Sliding and Integral Sliding Mode Control Design for Twin Rotor System

Muhammad Taimoor, Sohail Iqbal, Abdul Waheed
Institute of Space Technology (IST), Department of Aeronautics and Astronautics, Pakistan
muhtaimoor@gmail.com, siayubi@gmail.com, waheed@ist.edu.pk

Abstract—**The Sliding and Integral Sliding Mode control design anticipated to handle out the cross-coupling effects in a twin rotor system. These cross-coupling effects lead to tainted act during precise maneuvering of twin rotor system. These cross-coupling effects can be concealed by introducing decoupling procedures such as Sliding and Integral Sliding Mode Controls. These controllers propose sturdiness at the rate of achievement to triumph over the cross-coupling in order to achieve the desired performance. The performances of both controllers are compared. This initiative has been effectively proved by simulations.**

Keywords: Sliding Mode Control, Integral Sliding Mode Control, Nonlinear Control, Twin Rotor System

NOMENCLATURE

Symbol	Name	Units
ψ	*Elevation Angle*	rad
ϕ	*Azimuth Angle*	rad
I_1	*moment of inertia of vertical rotor*	$kg.m^2$
I_2	*moment of inertia of vertical rotor*	$kg.m^2$
a_1	*static characteristic parameter*	
b_1	*static characteristic parameter*	
a_2	*static characteristic parameter*	
b_2	*static characteristic parameter*	
M_g	*gravity momentum*	$N.m$
$B_{1\psi}$	*friction momentum function parameter*	$N.m.s/rad$
$B_{2\psi}$	*friction momentum function parameter*	$N.m.s/rad$
$B_{1\phi}$	*friction momentum function parameter*	$N.m.s/rad$
$B_{2\phi}$	*friction momentum function parameter*	$N.m.s/rad$
Kgy	*gyroscopic momentum parameter*	s/rad
k_1	*motor* 1 *gain*	
k_2	*motor* 2 *gain*	
T_{11}	*motor* 1 *deno* min *ator parameter*	
T_{10}	*motor* 1 *deno* min *ator parameter*	
T_{21}	*motor* 2 *deno* min *ator parameter*	
T_p	*cross reaction momentum parameter*	
T_0	*cross reaction momentum parameter*	
k_c	*cross reaction momentum* gain	

I. INTRODUCTION

The purpose of this paper is to reduce the cross-coupling effects in helicopter dynamics which is an airplane that is elevated, boosted and exercised by vertical and horizontal rotors. Most of the twin rotor structures cause soaring cross-coupling effects in all their direction of action. Mainly the gyroscopic cause on azimuth dynamics avoids defined maneuvers by the machinist stressing the need to balance the cross-coupling, the mission that is visibly include to the workload for the pilot if done physically [1]. The twin rotor structure basically shows the manners of a factual helicopter with smaller amount of number of independence. In factual helicopter the power is attained by orienting the blade of rotors appropriately by means of the help of combined and repeated actuators, by maintaining the rotor at constant rate. In order to shorten the motorized design of the structure, twin rotor structure group is deliberated in a little different way. In such situation, the blades of the rotors have a rigid position, and power is attained by calculating the rate of the rotors. As the outcome of this, the twin rotor structure has extremely nonlinear fixed dynamics. Furthermore, it has a tendency to be the smallest period structure demonstrating unsteady nil dynamics. This structure created very demanding difficulty of accuracy manipulation in the incidence of cross-coupling. It has been comprehensively examined in the existence of the algorithms starting from linear robust control to nonlinear control sphere [2]. Te-Wei et al [3] has designed time optimal control for twin rotor structure. In order to design the controller, the MIMO structure was partitioned into two SISO structures and pairing was considered as interruption or variation in structure constraints. In this case the controllers have been planned that can endure up to 50% transforms in the structure constraints. M.Lopez et al [4] have suggested techniques for twin rotor structure such as full state feedback linearization methods and I/O linearization procedures. These techniques have been employed in elevation dynamics and azimuth dynamics. Pathway control of 2 DOF airplane has been designed by Dukta et al [7] called nonlinear extrapolative control. The state-space generalized predictive control method has been used for nonlinear algorithm. The nonlinear H_∞ has been designed by M.Lopez et al [10] in order to control the coupling considered as disturbance. The designed controller called nonlinear H_∞ shows the characteristics of nonlinear PID with time varying constants according to structure dynamics. The investigational outcomes show that structure drills with condensed coupling. For the class of uncertain systems several techniques such as robust stabilization and H_∞ have been presented by Jun et al [8]. Standard H_∞ control problems were solved in order to propose the quadratic stabilizing controllers for vague structures.

