The Generic Investigation of Initial Alignment of SINS

Anwar Ali Gaho
Manager, GNC, SUPARCO, Karachi. Email: anwaraligaho73@gmail.com
Muhammad Shahid Qureshi
Institute of Space Technology, Karachi.Email: mmsdq2000@yahoo.com

***Abstract* - Inertial navigation systems are being broadly used in water, ground and aerospace applications to provide the navigation solutions. The initial alignment is first pace to be determined for the inertial navigation system (INS). This pace of alignment plays a vital role for carrying out the diverse navigation applications. In this respect different approaches have been made in the navigation research for determination of initial alignment of SINS by using the inertial accelerometers and gyros sensors. Since the gyros provide body attitude information where as accelerometers provide position information in the INS.**

This paper describes the three methods of generic investigation of initial alignment of Strapdown Inertial Navigation System (SINS) on the stationary state. These methods give the comparative analysis of SINS initial alignment. In the first method of section III, only the accelerometers are used to compute the initial alignment of SINS in north, east and down (NED) frame. This method uses the information of measured gravity, local gravity in NED frame and direction cosine matrix (DCM). In this case derivations for elevation and roll are presented. The required results of initial alignment are produced from the experimental and simulated data of SINS on stationary state. In the section 1V, two more systematic methods called Method 2 and Method 3 are used for SINS initial alignment. In these both methods, inertial accelerometers and gyros are used to work out for SINS initial alignment by using body and local level NED frames. Since on stationary condition, inertial navigation systems are entirely self-reliant and can align themselves by using measurements of the local gravity and earth rates in body and navigation frames. From both methods, DCM is analytically computed which relates the body frame to the navigation frame.

The techniques of SINS alignment on stationary condition have great privileges over the SINS on the rotating disc. The size, heating, computational time and zeroing of rotating disc of SINS are great dilemma and their consequences are usually reflected in navigation results. These Initial alignments, on stationary condition, provide the solution of many questions. These techniques also provide solution and capability of selecting various grades of gyros and accelerometers to be used for required application. The experimental and simulation results have been presented in the form of numerical tables at the end of each section which are sufficient to show the comparative analysis and validity of methods.

Index Terms: Alignment, DCM, SINS, earth rate, gravity,NED

I. INTRODUCTION

Inertial navigation systems are generally used for ground, air and space vehicles to provide the navigation parameters such as position, velocity and attitude information. From one of these navigation parameters, initial attitude, called initial alignment, plays a vital role for carrying out the navigation for specific navigation applications. Initial alignment requirement is related to the transformation of the sensors' output into best estimate of attitude, velocity, and position of a vehicle with respect to the reference trajectory [1].

Inertial navigation systems are entirely autonomous systems. They can align themselves by using the measurements of the local gravity and earth rate information on desired elevation and latitude. On the given surface of earth, the local gravity is always downward towards the center of the earth and alignment process can be carried out through DCM from navigation frame to body frame [5].

This paper is consisted of six sections. Section I is about introduction of initial alignment. Section II is about the skewed configuration of sensors and formation of DCM. Section III is consisted of derivations for elevation and roll by using information of local gravity and earth rates and DCM. Section IV comprises the two systematic methods called Method 2 and Method 3 for determination of the direction of cosine matrix. Section V is about the assortment of inertial sensors in terms of initial alignment with assumed biases and drifts. In the last section VI, conclusion is presented.

II. SKEWED CONFIGURATION OF SENSORS

The skewed configuration of inertial sensors in SINS is used for increased operating range & fault detection. Sensors to IMU coordinate transformations are represented by two matrices of transformation. First matrix T_g^m is used for transformation from gyro measurement axes to IMU frame and another T_a^m is used for transformation from accelerometers measurement axes to IMU frame. Both these transformation matrices are originated from IMU calibration on 3-axis motion simulator. On stationary state, raw signals of gyros and accelerometers are required along with the calibration matrices to give measured parameters of earth rate and local gravity in vector form.

$$\omega^b = T_s^b \omega^s \qquad (1)$$

$$A^b = T_s^b A^s \qquad (2)$$

Where s indicates sensor axis and b denotes IMU frame.

In a strap down system, initial alignment process consists of determining the transformation matrix C_n^b which relates the computational navigation frame to the IMU or body frame [6]. The attitude matrix which transforms navigation frame to body frame is given by the following relation.

$$X_nY_nZ_n \xrightarrow{\phi} \xrightarrow{\psi} \xrightarrow{\theta} X_bY_bZ_b \quad (3)$$

$$C_n^b = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \quad (4)$$

$$C_n^b = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi+\sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi+\cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \quad (5)$$

III. METHOD1: DERIVATIONS FOR ELEVATION AND ROLL

In order to find the elevation and roll angles, only accelerometers signals are required which are then to be transformed from sensors frame to SINS frame. On the stationary state of the SINS, local gravity measured by accelerometers, is used to compute elevation and roll [4].

$$f^b = C_n^b g^n \quad (6)$$

$$\begin{bmatrix} f_x^b \\ f_y^b \\ f_z^b \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi-\sin\phi\sin\psi\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \quad (7)$$

Where f^b is a gravity measurement vector in body frame and ψ, θ and ϕ are yaw, elevation and roll respectively making body attitude angles with respect to local navigation frame.
After simplification, relation (7) can be written as,

