Cracked Isotropic Plate Structures – A Review

Asif Israr
Department of Mechanical Engineering, Institute of Space Technology, P O Box 2750, Islamabad – 44000, Pakistan,
Email asif.israr@ist.edu.pk

ABSTRACT - Structural components potentially require continuous monitoring for the detection of cracks, crack growth, and damage, for ensuring an uninterrupted service in critical installations. Cracks can be present in structures for various reasons such as impact, fatigue, corrosion, external & environmental factors such as temperature, relative humidity, rainfall, and because of the effects of the general properties of structures over time. The presence of small irregularities in the form of a damage or crack does not only cause a local variation in the stiffness, but can affect the mechanical & dynamical behaviour of the complete structure. Cracks and damage present in vibrating components can lead to a complete loss of equilibrium and cause catastrophic failure. For these reasons, it is necessary to get insight of the dynamics of plate structures in the presence of damages or cracks. There is a limited literature on this topic, and the review attempts to describe the current literature on this subject.

Keywords: Cracks, Plates, Damages, Vibrations, Finite Element Method, Stress Intensity Factor, Non-Destructive Assessment.

1. OVERVIEW

Damages or cracks cause a severe threat to the performance of plate structures. Generally structures are failed due to corrosion and material fatigue. Hence, it is utmost important for an engineers and designers to develop methods that allow the early detection and localization of cracks for the smooth running and longevity of machines and structures. The dynamical behaviour of plate structures with minor irregularities in the form of cracks under various loading conditions and boundary conditions have been studied earlier by many scientists and engineers, and different analytical & experimental approaches have been proposed to examine the problems. Nevertheless limited material has been published for obtaining the dynamic/vibration characteristics of cracked plate structures. Dimarogonas (1996) gave a brief overview on the vibration of cracked structures, and covered a wide variety of areas that includes cracked beams, coupled systems, flexible rotors, shafts, turbine rotors & blades, pipes & shells, empirical diagnoses of machinery cracks, and bars & plates. However, he presented a mere list of publications on vibration of cracked plates. This review summarizes the literature in the following categories: use of vibration analysis for crack detection, determination of stress intensity factor for crack detection, and non-destructive assessment for damage detection in plate structures followed by concluding remarks.

2. USE OF VIBRATION ANALYSIS FOR CRACK DETECTION

In the last three decades, the main focus was developing of online structural health monitoring systems to replace conventional non-destructive techniques that require substantial down-time, cost and human effort. Vibration based damage detection is one of the most promising techniques for implementation in monitoring of plate structures. In these methods, the presence of damage is detected by monitoring changes in one of the dynamic/vibration parameters of the structure such as resonant frequencies, damping or mode-shapes. Frequency based methods have the advantage over the mode-shape methods are that measurements need to be taken only at a single location. Therefore, the measuring of the degree of variation in frequencies can provide a means for the detection of the location and the extension of damage. For these reasons, the variation of the natural frequencies in the cracked plate elements has been the subject of many researchers.

Lynn & Kumbasar (1967) were one of the first investigators who worked to find the vibrations of a cracked rectangular plate. They used a Green's function to obtain a homogenous Fredholm integral equation of the first kind which satisfied the mixed edge boundary conditions along the crack boundaries. Lynn & Kumbasar used the Krylov & Bogolivbov method to solve the integral equation for narrow cracks in the plate of all sides simply supported. The result was presented in terms of frequencies variations with respect to different crack lengths, and the relative moment distributions along the un-cracked segments. The finite element displacement method was applied by Petyt in 1968 to obtain the dynamic/vibration characteristics of plates with crack located at the centre. In addition, the effect of an increase in the width of the plate was described and this showed the decrease in amplitude in the area of the crack and the increase in the curvature of the plate in the region of the crack tip. The results were presented in terms of frequencies verses crack length, and stresses &

deflections verses length of the plate. Stahl & Keer (1972), and Aggarwala & Ariel (1981) used homogeneous Fredholm integral equations of the second kind and solved the eigen-value problem of simply supported plates by assuming the singularity of stress at the tips of the crack. These authors presented a method which was limited to crack locations, and reduced the problem to a dual series equation; however this method involved lots of computation. Aggarwala & Ariel pointed out that the frequency appeared to be higher when the cracks started from outside as compared to the case when cracks started from inside. The reason for this discrepancy was explained for the cracks starting from outside, that in the limiting case when they reached the centre, on account of symmetry the condition $\partial w/\partial x$ (where w represents the transverse displacement of the plate along x-axis) must be satisfied at the centre, which was not applicable when the cracks started from the centre. Hirano & Okazaki (1980), and Nezu (1982) solved the differential equations by applying the finite Fourier transform governing the problem of the cracked plates in which one pair of simply supported edges and the line of the crack was perpendicular to each other. They obtained integral equations which possessed the unknown discontinuities in terms of the slope and deflection across the crack. These unknown quantities were expanded into a Fourier series, and solved symmetric & anti-symmetric cases of their proposed models. Hirano & Okazaki presented their results for the mixed cases of plate supports i.e. simply supported, clamped, and free edges. In all cases, if plotted against frequency versus aspect ratio of the plate showed the decreasing trend. Similarly, Nezu summarized that the eigenvalues decreases as the notch length increases. Their decreasing rates were different depending on vibration modes. Secondly, the internal moment in the symmetric first mode and the shearing force in the anti-symmetric first mode increased rapidly toward the notch tip and as the notch became long, their increasing rates became large. Nezu also observed that if a short notch is located in the neighbourhood of an edge of the plate the internal moment in the symmetric first mode increases rapidly toward the edge, but if it is located at the centre of the plate, the moment increases toward the notch tip. Lee (1992) introduced an useful technique for determining the fundamental frequency of annular plates by considering the sub-section of the plate domain under different boundary condition. The method was based on Rayleigh's principle. The results showed that the fundamental vibration frequency increases when the crack is located near one of the two edges and it decreases with increasing crack length for a crack located near the centre between the two edges. Liew et al. (1994) used the decomposition method for obtaining the natural frequencies of plates with cracks by assuming the domain of the plate was the assemblage of small sub-domains with any set of boundary conditions. They used the appropriate functions to obtain a governing eigenvalue equation. The results were plotted and compared with those of other authors for a symmetric & anti-symmetric plate mode, and for two sets of crack locations. The main advantage of this method over the other discretization methods is that this method used very few meshes in the formulations. Ramamurti & Neogy (1998) analyzed a cracked blade of a turbo-machine modelled as a non-rotating cantilevered plate. They used Rayleigh Ritz method for finding the frequencies of this specific structure. It was found through their results that the cracked vibration frequencies were lower than the corresponding un-cracked vibration frequencies, although the authors were not satisfied with these findings and argued that the natural frequency is not a good criterion for the detection of damages. Similarly, Neogy & Ramamurti (1997) further extended their studies and proposed a model of a damaged blade, and the damage was in the form of non-propagated crack for determining the natural frequency as one of the criteria for the detection of cracks, however at this time they modelled a blade as a rotating shell. In their findings, they observed that the effect of rotation in the first mode was very pronounced but the effect of the crack was marginal, and so rotation failed to alter considerably the pattern of frequency reduction. In the case of the second and third modes both the crack and the rotation produced noticeable effects and the relatively larger effect of rotation to the decrease in frequency was observed. Therefore, it was concluded from these results that small cracks in turbine blades, when modelled as a cantilevered plate, cannot easily be detected by the natural frequency criterion.