In this paper, the authors have presented procedure for twin rotor system in order to attain the desired performance in both vertical and horizontal planes in the existence of cross-coupling. The cross-coupling in structure has been controlled beside with the desired outcome and feat. The twin rotor system dynamics have been explained in Section II.

II. SYSTEM MODEL DESCRIPTION

The elevation and azimuth dynamics of the helicopter can be explain by considering the dynamics of twin rotor system. The free body diagram of vertical airplane dynamics shown in figure.1

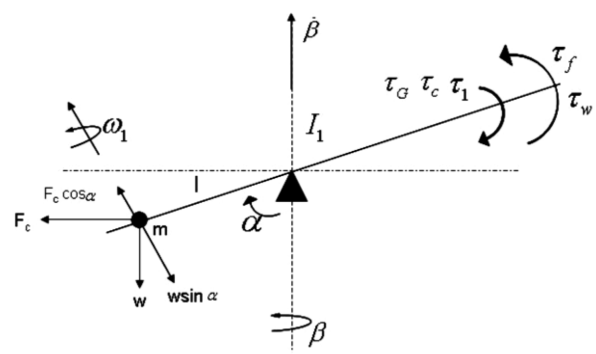

Fig.1 Free Body Diagram of Vertical Plane Dynamics

The mathematical expression of total torque created in the vertical airplane is given below:

$$I_1\ddot{\alpha} = \tau_1 + \tau_c + \tau_G - \tau_w - \tau_f \qquad (1)$$

The free body diagram of horizontal airplane dynamics shown in figure.2

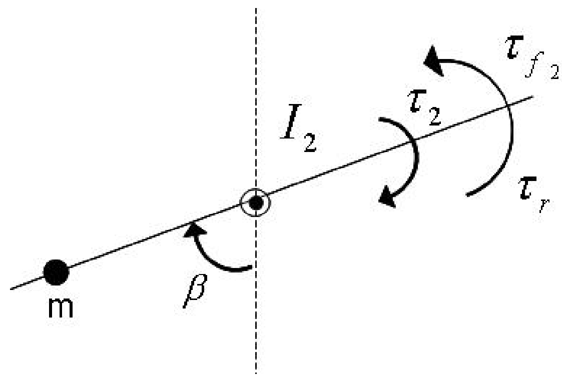

Fig.2 Free Body Diagram of Horizontal Plane Dynamics

The mathematical expression of total torque created in the horizontal airplane is given below:

$$I_2\ddot{\beta} = \tau_2 - \tau_r - \tau_{f2} \quad (2)$$

The nonlinear model of the system has been developed by utilizing the net torque equations (1) and (2) [19]. Finally the different states and the yields of the structure are given in matrices form as in (a) and (b) [19].

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} \textit{Main motor speed} \\ \textit{Elevation Angle} \\ \textit{Angular speed in Elevation} \\ \textit{Side motor speed} \\ \textit{Azimuth Angle} \\ \textit{Angular speed in Azimuth} \\ \textit{Angular Momentum} \end{bmatrix} \quad (a)$$

$$Y = \begin{bmatrix} x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} \textit{Elevation Angle} \\ \textit{Azimuth Angle} \end{bmatrix} \quad (b)$$

The equation (5) represents the elevation dynamics and the equation (8) represents the azimuth dynamics of the twin rotor system, while the equations (3) and (6) represent the dynamics of the key and side motor. The cross-coupling effect is represented by the equation (9). The values can be taken from [19].

$$\dot{x}_1 = \frac{1}{T_1}\left(-x_1 + u_1\right) \quad (3)$$

$$\dot{x}_2 = x_3 \quad (4)$$

$$\dot{x}_3 = \frac{1}{I_1}\left(\left(a_1 x_1\right)^2 + b_1 x_1 - B_1 x_3 - T_g sinx_2 - K_{gyro} u_1 x_6 \cos x_2\right) \quad (5)$$

$$\dot{x}_4 = \frac{1}{T_2}\left(-x_4 + u_2\right) \quad (6)$$

$$\dot{x}_5 = x_6 \quad (7)$$

$$\dot{x}_6 = \frac{1}{I_2}\left(\left(a_2 x_4\right)^2 + b_2 x_4 - B_2 x_6 + T_{pr} x_7 - K_r T_{or} u_1\right) \quad (8)$$

