and its representation in form of line graph.

Geometry

Many of the teachers were not confident in teaching geometry due to variety of the reasons; like they had not taught themselves geometry by their teachers, the students did not bring geometry boxes with them, geometry is boring for them. It was disclosed during the interviews that the students had very little experience of using foot ruler, protractor, divider, compass etc. to measure line segments, angles and areas. The teachers did not use geometry box for writing boards and had not taught using the tools of drawing line segments, angles or triangles etc. Without doing with hands it seemed impossible a student can draw geometrical figures correctly.

During the next part of the study, a remedial effort to rectify the students' misconceptions and errors would be reported. Finally, in the third part of the study the effect of the remedial effort would be studied.

Recommendations

Summary of misconceptions and errors

From both quantitative analysis of students' pre-test scores and qualitative analysis of the interviews of the 48 students, following deductions may be made:

- The students have displayed errors in reading names of numbers, arranging given numbers into ascending & descending order and place value of numerals in a number. A few teachers also have difficulty in understanding place value and ordering of numbers.

- The students displayed errors in comparing two fractions as which one of them is bigger and solving problems of addition and subtraction of fractions. Most of the students have faced difficulties in multiplication and division of simple fractions. The teachers also have those errors and students' errors may be the result of teachers' errors.
- In decimal, it was found that the place value of the digits was clear to few students and teachers.
- The questions on measurement had posed many problems to the students. Both the teachers and students displayed difficulty in estimating the quantities, like 250 grams, 2 meters etc. Therefore, the concepts of measurement of basic quantities (length, weight and mass) were not well entrenched.
- Geometry and graph were also very weak areas and the students have little experience of using simple geometry box tools like foot ruler (straight edge), protractor and compass etc.
- The students could tell after observing the bars in a given graph which one is maximum, but they had difficulty in reading value of the bar and difference between the quantity shown by two bars. Further, reading of line graph was more difficult for both the teachers and students.

On the basis of the findings regarding reasons responsible for errors students made and misconceptions developed by them may be due to teachers' insufficient knowledge. In order to design a remedial effort, intervention, through this study, following guidelines for training of teachers were developed:

1. Teachers' knowledge of number systems needs to be consolidated and logical connections between various types of numbers like natural, whole, even, odd, prime, composite, integers, rational and irrational required to be developed.
2. Place value is very important to be understood properly and for this purpose base 10 number system may be compared with different basis. This comparison and contrast would be helpful in developing well founded concept of the number system and in turn would be helpful in instruction of the concepts of 'number' and 'operation on numbers'.
3. Teachers seldom provide opportunity to the students to actively participate in doing mathematics themselves and they did little to relate the new concepts to the existing knowledge of students or make connections with the environment.
4. Students have learnt denominators and numerators by rote without having understanding of fractions. For example most of the students were unable to

understand half a “roti” (bread) is bigger than the one third of the ‘roti’. The teachers should be properly trained in understanding of ‘fractions’ and ‘operations on fractions’ by relating these concepts with geometrical meaning or with the daily life experience of ‘part of a whole’ before shifting to mathematical representation of the fractions. Similar efforts may be made to develop concept of ‘decimal’, particularly with the help of currency.

5. Very few of the students had understanding of measurement of quantities while those are expressed in units and subunits. For example, most of the students find it difficult to understand ‘how many centimetres constitute a meter and if the lengths are given in meters and centimetres, how these should be added or subtracted? The real misconception disclosed was that most of the students fail to estimate the quantities in everyday life.
6. Almost all of the sample students were very weak in drawing. Hence, they were very weak in reading graphs especially scale of the graph. It was also found that they had seldom used geometry box for drawing lines, circles or other geometrical figures.

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