Khadem & Rezaee (2000(a)) introduced a new technique for the vibration analysis of plates in the presence of cracks and considered the effect of compliance due to bending only. Later in 2000(b), they established an analytical approach for damage in the form of a crack in a rectangular plate using the application of external load for different boundary conditions. They concluded from their results that a crack at a specific depth and location would affect each of the natural frequencies differently. Wu & Shih (2005) theoretically analyzed the nonlinear response and dynamic instability of simply supported cracked plates subjected to a periodic in-plane or membrane load. They used the incremental harmonic balance method for solving this model. Their results indicated that the stability behaviour and the system response were governed by the crack location, in-plane or membrane loading, the aspect ratio, and the vibration amplitude. They also explained that increasing the crack ratio i.e. the ratio of the crack length to the edge length of the plate where the crack lies and/or the static component of the membrane load decreases the dynamic characteristics of the entire structure under consideration. Israr et al. (2006) proposed a mathematical model for the identification of damages in the form of crack in an aircraft panel structure. The panel was modelled as an isotropic cracked plate, and plate was subjected to free vibration for the measurement of system response in terms of frequencies & amplitudes. Further studies show that the literature doesn't appear to contain much reference to analytical or mathematical models for plates with crack undergoing forced vibration. By taking this into consideration Israr (2008), (2009) & in (2011) developed an approximate technique for obtaining the vibration frequencies of the plates in the presence of crack that extends partially through the thickness. Initially, certain simplifying assumptions such as the plate is perfectly elastic, homogeneous, isotropic material and has a uniform thickness throughout, allow Hooke's law to hold, etc. were made for the solution of this class of vibration problems. A term containing a cubic nonlinearity was introduced into the model by the use of Berger's formulation (1955), and reduced the equation into the well-known Duffing equation. They studied both theoretically and experimentally the nonlinear responses, amplitudes, the vibration frequencies and modes of vibration. The perturbation method of multiple scales was used, and found that by increasing the length of the crack, the frequency of vibration decreases and the vibration amplitude increases. Furhter studies shows that Israr & Cartmell (2011) evaluated global dynamics of an isotropic cracked rectangular plate, subjected to SSSS, CCSS and CCFF boundary conditions under the influence of transverse harmonic excitations, shown to be convertible to classical Duffing Equation. Crack consisted of a continuous line in the middle and along the x- axis of the plate. Computational procedure has been implemented using Dynamics 2 and NDSolve integrator within *Mathematica*. Bifurcation studies, estimation of Lyapunov Exponent and Poincare maps have been used for analysis of nonlinear dynamical system. Results showed that system response could be extremely susceptible to changes in control parameters. Later in 2012, Israr & Zulfiqar proposed an approximate vibration response of cracked plate subjected to transverse loading through higher order perturbation method of multiple scales. It was found that excitation frequency of cracked plate under higher order perturbation method of multiple scales had less error as compared to resonance frequency of low order perturbation. Huang & Leissa (2009) applied Ritz method with special displacement function for the analysis of a rectangular plate with a side crack subjected to free vibration. A detailed convergence studies was carried out to analyze a rectangular plate of simply supported edge with horizontal crack. The results showed that proposed displacement function convergences the numerical solution particularly when a crack was large. It was observed that this approach is capable for measuring the vibration frequencies and mode shapes in case of free vibration of simply supported & square plate of all side free with side cracks to monitor the effects of length, location and orientations of side cracks including cracks which are not along a symmetry axis are skewed. Sh et al. (2010) proposed a closed form of equations derived from Mindlin plate theory for the vibration analysis of moderately thick rectangular plates having an arbitrary number of all-over part-through cracks with different possible boundary conditions. The elastic behaviour of this crack was described by a continuously distributed line spring model. Theoretical results were compared with the results produced through finite element analysis software. A parametric study of moderately thick rectangular plates was carried out in the form of graphs and tables under different possible boundary conditions to monitor the effects of crack thickness to length ratio and number of cracks on the natural frequencies. Saleh (2012), carried out free vibration analysis of variously cracked square thin Simply supported plates and investigated the effects of different crack parameters (i.e. length, location and orientation) on natural frequencies and corresponding mode shapes. They conducted Eigen values vibration analysis for edge, internal and corner cracked plates and used FE software package *ANSYS* for computation. Investigations revealed that crack parameters significantly influence natural frequencies and internally cracked plate with crack orientation of 45° showed highest reduction in frequency parameter. Ismail & Cartmell (2012) and Ismail (2012) extended the work of Israr (2008) and proposed an analytical model of an aircraft panel structure, in the form of thin isotropic cracked flat plate consisting of different surface crack orientation & subjected to CCFF, CCSS and SSSS boundary conditions for vibration analysis. Crack formulation in the form of simplified line spring model is subjected to transverse harmonic excitation. Cubic nonlinear system obtained by employing Berger's formulation is investigated by the use of amplitude frequency and multiple scale perturbation method. Orientation of crack in the plate greatly affected the vibration and nonlinear characteristics of the plate structure.

Bending of a rectangular plate with cracks was initially explored by Keer & Sve in 1970. They limited their analysis to such a crack location that confines to a position along the symmetry axis, and reduced the system of equations into a dual series equation. The solution to this particular case was based on Fredholm integral equation of the second kind initially proposed this technique by Westmann & Yang in 1967. Keer & Sve studied three cases comprising an internal centrally located crack, two equal length collinear external cracks and a single external crack. In all these cases the two plate boundaries perpendicular to the crack edges were simply supported & clamped. The authors found that there was a strain energy changes for both support cases, and for different aspect ratios of the plate. By comparing the two support conditions it was observed that the clamping tends to reduce the possibility of fracture since the release rate of strain energy was smaller. A similar technique was proposed by Stahl & Keer (1972) for the stability and vibration/dynamic analysis of rectangular plates with cracks and was bounded by the same restrictions encountered by Keer & Sve (1970). Solecki (1975) took this challenge and removed these limitations by proposing a more feasible method that allowed the study of rectangular plates with arbitrarily located cracks. And, in 1983, Solecki extended his work by using the generalized Green-Gauss theorem and finite Fourier transformation for studying the natural flexural vibration of simply supported cracked rectangular plates. The arbitrarily located crack was parallel to one of the plate edge. A centrally located, and off-centre, crack was also discussed and explained. Analytical results were not obtained because of the reasons that the curvature singularity at the cracks tips was not explicitly isolated. Saito et al. (2008) and in 2009 analyzed vibration response (linear and nonlinear) of a cantilevered plate by varying the crack location or crack length. In particular, they studied the veering phenomena and obtained the vibration frequencies of a plate containing a crack. It was found that the veering due to the variation in the length of the crack entails the mode shapes changes. The nonlinear behaviour of the plate due to the presence of the crack surfaces was also examined. A hybrid frequency and time domain (HFT) approach based on the method of harmonic balance was employed to the estimation of nonlinear response in terms of resonant frequencies at crossing or veering regions. It was observed that the nonlinearity caused by the crack affects the characteristics of veering or crossings. In addition, a method for obtaining the nonlinear resonant frequency was introduced by extending the approximation in bilinear frequency. It was found that both methods bilinear frequency approximation and HFT produced similar results.