$$\dot{x}_7 = -T_{pr} x_7 + K_r T_{or} u_1 \quad (9)$$

The response of the structure can be examined by using the phase portrayals of the vertical and horizontal airplane dynamics for both sliding and integral sliding mode controller designs. The phase portraits for sliding mode are given in Figure 3 and Figure 4. Elevation states are not converging to origin, in fact the angular position of the elevation is governed its speed of transform, as the speed of transform of the angular point approaches to zero so the system approaches to that angular position in vertical plane. The response of the azimuth dynamics is asymptotically stable and converges to zero. The phase portraits of integral sliding mode are given in Fig. 5 and Fig. 6. The response of elevation and azimuth dynamics for integral sliding mode are asymptotically stable and converges to zero.

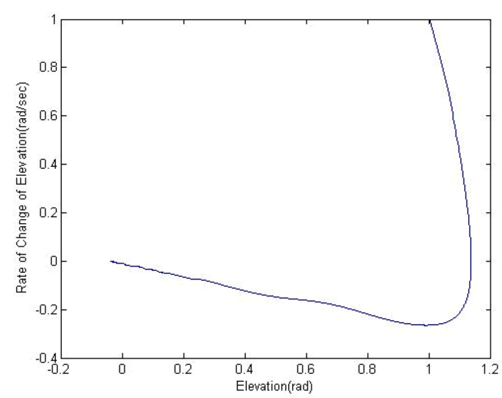

Fig.3 Elevation Phase portrayal of Sliding Mode

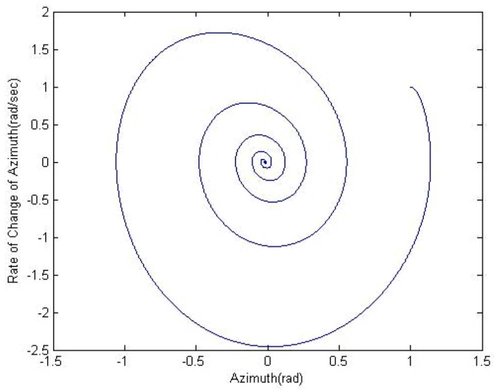

Fig.4 Azimuth Phase portrayal of Sliding Mode

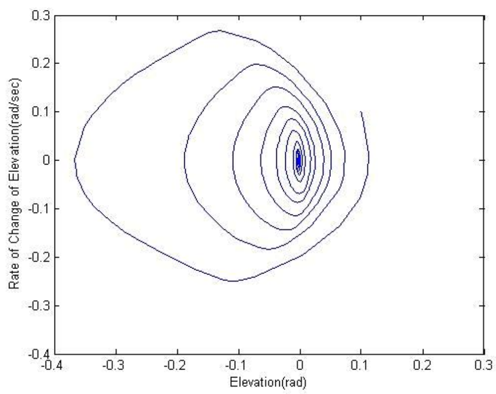

Fig.5 Elevation Phase portrayal of Integral Sliding Mode

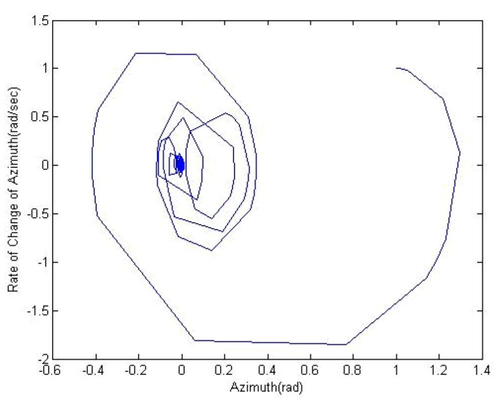

Fig.6 Azimuth Phase portrayal of Integral Sliding Mode

III. NONLINEAR CONTROLLER SYNTHESIS

The hyper-plane is distincted as the sliding-surface for the sliding mode and integral sliding mode controller design of twin rotor structure. This procedure consists of two steps; the first step is the attainment step and the second step is named as the sliding step. In attainment step, by designing a suitable control law the states are determined to a steady diverse whereas in the sliding step states approach to a balance position. The benefits of such procedure is that the nonlinear expressions which act as interruption or ambiguity are rejected and the structure acts like reduced order system, this means that there is no overshoot when the system is regulated from random preliminary form to the deliberated balance position. The designing of controllers for twin rotor structure are conceded out by dividing the MIMO structure into SISO structure, sliding surfaces are deliberated for every SISO structure based on the inaccurate dynamics distincted as in eq (9).