In summary, it has been demonstrated from previous studies that the measurement of frequency changes and mode shapes can be used effectively to identify and assess the crack size and location in plates by the help of different approximate analytical techniques. Liu & Lam (1994) suggested that the strip element method can be used with few meshes in the formulation as compared to other discretization methods. Neogy & Ramamurti (1997) concluded that cracks cannot easily be detected by the natural frequency criterion. However, it is concluded from the above discussion that frequency and mode shapes are best suited analytical approaches among others for the identification of cracks and damages in plate structures.

3. DETERMINATION OF STRESS INTENSITY FACTOR FOR CRACK DETECTION IN PLATE STRUCTURES

Stress intensity factor is usually used in linear elastic fracture mechanics to depict the stress state near the crack tip by considering the geometry, size and location of the crack. When stress intensity factor exceed the critical value, crack propagation occurs. Therefore, it can give an idea whether the stresses in an elemental part will cause cracks to propagate. Detection of crack in plate structures by determining the stress intensity factor have been employed by many researchers, and a brief review on this subject is as follow:

The Line Spring Model (LSM) has been rigorously treated by many researchers since it was initially proposed by Rice & Levy in 1972. The Rice & Levy model reduces the truly three dimensional problem to two dimensional plate or shell theory. This approach also reduces the computational time as compared to full three dimensional models, and within certain limits, offers acceptable accuracy. They employed two dimensional generalised plane stresses, and used Kirchhoff's plate bending theories with a line spring to represent a part-through crack, and also chosen compliance coefficients to match those of an edge-cracked strip in plane strain. The line of discontinuity was of length 2a and the plate was subjected to remote uniform stretching and bending loads along the far sides of the plate. These authors computed the force and moment across the cracked section to find out the stress intensity factor, and the solution to the problem was characterised in terms of the Airy stress function. Their results showed that the ratio of the stress intensity factor for an all-over crack, and the stress intensity factor of an edge crack in plane strain for the same depth of the crack to the thickness of the plate, for remote tensile or bending load, approaches unity with an increase in the ratio of the length of the crack to the thickness of the plate. Furthermore, at small values of the depth of the crack to the thickness of the plate, the ratio of the stress intensity factor approaches unity for small values of the length of the crack to plate thickness. Moreover, they pointed out that this enlightened approximation is most appropriate along those regions where the depth of the crack fluctuates gradually. Wen & Zhixe (1987) suggested a modified line spring model for the analysis of fracture in plates containing a slender and part-through cracks under tension and shearing by taking into account the non-local deformation effect for the line spring. These authors investigated the accuracy of the problem and the relationship of the line spring model between

Kirchoff's theory and Reissner's theory of plates and shells, and calculated the stress intensity factors for different loading and cracked geometry. Two sets of problems were discussed; an infinite plate with a symmetric elliptical crack under tension, and a rectangular surface crack under uniform shearing. It was observed that the relative error increased with the increase of the ratio of the depth of the crack to plate thickness. Shawki et al. (1989) formulated the power-law line spring model for fully-plastic semi-infinite body with deep cracks subjected to remote bending and tension to find the stress intensity factors. They used finite element program for implementing this model. These authors applied this model to three dimensional problems with semi-elliptical surface cracks. It was shown that the loading axis shifts with increasing loads and indicated the change in the local ratio of bending to tension. Zeng et al. (1993) developed a new line spring model based on the boundary element method to find the surface flaws in the form of crack in the plate structure. The virtual crack extension method was employed to obtain the stress intensity factor at the edges of the crack front-free surface. These authors found by detailed investigations that the stress intensity factor at the edges of the crack front-free surface is barely acceptable. The dynamic stress intensity factor (DSIF) in structures having a crack can give information about the propagation of crack as reported by Polyzos et al. (1994). They employed a frequency-domain boundary element method along with the numerical Laplace transform and the corresponding principle of linear viscoelasticity in conjunction with quarter-point boundary elements for determining the DSIF for viscoelastic plates with crack. Their results showed that viscoelasticity generally reduces the plate response. Cordes & Joseph (1995) analyzed the LSM and used it to compute the stress intensity factors for the internal and surface cracks in a Reissner's plate that contained residual stresses. Such stresses are usually caused by deliberate or un-deliberate activities during fabrications and installation, and need to be examined in detail to ensure that the material responds in a secure and conventional manner throughout its life span. These authors presented a series of results for different crack length and depth, and compared their results with the LSM classical theory (the Irwin model of 1962) and the finite element model, which showed that the current model results ranged from 0.6-0.8% higher, whereas the average percentage difference was found to be 4.2%. In addition, upon increasing of loading, the discrepancy in their results also increased slightly.

Gross et al., in 1964, described in their technical report that stress intensity factors may be obtained by a boundary value collocation method applied to the Williams stress function for a single edge notch tension specimen. The results were presented in terms of non-dimensionalized stress intensity factors & the ratio of the crack length to the specimen width, and this showed an increasing trend with the increase of crack ratio. Moreover, at small ratios of the length of the crack to specimen width (0.15-0.40) the results were in good agreement with other authors' results, and above 0.40 the differences in their results were implicitly due to bending, which was not taken into account in their analytical solutions. Rossmanith (1985) used the Westergaard stress function approach to plate bending problems with centrally located crack for determining the stress intensity. Wang et al. (2003) extended a boundary collocation method based on complex variable theory which was proposed for obtaining the stress intensity factors of cracks in a plate of finite length. Five examples including a rectangular/circular plate with a central crack, a rectangular plate with a slanted crack, a simply supported plate with a central crack and a plate with two cracks were discussed, and it was concluded that good agreement for short cracks was evident with those obtained by other methods. Purbolaksona et al. (2006) proposed a dual boundary element method for the geometrical nonlinear analysis of a square cracked plate with fully clamped and simply supported boundary conditions. It was shown that the normalised stress intensity factors in the membrane of the plate increased significantly after a few increments of the load. However, in bending the normalised stress intensity factors decreased if compared with the linear results. Guimarães & Telles (2007) extended the numerical Green's function technique to the problems of fracture mechanics in conjunction with Reissner's plate theory. The technique produced a fundamental plate bending Green's function that included cracks to be used in the classical boundary element method for the solution of such configurations. The solution was presented in terms of normalized stress intensity factors to analyze the behaviour of the structure.

Cracked plates can be modelled in many ways using Finite Element Method (FEM). Israr (2008), Israr & Atepor (2009) performed FE analyses for a nonlinear vibrating plate with crack for a Clamped Clamped Free Free (CCFF) boundary condition. Three dimensional profiles for mode I, II & II, deflection, and excitation frequency were obtained and showed that these results are fully in line with the analytical approach based on the perturbation method as proposed by Israr et al. (2009). Raju & Newman in 1979 used a three dimensional FE model to analyze an elliptical circular, and penny-shaped crack completely embedded in a finite thickness plate under uniform tension. Singular and isoparametric elements were used in combinations to model elastic bodies with cracks. Stress intensity factors were obtained and compared with the exact solutions for the same configuration to validate the results of FEM as used, and concluded that the calculated stress intensity factors for embedded elliptical and circular cracks were 0.4-1% lower than the exact solutions. However, the calculated stress intensity factors in the sharpest curvature region of the ellipse in case of elliptical crack were 3% above the exact solution. Alwar & Nambissan (1983) obtained the stress intensity factors for finite rectangular plates by the use of FE techniques. Three dimensional isoparametric singular brick elements were used to analyse the bending of plates with cracks. This analysis showed the values of stress intensity factor to be 5-10% higher than the two dimensional analysis and these authors found that the stress intensity factor varied in a nonlinear fashion across the plate thickness. Fujimoto & Sumi (1987) used a FEM to identify damages in the form of crack for a rectangular plate with centrally located crack. One quarter of the plate was analyzed due to symmetry using the triangular elements. Very fine meshes along the crack tips were used to account for the high localized stress concentration. The results were compared with the experimental data obtained by the laser holography technique. Aluminium alloy 7075T6 was used as the material of the plate of aspect ratio 0.5 and obtained numerical results for the same configurations with crack ratio of 0.4. It was found that both the finite element and experimental results were in good agreement. Qian et al. (1991) built a FE model of plates with crack using the integral of the stress intensity factor, and applied this to solve the damaged plate vibration problems for different cases such as centrally located crack of a simply supported plate and a cantilever type square plate. In all these cases mesh subdivision in the neighbourhood of the crack tip was not made that reduced the numerical computations. These authors compared their results with the model of Solecki (1983) with good agreement. Krawczuk (1993) presented a finite element model and used stiffness matrix for analyzing a rectangular plate and shell consisting of a through, non-propagating, open crack. He studied the effects of the length and location of the crack on the changes of the eigen-frequencies of a cantilever and simply supported rectangular plate. The centre of the crack was located in the middle of the element, and assumed a linear normal stress & constant shear stress distributions across the crack region. It can be concluded from this work that decreasing natural frequencies are a function of crack's length and location, the modes shape, and plate's boundary conditions. Later, in 1994 similar findings were obtained except that Krawczuk & Krawczuk et al. showed that the length and location of crack greatly influences on the amplitudes of the transverse forced vibration in case of cantilevered cracked plates and beam made of aluminium. They found that an increase in the amplitude of the transverse forced vibration amplitude of the damaged plate is a function of the length and location of the crack. Further studies show that Krawczuk et al. introduced elasto-plastic properties into cracked plates in 2001, and found similar findings. Su et al. (1998) extended the fractal two level finite element method (F2LFEM) for the free vibration analysis of a thin plate containing a crack with simply supported boundary conditions subjected to edge moments for three modes of vibrations. The normalised vibration frequencies for simply supported square centre cracked plate was compared with the model of Stahl & Keer (1972), and found close agreement between the two methods. Goncalves & Castro reviewed the LSM in 1999, and its implementation in finite element software. A plate containing a part-through crack was studied and obtained the stress intensity factors for the cases of pure tension and pure bending. It was emphasized that the LSM is not a good approximation in cases where the crack depth varies rapidly and near the free surfaces of the crack because the curvature of the crack front was not taken into account. Fujimoto et al. (2003) performed a vibration analysis of cracked rectangular plates. A tensile load was applied to the structure to be analyzed. A crack was located at the centre of the plate and along the line of symmetry perpendicular to the direction of the tensile load. These authors used the FEM for finding the eigenvalue analysis, and the body force method was used for in-plane stress analysis associated with the eigenvalue analysis. Their numerical and experimental results confirmed that the natural frequencies of all vibration modes increased monotonously with increase in tensile load, whereas the rate of increasing frequency depends upon the mode shapes. It was also observed that this method was well suited to a plate with a long crack and showed noticeable variations in mode shapes for a small range of tensile load. The assessment of failure of a cracked plate subjected to biaxial loading was presented by Kim et al., in 2004. Their analyses were based on two and three dimensional elastic-plastic finite element analysis for analyzing of plates with through-thickness and semi-elliptical surface crack. It was found that for a thicker plate with semi-elliptical surface cracks the biaxiality effect on crack tip stress triaxiality was more pronounced near the surface points along the crack front, and reported that the aspect ratio of the crack was found to be more vital as compare to the relative depth of the crack. The same subject has also been reported by Wilson & Thompson (1971) and Sathari-Far (2004). Horibe & Watanabe (2006) proposed an inverse method that used a genetic algorithm for the identification of width and position of the cracks derived from changes in the natural frequencies in plate structures. In this method, the natural frequencies were calculated by finite element method which was based on the Bogner, Fox & Schmidt model and employed the response surface method for minimising the processing time. They discussed two types of crack in the analysis comprised internal and edge cracks for a cantilever, and a plate with two ends clamped. The results showed that the proposed method provides satisfactory identification of cracks in plates and explained that the problem within the approximation accuracy of the interpolation function, and rapid changes in natural frequency could not be resolved due to coarse lattice-point intervals. Cheung & Song (2009) introduced bending crack strip (BCS) that combines the shape functions that govern the deflection around a crack of the spline finite strip with the eigenfunction solutions of the differential equations. A three-dimensional cracked plate structure with embedded cracks that are perpendicular to the longitudinal axis of the strips was analyzed with this new technique and finite strip method (FSM) to compute the stress intensity factors. The authors illustrated findings through examples of their proposed model for monitoring of damages. This technique drastically reduces the time required for analysing the structure without disrupting the degree of accuracy which is the advantage of this model.

Similarly, Irwin (1962) examined a plate containing a part-through crack subjected to tension and derived a relation for the crack stress-field parameter and the crack extension force at the boundaries of a flat elliptical crack. Approximate and experimental findings were made and found encouraging results. Matbuly et al. (2008) introduced an iterative hybrid technique for solving two dimensional crack problems by employing the boundary element method and distributed dislocation method. Iterations were carried out between the crack faces and the plate boundaries unless and until all of the assumed boundary conditions are satisfied. Matbuly et al. applied this technique to solve three cases, with a crack at the centre subject to mode I, a kinked crack of two branches, and a kinked crack of three branches.

Above all, stress intensity factor is a useful criterion for evaluating failure or damages in the materials in the presence of a crack. It describes mathematical stresses and deformation distribution around a crack tip. It is observed that analytical expressions for stress intensity factor exist for simple geometries, however, it has become difficult and more computationally expensive to predict stress intensity factors theoretically, which is because of the complexity of modern structures.

4. NON-DESTRUCTIVE ASSESSMENT FOR DAMAGE DETECTION IN PLATE STRUCTURES

Non-destructive assessment (NDA) is widely used in industry to evaluate the structural integrity of civil, aero and mechanical structures. Conventional methods for NDA crack detection provide level one i.e. existence of damage and limited level two information i.e. location of damage. These methods not only provide the information that the structure is damaged, but also the location and severity of the damage. Many NDA methods are available and are currently being used commercially such as ultrasound, vibration methods, thermal wave, surface waves, Eddy currents, and acoustic emission. A study based on these methods is presented next for the damage detection assessment in cracked plates.