$$E = X - X_{eq} \tag{9}$$

Where X_{eq} are the preferred principles of the structure. The sliding surface for the sliding mode controller is defined as in eq (10).

$$S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} e_3 \\ e_6 \end{bmatrix} + \begin{bmatrix} c_1 e_1 + c_2 e_2 \\ c_4 e_4 + c_5 e_5 \end{bmatrix} \tag{10}$$

The above equation in (10) can be written as

$$\begin{bmatrix} e_3 \\ e_6 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} - \begin{bmatrix} c_1 e_1 + c_2 e_2 \\ c_4 e_4 + c_5 e_5 \end{bmatrix} \tag{11}$$

Similarly, the sliding surface for the integral sliding mode controller is defined in (12).

$$S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} e_3 \\ e_6 \end{bmatrix} + \begin{bmatrix} c_1 \int_0^t e_1 dt + c_2 e_2 \\ c_4 \int_0^t e_4 + c_5 e_5 \end{bmatrix} \tag{12}$$

The above equation can be written as in eq (13).

$$\begin{bmatrix} e_3 \\ e_6 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} - \begin{bmatrix} c_1 \int_0^t e_1 dt + c_2 e_2 \\ c_4 \int_0^t e_4 + c_5 e_5 \end{bmatrix} \tag{13}$$

The system given in eq (11) and eq (13) is steady if and only if S=0 and the speed of junction of the system will be governed by various dynamics of the system. The surfaces of the system given in eq (10) and eq (12) which are based on Liapunov function [18] can be defined as

$$V_1 = \frac{1}{2} s_1^{\,2} \tag{14}$$

$$V_2 = \frac{1}{2} s_2^{\,2} \tag{15}$$

The equivalent controls for both the elevation and azimuth dynamics are defined in eq (16) and (17)

$$u_{1eq} = -\frac{T_1}{c_1}[\frac{c_1}{T_1}(-x_1) + ([\frac{1}{I_1}((a_1 x_1)^2 + b_1 x_1 - B_1 x_3 - T_g \sin x_2)] + c_2 x_3)] \tag{16}$$

$$u_{2eq} = -\frac{T_2}{c_4}[\frac{c_4}{T_2}(-x_4) + ([\frac{1}{I_2}((a_2 x_4)^2 + b_2 x_4 - B_2 x_6)] + c_5 x_6)] \tag{17}$$

The control law 'U' that will force the system to converge at S=0 is defined in eq (18) this law will guarantee that the system will converge to the sliding surface and will show robustness against the cross-couplings effects

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_{1eq} - K_1 sign(s_1) \\ u_{2eq} - K_2 sign(s_2) \end{bmatrix} \tag{18}$$

The negative rate of change of energy of the system ensures the stability of the system and the equations for both the elevation and azimuth dynamics are given in (19) and (20).

$$\dot{V}_1 = s_1 \dot{s}_1 \tag{19}$$

$$\dot{V}_2 = s_2 \dot{s}_2 \tag{20}$$

$\dot{V}_1$ and $\dot{V}_2$ will always be negative and guarantee the stability of the twin rotor system.

Table 1 System Parameters

System Outputs	50° in Elevation $\pm 40^\circ$ in Azimuth
Main Motor '1'	DC Motor with Permanent Magnet Max Voltage 12V Max Speed 9000 RPM
Side Motor '2'	DC Motor with Permanent Magnet Max Voltage 6V Max Speed 12000 RPM
System Parameters	T_1 =1.2 s a_1 =0.0135 N.m/MU b_1 =0.0924 N.m/MU^2 I_1 =6.8e-2 Kg.m^2 B_1 =1e-1 Kg.m^2/s T_g =0.32 N.m T_2 =1 s a_2 =0.02 N.m/MU b_2 =0.09 N.m/MU^2 T_{or} =3.5 s T_{pr} =2 s K_r =-0.2 N.m/MU I_2 =2e-2 Kg.m^2 B_2 =1e-3 Kg.m^2/s K_{gyro} =0.05 Kg.m/s
Controller Parameters	K_1 =0.105 K_2 =0.105