Cawley & Adams (1979(a)) proposed a method which described a non-destructive assessment of the structures by measurement of the vibration frequencies. The model reduced the elastic coefficients of the element at the location of the crack. These reduced elastic coefficients were not related to the real crack dimensions, which is the disadvantage of this method. It was found how measurements made at a particular position in the structure could be utilized to identify, locate and roughly to quantify damage in a wide variety of one and two dimensional structures. In 1979(b) these authors proposed a method based on experiments to measure the damage location and its depth from the changes in the vibration frequencies. Cornwell et al. (1999) employed a strain energy approach to identify and trace damage in plate structures. This technique required the structural mode shapes before and after damage. The algorithm was effective in determining defective areas with reduction of stiffness as low as 10%. Żak et al. (2000) postulated a non- destructive method based on vibration technique which was able to investigate experimentally for establishing changes in the first three bending vibration frequencies in the beam and plate element due to delaminations. The drop in bending vibration frequencies due to delaminations were the functions of the delamination length & mode of vibration, and finally it can be concluded that as the size of the failure increased the reduction in the vibration frequencies also increased. Yan & Yam (2002) identified damages in composite plates by using spatial wavelet theory initially proposed by Liew & Wang in 1998 to decompose the dynamic responses. The damage was in the form of crack and considered a simply supported beam with a transverse on-edge non-propagating open crack for modelling the problem. They obtained the wavelets along the beam length depending upon the numerical solution of the beam deflection. The location of the damage was then indicated by peaks along the beam length. Krawczuk et al. (2003), and later in (2004), applied a versatile numerical approach for the analysis of wave propagation and damage detection within cracked plates. They considered the spectral plate element as a tool for the investigation of such phenomena and showed that when a propagating wave runs to the crack location of the plate it divides itself into two signals, which show an indication of the damage section. Chang & Chen (2004) presented a spatial wavelet approach for damage detection in a rectangular plate with clamped edges on four sides. In this method spatially distributed signals in terms of mode shapes or displacement of the rectangular plate after damage were used. These spatially distributed signals were obtained by FEM and analysed by wavelet transformation. The results showed that the position of damage can be identified by showing a peak at this location. However, some indications of damage were also observed at the clamped edges of the rectangular plate. Therefore, it was concluded that it was very hard to detect the crack position at the edges. Epureanu & Yin (2004) monitored structural health while employed vibration based damage detection. They investigated a panel excited by flow induced loads and considered the nonlinearity resulted from the bending and stretching of the panel. The panel was assumed to be a one dimensional, homogenous, isotropic and elastic thin plate with spring-supported end points. This method used probability density functions for determining the structural response. These authors argued that nonlinearities interfered with linear behaviour, and small changes in parameter were not easy to detect using linear methods, however, linear methods minimize the influence of nonlinearities. Park et al. (2005) examined a composite plate subjected to damage introduced by firing a small projectile out of a gas gun caused a delamination in a plate. Frequency response functions were measured from piezoelectric Macro-Fiber Composite (MFC) that exerts an excitation at high frequency ranges (5-20 kHz) on the plate to measure and detect the delimination in a composite plate. Frequency response functions and damage indicator features were derived from the signals and used to monitor the plate conditions. In addition, this method was validated by ultrasonic scanning methods, where the delaminated area was well correlated to the damage indicator feature. Trendafilova (2005) carried out vibration based analysis of an aircraft wing scaled model to identify and trace the damages. In this study localised and distributed damage was considered, and a simplified finite element model was used to model the problem for the vibration response. The wing was split into five volumes for the purpose of analysing the damage detection for the first ten natural frequencies. It was shown that the cracks of length less than half of the wing width are undetectable in the case of localised damage, whilst in the case of distributed damage damage less than 30% in any of the volumes was not detectable using natural frequencies. The author proposed in her concluding remarks that changes in the lower modal frequencies were affected by damage close to the wing root, and their changes decreased when the damage moved towards the wing tip, or conversely the higher frequencies were more affected by damage close to the wing tip and their changes increased when damage moved from the wing root towards the tip. Later, Trendafilova et al. (2006) applied a similar technique for vibration based damage detection in aircraft panels modelled as isotropic plates with a crack at some specified location, and they obtained extremely good results. Gorman et al., in 2006 used the modal frequencies approach for the vibration health monitoring of a coupled plate/fluid interacting system. A Galerkin method was used to develop the analytical model for the frequency model analysis and the theory was further expended to explain the natural coupled modes in terms of the relative energy associated with each of the two sub-systems of the plate and the fluid. Budipriyanto et al. (2006) studied the root mean square (rms) responses of the model of a cross-stiffened plate of a ship structure, and examined to detect the location and size of the cracks. An aluminium plate was used, with two same sizes of web frames and four horizontal of three different sizes. The identification of the damages was carried out at four different crack locations. They showed that the normalised rms of the strain and acceleration responses between different frequency ranges are capable for the identification of cracks. Hadjileontiadis & Douka (2007) presented a technique based on kurtosis analysis for the identification of crack in an isotropic thin rectangular plate having a crack parallel to one of its edges with simply supported boundary conditions. The abrupt changes derived from kurtosis signal were used to detect crack length and location. The accuracy of this model was also investigated by the use of noise stress test for the added noise induced in the structure. The authors believed that this method is capable to identify the crack size, location and depth in the presence of noise, and can be used in variety of practical applications. Nguyen & Olatunbosun (2007) proposed a method based on wavelet transform and the breathing crack phenomenon for the identification and remote monitoring of a crack. During vibration the crack opens and closes which caused the crack edges come into and out of contact and produced non-linear effects due to changes in the structural stiffness. Wavelet transform was used to identify such nonlinearitly in the spatial variation of the response signals obtained from the structure. The results showed that the technique is quite helpful for remote health monitoring of the structures. Kannappan & Shankar (2007) presented an analytical approach for identifying damages in plates using combination of frequency measurements and mode shapes. Damage was modeled as a spring with finite stiffness which was less than that of the undamaged structure. The theory to deduce the size of through thickness crack had also been extended for plate structures. The accuracy of the method in locating and characterising the damage was demonstrated with frequencies obtained from finite element analysis. The maximum error in prediction of location and crack length was less than 5%. Semperlotti & Conlon (2010) presented an approach for the identification of defects displaying nonlinear vibration behavior. They investigate the nonlinear structural dynamic response exhibited by a riveted joint with loosened fasteners connecting a stiffener with a flat panel. The excitation generated elastic waves with dominant bending components, triggered the nonlinear contact between the plate and the stiffener induced a dynamic response with nonlinear harmonics. Experimental structural intensity maps were estimated at the super-harmonic frequencies. This technique provides an experimental approach for the characterization and two-dimensional visualization of nonlinear defects. Khaji & Noureini (2011) presented a numerical method i.e. spectral finite element method (SFEM) to simulate transient wave scattering phenomena in isotropic material plates. Classical finite element method (FEM) and spectral elements were combined to attain a tool handling large scale wave propagation problems. 2D spectral Finite elements were formulated using Lagrange interpolation function supported on Legendre- Gauss- Lobatto points in conjunction with LGL integration rule. SFEM offered special features over classical FEM in solving large scale transient wave propagation problems. As discussed, several non-destructive assessments have been proposed based on different approximate techniques. It can be seen that the damage in a structure changes its dynamic/vibration characteristics like natural frequencies, damping and modes of vibration, the damage can be detected and characterised using these parameters. Iwaniec et al. (2012) analyzed vibration responses and detected changes in determinism of undamaged and cracked Aluminum plates by the use of recurrence plots and recurrence quantification analysis. Plates have been vibrated by harmonic excitation of frequencies corresponding to structural resonance. Major focus is on nonlinear properties resulting from different crack behavior under vibration excitation. Results revealed that dynamic behavior of plates changes with respect to various excitation frequencies and crack modes.

Finally, solutions for cracked and damaged plates have been investigated by means of various methods by many researchers. Each solution technique is of some form of special relevance and treatment involves some particular type of approximations. Therefore, the users have the choices to select and implement the most appropriate method which fulfil the requirement and nature of the problems. The finite displacement method was used by Petyt (1968) and the finite Fourier transform was used by Hirno & Okazaki (1980), Nezu (1982) & Solecki (1975) for their analyses. Rayleigh-Ritz method was used by Ramamurti & Neogy (1998) for identifying the damages in the form of cracks in the plate structures. The finite element method is a fast growing technique which has also been applied to cracked plates by many researchers, such as Raju & Newman (1979), Qian et al. (1991), Krawczuk (1992), Su et al. (1998), Fujimoto (2003), and Trendafilova (2005) etc. The spatial wavelet theory is used by Krawczuk (2003) and (2004), Yan & Yam (2002) and Chang & Chen (2004) and perturbation method of multiple scales is employed by Israr et al. (2006), (2008), (2012) and (2009) and Ismail et al. (2012).

In practice, it is impossible to come across a crack problem in which only a single crack is involved. Many engineering practices simplifies the multiple cracks problem into single crack problem because of the reasons that there is a lack of effective means to solve the original problem and due to the desire for a quick and simple solution based on the safe design consideration. Therefore, in general, single crack problems are limited to laboratory tests which are often conducted on plates or beams to acquire desirable parameters and to verify new computational theories & behaviours under specified initial conditions, load conditions and boundary conditions.

5. CONCLUSIONS

The effect of cracks in vibrating components can lead to catastrophic failure, which ultimately reduces the local flexibility of the plate structures. Therefore early detection is necessary for better overall performance.

There are several theoretical and experimental approaches that exist for dealing with the linear analysis of the cracked plates, however, relatively little material has been published for the case of nonlinear analysis of cracked plates, and this generally needs more attention. One can find some interesting phenomena in structures in the presence of nonlinearities which are overlooked when considering linear models. Therefore, it is utmost important to study and model the system's nonlinearity for better understanding of the dynamical behaviour of a structural system with small damages in the form of cracks under different loading and boundary conditions.

ACKNOWLEDGMENTS

The author wish to acknowledge the support and services provided for this research by Professor Matthew P Cartmell, University of Glasgow, Glasgow, UK.

REFERENCES

[1] Aggarwala BD, Ariel PD. Vibration and Bending of a Cracked Plate. *Engineering Transactions* 1981; **29**: 295-310.

[2] Alwar RS, Nambissan KNR. Three-Dimensional Finite Element Analysis of Cracked Thick Plates in

Bending. *International Journal for Numerical Methods in Engineering* 1983; **19**: 293-303.

[3] Berger HM. A New Approach to the Analysis of Large Deflections of Plates. *Journal of Applied Mechanics* 1955; **22**: 465-472.

[4] Budipriyanto A, Haddara MR, Swamidas ASJ. Crack Identification in a Cross-stiffened Plate System using the Root Mean Square of Time Domain Responses. *Canadian Journal of Civil Engineering* 2006; **33**: 989-1004.

[5] Cawley P, Adams RD, The Location of Defects in Structures from Measurements of Natural Frequencies. *Journal of Strain Analysis* 1979(a); **14**(2): 49-57.

[6] Cawley P, Adams RD. Defect Location in Structures by a Vibration Technique. *American Society of Mechanical Engineers Design Engineering Technical Conference*, St. Louis 1979(b), paper 79-DET-46.

[7] Chang C-C, Chen L-W. Damage Detection of a Rectangular Plate by Spatial Wavelet Based Approach. *Applied Acoustics* 2004; **65**: 819-832.

[8] Cheung Moe MS, Song Z. Finite-Strip Method for the Analysis of Cracked Plates with Application to Plate-Girder Bridges. *Journal of Structural Engineering*, February 2009; **135**(2): 198-205.

[9] Cordes RD, Joseph PF. Surface and Internal Cracks in a Residually Stressed Plate. *International Journal of Fracture* 1995; **68**: 287-314.

[10] Cornwell P, Doebling SW, Farrar CR. Application of the Strain Energy Damage Detection Method to Plate-Like Structures. *Journal of Sound and Vibrations* 1999; **224**(2): 359-374.

[11] Dimarogonas AD. Vibration of Cracked Structures: A State of the Art Review. *Engineering Fracture Mechanics* 1996; **55**(5): 831-857.

[12] Epureanu BI, Yin S-H. Identification of Damage in an Aeroelastic System Based on Attractor Deformations. *Computers and Structures* 2004; 82: 2743-2751.

[13] Fujimoto T, Sumi S. Vibration Characteristics of Center Cracked Plates under Tension (*In Japanese*). *Bull. Jpn Soc. Mech. Engrs* 1987; **53**: 1124-1131.

[14] Fujimoto T, Wakata K, Cao F, Nisitani H. Vibration Analysis of a Cracked Plate Subjected to Tension using a Hybrid Method of FEM and BFM. *Materials Science Forum*. Trans. Tech. Publications, Switzerland: 2003. **440-441**, p. 407-414.

[15] Goncalves JPM, de Castro PMST. Application of the Line Spring Model to Some Complex Geometries, and Comparison with Three-dimensional Results. *International Journal of Pressure Vessels and Piping* 1999; **76**: 551-560.

[16] Gorman DG, Trendafilova I, Mulholland A, Horáček J. Vibration of a Coupled Plate/Fluid Interacting System and its Implication for Modal Analysis and Vibration Health Monitoring. *Applied Mechanics and Materials* 2006; **5-6**: 323-330.

[17] Gross B, Srawley JE, Brown WF Jr. Stress-Intensity Factors for a Single-Edge-Notch Tension Specimen by Boundary Collocation of a Stress Function. *NASA Technical Note D-2395*, August 1964.

[18] Guimarães S, Telles JCF. On the Numerical Green's Function Technique for Cracks in Reissner's Plates. *Computer Methods in Applied Mechanics and Engineering* 2007; **196**: 2478-2485.

[19] Hadjileontiadis LJ, Douka E. Kurtosis Analysis for Crack Detection in Thin Isotropic Rectangular Plates. *Engineering Structures* 2007; **29**(9): 2353-2364.