SIMULATION RESULTS

IV Controller Implementation

The output state responses of both sliding mode controller and integral sliding mode controllers implemented in equation (18) are shown in fig.7 and fig.8. The comparison of both controllers is shown in figure.9. By comparing the equilibrium positions in both integral and sliding mode controllers it can be seen that the equilibrium is achieved very fast in integral sliding mode as compared to sliding mode. The elevation state comes to its equilibrium position in 7 sec in case of integral sliding mode whereas it is achieved in 9 sec in case of sliding mode. Similarly, by comparing the azimuth state, it comes to its equilibrium position in 10 sec by using the integral sliding mode whereas by using the sliding mode it is achieved in 20 sec. Similarly, by comparing the overshoot ratio of both controllers, in case of elevation, it is 5% by using integral sliding mode whereas it is 20% by using sliding mode. Similarly, in case of azimuth, it is 10% by using integral sliding mode whereas it is 20% by using sliding mode.

Table 2 Simulation Results

	Azimuth	Elevation		
	ISMC	SMC	ISMC	SMC
Overshoot (%)	10	20	5	20
Settling Time (sec)	10	20	7	9

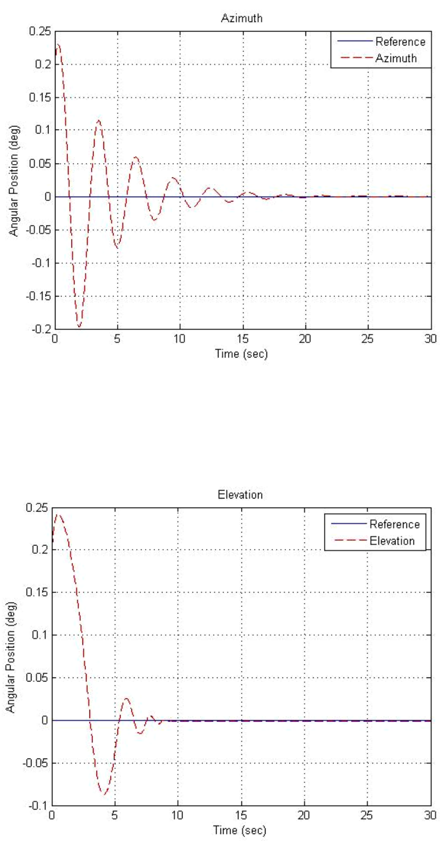

Fig.7 System Response Initialized in nonlinear Range (SMC)

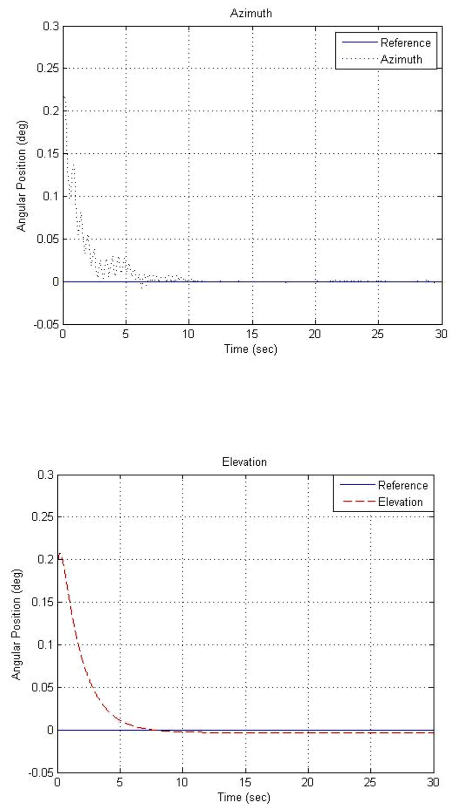

Fig.8 System Response Initialized in nonlinear Range (ISMC)

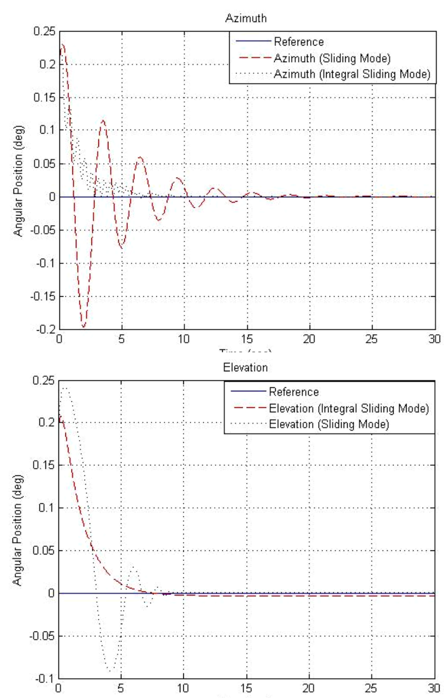

Fig.9 System Response Initialized in nonlinear Range (Comparison of ISMC & SMC)