[20] Hirano Y, Okazaki K. Vibration of Cracked Rectangular Plates. *Bulletin JSME* 1980; **23**: 732-740.

[21] Horibe T, Watanabe K, Crack Identification of Plates using Genetic Algorithm. *JSME International Journal* 2006; **49**(3): 403-410.

[22] Huang CS, Leissa AW. Vibration Analysis of Rectangular Plates with side Cracks via the Ritz Method. *Journal of Sound and Vibration*, June 2009; **323**(3-5): 974-988.

[23] Irwin GR. Crack Extension Force for a Part through Crack in a Plate. *Journal of Applied Mechanics* 1962; **29**: 651-654.

[24] Ismail R, Cartmell M.P. An investigation into the Vibration Analysis of a Plate with a Surface Crack of Variable Angular Orientation, *Journal of Sound and Vibration* 2012; **331**: 2929–2948.

[25] Ismail R. Vibration Analysis of a Plate with an Arbitrarily Orientated Surface Crack. PhD Thesis, University of Glasgow, UK, (2012).

[26] Israr A, Atepor L. Investigation of the Nonlinear Dynamics of the Partially Cracked Plate. *Journal of Physics*: *Conf. Sr.* 2009; **181**: 012087.

[27] Israr A, Cartmell M.P. Dynamical System Analysis from nonlinear Transition to Chaos for a cracked plate. *J. Engg. and Appl. Sci*. 2011; **30**(1): 1-13.

[28] Israr A, Cartmell MP, Krawczuk M, Ostachowicz WM, Manoach E, Tremdafolova I, Shishkina EV, Palacz M. On Approximate Analytical Solutions for Vibrations in Cracked Plates *Modern Practice in Stress & Vibration Analysis VI*, P.Keogh, eds., Trans Tech Publications Ltd, Switzerland 2006; **5-6**: p. 315-322.

[29] Israr A, Cartmell MP, Manoach E, Tremdafolova I, Ostachowicz WM, Krawczuk M, Żak A. Analytical Modelling and Vibration Analysis of Partially Cracked Rectangular Plate with Boundary Conditions and Loading. *ASME Journal of Applied Mechanics* 2009; 76(1).

[30] Israr A, Zulfiqar S. Nonlinear Vibration of Partially Cracked Plates Using Higher Order Perturbation Method. *Journal of Space Technology* 2012; **1**(1): 7-10.

[31] Israr A. Model for Vibration of Cracked Plates for use with Damage Detection Methodologies. *IST Journal of Space Technology,* June 2011; **1**(1): 17-25.

[32] Israr A. Vibration Analysis of Cracked Aluminium Plates. PhD Thesis. University of Glasgow, Glasgow, UK, (2008).

[33] Iwaniec J, Uhl T, Staszewski W.J, Klepka A. Detection of Changes in Cracked Aluminium Plate Determinism by Recurrence Analysis, Nonlinear Dyn DOI 10.1007/s11071-012-0436-9, Springer Science+Business Media B.V., (2012).

[34] Kannappan L, Shankar K. Non Destructive Inspection of Plates using Frequency Measurements. *5th Australasian Congress on Applied Mechanics, ACAM 2007*, 10-12 December 2007.

[35] Keer LM, Sve C. On the Bending of Cracked Plates. *International Journal of Solids and Structures* 1970; **6**: 1545-1559.

[36] Khadem SE, Rezaee M, An Analytical Approach for Obtaining the Location and Depth of an all-over Part-through Crack on Externally In-plane loaded Rectangular Plate using Vibration Analysis. *Journal of Sound and Vibration* 2000(b); **230**(2): 291-308.

[37] Khadem SE, Rezaee M, Introduction of Modified Comparison Functions for Vibration Analysis of a Rectangular Cracked Plate. *Journal of Sound and Vibration* 2000(a); **236**(2): 245-258.

[38] Khaji N, Noureini H.K. Detection of a Through – Thickness Crack based on Elastic Wave scattering in Plates Part I. Forward Solution 2012.

[39] Kim YJ, Chung KH, Kim JS, Kim YJ. Three-dimensional Elastic-Plastic Finite Element Analysis of Biaxially Loaded Cracked Plates. *Key Engineering Materials*, Trans. Tech. Publications, Switzerland 2004; **261-263**: 699-704.

[40] Krawczuk M, Ostachowicz WM. A Finite Plate Element for Dynamic Analysis of a Cracked Plate. *Computer Methods in Applied Mechanics and Engineering* 1994; **115**: 67-78.

[41] Krawczuk M, Palacz M, Ostachowicz W. Spectral Plate Element for Crack Detection with the Use of Propagating Waves. *Modern Practice in Stress and Vibration Analysis*, Materials Science Forum 2003; **440-441**: 187-194.

[42] Krawczuk M, Palacz M, Ostachowicz W. Wave Propagation in Plate Structures for Crack Detection. *Finite Elements in Analysis & Design* 2004; **40**: 991-1004.

[43] Krawczuk M, Żak A, Ostachowicz W. Finite Elements Model of the Plate with Elasto-platic Through Crack. *Computers and Structures* 2001; **79**: 519-532.

[44] Krawczuk M. A Rectangular Plate Finite Element with an Open Crack. *Computers and Structures* 1993; **46**(3): 487-493.

[45] Krawczuk M. Natural Vibrations of Rectangular Plates with a Through Crack. *Archive of Applied Mechanics* 1993; **63**: 491-504.

[46] Krawczuk M. Rectangular Shell Finite Element with an Open Crack. *Finite Elements in Analysis and Design* 1994; **15**: 233-253.

[47] Lee HE. Fundamental Frequencies of Annular Plates with Internal Cracks. *Computers and Structures* 1992; **43**: 1085-1089.

[48] Liew KM, Hung KC, Lim MK. A Solution Method for Analysis of Cracked Plates under Vibration. *Engineering Fracture Mechanics* 1994; **48**(3): 393-404.

[49] Liew KM, Wang Q. Application of Wavelet Theory for Crack Identification in Structures. *Journal of Engineering Mechanics* 1998; **124**: 152-157.

[50] Lynn EE, Kumbasar N. Free Vibration of Thin Rectangular Plates having Narrow Cracks with Simply Supported Edges. *10th Midwestern Mechanics Conference, Fort Colins:* 1967, p. 911-928.

[51] Matbuly MS, Mohammad SA., Osman T. Analysis of Cracked Plates using an Iterative Hybrid Technique of Boundary Element Method and Distributed Dislocation Method. *Engineering Fracture Mechanics* 2008; **75**: 1535-1544.

[52] Nagaya K. Simplified method for solving problems of vibrating plates of doubly connected arbitrary shape, part I: derivation of frequency equation. *Journal of Sound and Vibration* 1981; **74**: 543-551.

[53] Nagaya K. Simplified method of solving problems of vibrating plates of doubly connected arbitrary shape, part II: applications and experiments. *Journal of Sound and Vibration* 1981; 74: 553-564.

[54] Neogy S, Ramamurti V. Effect of Crack on the Natural Frequency of Rotating Twisted Blades. *Proceeding of DETC'97 ASME Design Engineering Technical Conferences*, Sacramento, California: 14-17 Sep. 1997.

[55] Nezu K. Free Vibrations of a Simple Supported Rectangular Plate with a Straight Through-notch. *Bulletin JSME* 1982; **25**: 16-23.

[56] Nguyen VK, Olatunbosun OA. A Method for Remote Monitoring of Structural Health Based on the Nonlinear Phenomenon in Dynamic Response of Damaged Structures. *Proceedings of the World Congress on Engineering*, Vol. II, London, UK, July 2-4, 2007.

[57] Park G, Rutherford AC, Wait JR, Nadler B, Farrar CR, Claytor TN. High-Frequency Response Functions for Composite Plate Monitoring with Ultrasonic Validation. *AIAA Journal* 2005; **43**(11): LA-UR-04-7688, 2431-2437.

[58] Petyt M. The Vibration Characteristics of a Tensioned Plate Containing a Fatigue Crack. *Journal of Sound and Vibration* 1968; **8**(3): 377-389.

[59] Polyzos D, Stamos AA, Beskos DE. BEM Computation of DSIF in Cracked Viscoelastic Plates. *Communications in Numerical Methods in Engineering* 1994; **10**: 81-87.

[60] Purbolaksona J, Dirgantara T, Aliabadi MH. Nonlinear Analysis of Cracked Plates. *Key Engineering Materials*, March 2006; **306-308**: 661-666.

[61] Qian GL, Gu SN, Jiang JS. A Finite Element Model of Cracked Plates Application to Vibration Problems. *Computers and Structures* 1991; **39**: 483-487.

[62] Raju IS, Newman JC Jr. Stress-Intensity Factors for a Wide Range of Semi-elliptical Surface Cracks in Finite-thickness Plates. *Engineering Fracture Mechanics* 1979; **11**: 817-829.

[63] Ramamurti V, Neogy S. Effect of Crack on the Natural Frequency of Cantilever Plates—a Rayleigh–Ritz Solution. *Mechanics Based Design of Structures and Machines: an International Journal* 1998; **26**(2): 131-143.

[64] Rice JR, Levy N. The Part Through Surface Crack in an Elastic Plate. *Journal of Applied Mechanics* 1972; **3**: 185-194.

[65] Rossmanith HP. On Cracked Plates Subjected to Classical Bending. *Journal of Applied Mathematics and Physics*, January 1985; **36**: 166-171.

[66] Saito A, Castanier MP, Pierre C. Estimation and Veering Analysis of Nonlinear Resonant Frequencies of Cracked Plates. *Journal of Sound and Vibration*, October 2009; **326**(3-5): 725-739.

[67] Saito A, Castanier MP, Pierre C. Vibration Response of Cracked Cantilevered Plates near Natural Frequency Veerings. *49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Schamburg, IL, AIAA 2008-1872, 7-10 April 2008.

[68] Saleh N.A.H. Free Vibration Analysis of Squared Simply Supported Plates Containing Various Crack Configurations, *Al-Qadisiya Journal For Engineering Sciences* 2012; **5**(3): 265-279.

[69] Sattari-Far I, Dillström P. Local Limit Load Solutions for Surface Cracks in Plates and Cylinders using Finite Element Analysis. *International Journal of Pressure Vessels and Piping* 2004; **81**: 57-66.

[70] Semperlotti F, Conlon SC. Structural Damage Identification in Plates via Nonlinear Structural Intensity Maps. *Journal of the Acoustical Society of America*, February 2010; **127**(2)L EL48-EL53.

[71] Sh. Hosseini-Hashemi., Heudar Roohi Gh, Hossein Rokni DT. Exact Free Vibration study of rectangular Mindlin Plates with all-over Part-through Open Cracks. *Computers and Structures*, September 2010; **88**(17-18): 1015-1032.

[72] Shawki TG, Nakamura T, Parks DM. Line-spring Analysis of Surface Flawed Plates and Shells using Deformation Theory. *International Journal of Fracture* 1989; **41**: 23-38.

[73] Solecki R. Bending Vibration of a Simply Supported Rectangular Plate with a Crack Parallel to one Edge. *Engineering Fracture Mechanics* 1983; **18**: 1111-1118.

[74] Solecki R. Isotropic Finite Plates with Cracks. *12th Annual Meeting of the Society of Engineering Sciences*, Austin, Texas, US: 20-22 October 1975. p. 373-380.

[75] Stahl B, Keer LM. Vibration and Stability of Cracked Rectangular Plates. *International Journal of Solids and Structures* 1972; **8**: 69-91.

[76] Su RKL, Leung AYT, Wong SC. Vibration of Cracked Kirchhoff's Plates. *Key Engineering Materials* 1998; **145-149**: 167-172.

[77] Trendafilova I, Manoach E, Cartmell MP, Krawczuk M, Ostachowicz WM, Palacz M. On the Problem for Damage Detection of Vibrating Cracked Plates. *Modern Practice in Stress and Vibration Analysis VI*, P.Keogh, Eds., Trans Tech Publications Ltd, Switzerland: (2006); **5-6**: 247-254.

[78] Trendafilova I. A Study on Vibration-Based Damage Detection and Location in an Aircraft Wing Scaled Model. *Applied Mechanics and Materials*, Trans. Tech. Publications, Switzerland: August 2005; **3-4**: 309-314.

[79] Trendafilova I. An Investigation on Vibration-based Damage Detection in an Aircraft Wing Scaled Model. *Key Engineering Materials,* September 2005; **293-294**: 321-328.

[80] Wang YH, Tham LG, Lee PKK, Tsui Y. A Boundary Collocation Method for Cracked Plates. *Computers and Structures* 2003; **81**: 2621-2630.

[81] Wen YS, Zhixe J. On the Equivalent Relation of the Line Spring Model: A Suggested Modification. *Engineering Fracture Mechanics* 1987; **26**(1): 75-82.

[82] Westmann RA, Yang W H. Stress Analysis of Cracked Rectangular Beams. *Journal of Applied Mechanics* 1967; **34**: 693-701.

[83] Wilson WK, Thompson DG. On the Finite Element Method for Calculating Stress Intensity Factors for Cracked Plates in Bending. *Engineering Fracture Mechanics* 1971; **3**: 97-102.

[84] Wu GY, Shih YS. Dynamic Instability of Rectangular Plate with an Edge Crack. *Computers and Structures* 2005; **84**: 1-10.

[85] Yan YJ, Yam LH. Online Detection of Crack Damage in Composite Plate using Embedded Piezoelectric Actuators/Sensors and Wavelet Analysis. *Computer and Structures* 2002; **58**: 29-38.

[86] Żak A, Krawczuk M, Ostachowicz W. Numerical and Experimental Investigation of Free Vibration of Multilayer Delaminated Composite Beams and Plates. *Computational Mechanics* 2000; **26**: 309-315.

[87] Zeng Z-J, Dai S-H, Yang Y-M. Analysis of Surface Cracks using the Line-spring Boundary Element Method and the Virtual Crack Extension Technique. *International Journal of Fracture* 1993; **60**: 157-167.