V CONCLUSION

The designed sliding mode surfaces show virtuous results to contract with cross-coupling effects in twin rotor system dynamics. The sliding and integral sliding mode controllers show the required performance along with the robustness against the coupling stated as interruption. By comparing the results of both controllers with respect to overshoot and settling time it can be conclude that the results of integral sliding mode controller are better than the sliding mode controller. Thus, the integral sliding mode controller technique is better than the sliding mode controller.

REFERENCES

[1]. Gareth D. Padfield, "*Helicopter Flight Dynamics The Theory and Application of Flying Qualities and Simulation Modeling*" Second Edition Blackwell Publishing.

[2]. Qadeer Ahmed, "*Nonlinear Robust Decoupling Control Design for Twin Rotor System"*

[3]. Te-Wei, Peng Wen, "*Time Optimal and robust control of Twin rotor system*" Proc. of 2007 IEEE Int. Conf. on Control and Automation, Guangzhou, China, 2007.

[4]. M. Lopez-Martinez, F.R Rubio, "*Control of a laboratory helicopter using feedback linearization*"

[5]. M. L´opez-Martinez, J.M. D´ıaz, M.G. Ortega and F.R. Rubio, "*Control of a Laboratory Helicopter using Switched 2-step Feedback Linearization*", Proc. of the American Control Conf. (ACC'04),2004.

[6]. M. Lopez-Martinez, F.R Rubio, "*Approximate feedback linearization of a Laboratory Helicopter*"

[7]. Arkadiusz S. Dutka, Andrzej W.Ordys, Michael J.Grimble "*Non-linear Predictive Control of 2 dof helicopter model*" Proc. of the 42nd IEEE Conference on Decision and Control Maui, Hawaii USA, 2003

[8]. Jun Yoneyama, Yukihisa Kikuchi, "*Robust control for uncertain systems with application to helicopter model*" Proc. of the SICE 2002, Osaka.

[9]. M. Lopez-Martinez, M.G. Ortega and F.R. Rubio, *"An H∞ Controller of the Twin Rotor Laboratory equipment"*, Proc. of 11th IEEE International Conference on EmergingTechnologies and Factory Automation (ETFA'03), 2003.

[10]. M. Lopez-Martinez, C. Vivas, M. G. Ortega, "*A multivariable nonlinear H∞ controller for Laboratory helicopter*". Proc. of 44th IEEE conference on Decision and Control and European Control Conference 2005, Seville Spain 2005.

[11]. *Tomáš Koudela, Renata Wagnerová, "Position Control with Robust Algorithms" Proc. of the Portuguese conf. on Automatic Control, 2000.*

[12]. Gwo-R.Y, H.T.Lui, Sliding mode control of a two degree of freedom helicopter via Linear Quadratic regulator.

[13]. Juhng-Perng Su Chi-Ying Liangt and Hung-Ming Chent, "*Robust*

Control of a Class of Nonlinear System and Its Application to a Twin lbtor MIMO System" IEEE ICIT'O2, Bangkok, Thailand.2002.

[14]. Q. Ahmed , A. I. Bhatti, S. Iqbal, *Robust Decoupling Control Design for Twin Rotor System using Hadamard Weights*, to be presented in CCA, MSC 2009, St. Petersburg, Russia

[15]. F.Van Diggelen and K. Glover, "*A Hadamard weighted loop shaping design procedure for robust decoupling*" Automatica Vol. 30, No. 5, pp 831-845 1994.

[16]. Q. G. Wang, "*Decoupling control"* LNCIS 285,pp 115-128 2003

[17]. Eds. K. Warwick,D. Rees, **"***Multivariable Control:An Introduction To Decoupling Control*" Industrial Digital Control Systems IEE Control Engineering series, Peter Peregrinus 1988.

[18]. Slotine, J. J. and Li W. (1991), *Applied Nonlinear Control*, Prentice Hall, ISBN 0-13-040-890-5

[19]. *Humusoft*, *CE 150 helicopter model: User's Manual* ,*Humusoft*, Prague, 2002