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} g\sin\theta \\ -g\sin\phi\cos\theta \\ -g\cos\phi\cos\theta \end{bmatrix} \quad (8)$$

Where

$$f_x = g\sin\theta \quad (9)$$

$$f_y = -g\sin\phi\cos\theta \quad (10)$$

$$f_z = -g\cos\phi\cos\theta \quad (11)$$

First computing for elevation from equation (9)

$$\theta = \sin^{-1}(fx/g) \quad (12)$$

With the substitution of numerically computed elevation from (12) into equation (10), roll ϕ can be calculated from the sine inverse function,

$$\phi = \sin^{-1}\left(-\frac{f_y}{g\cos\theta}\right) \quad (13)$$

In the above relations g can be considered as a normalized value and it can be calculated as,

$$g = \sqrt{\left(f_x^b\right)^2 + \left(f_y^b\right)^2 + \left(f_z^b\right)^2} \quad (14)$$

The roll ϕ can also be obtained from tangent inverse function by division of respective sides of (10) and (11) equations, i.e.

$$\phi = \tan^{-1}\left(\frac{f_y^b}{f_z^b}\right) \quad (15)$$

The elevation can also be found from tangent inverse function, which is obtained from equations (9), (10) and (11). By squaring both sides of (10) and (11) equations

$$f_y^2 = g^2\sin^2\phi\cos^2\theta \quad (16)$$

$$f_z^2 = g^2\cos^2\phi\cos^2\theta \quad (17)$$

By adding their respective sides &substituting $\sin^2\phi+\cos^2\phi=1$

$$g\cos\theta = \sqrt{f_y^2+f_z^2} \quad (18)$$

Now dividing the respective sides of equation (9) and (18),

$$\theta = \tan^{-1}\left(\frac{f_x}{\sqrt{f_y^2+f_z^2}}\right) \quad (19)$$

Trigonometric inverse functions from equations 12, 13, 15 and 19 are shown in Table 1.

TABLE 1

EL	ROLL
$\sin^{-1}(fx/g)$	$\sin^{-1}\left(-\frac{f_y}{g\cos\theta}\right)$
$\tan^{-1}\left(\frac{f_x}{\sqrt{f_y^2+f_z^2}}\right)$	$\phi=\tan^{-1}\left(\frac{f_y^b}{f_z^b}\right)$

The experimental accelerometers data of SINS as shown in the following Fig 1(a)

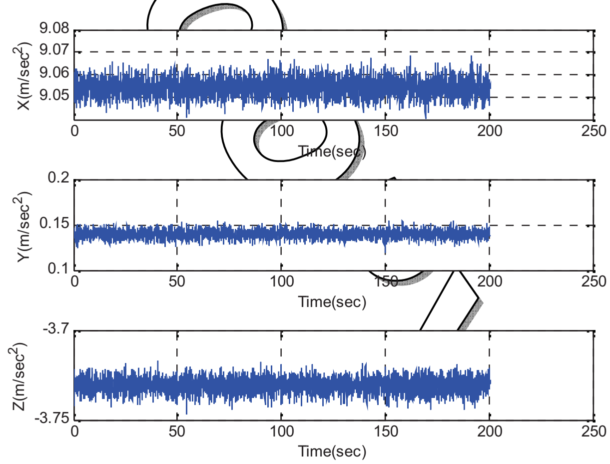

Fig.1 (a) Accelerometers data at certain elevated position

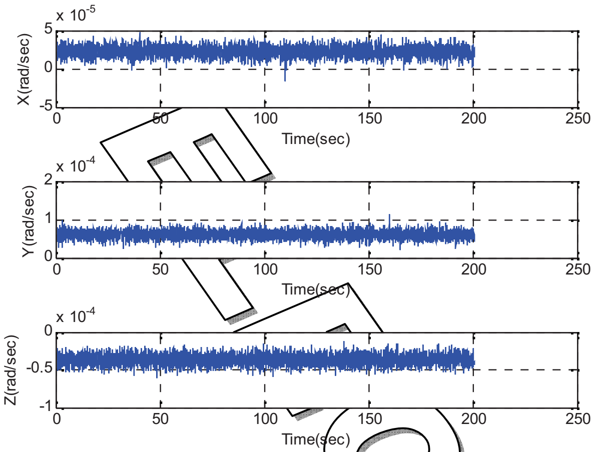

Fig1 (b) Gyros data at certain elevated position

The experimental results of initial alignment at certain elevation and latitude for first method are shown in Table 2.

TABLE 2

METHOD 1: EXPERIMENTAL RESULTS FOR ELEVATION AND ROLL

First Method	
El(deg)	Roll(deg)
68.055	0.015809

Table 3 shows the simulated result of initial alignment for different grades of sensors by using trigonometric functions of Table 1.

TABLE 3

METHOD1: SIMULATED RESULTS FOR ELEVATION AND ROLL

EL (deg)	Roll (deg)
68	0
68.0758	0
68.0765	-0.0002
68.1501	-0.0213
68.2243	-0.0424
68.2983	-0.0634
68.3720	-0.0842
68.4455	-0.1049

IV. DIRECTION COSINE MATRIX

Directions Cosine Matrix can be computed form the combination of the gravity and earth rate vectors in body and navigation frame respectively. The local gravity and earth rate vectors can be sensed by accelerometers and gyros respectively and can be transformed through DCM in navigation frame [3].

A. METHOD 2

To computer DCM, the navigation axes are supposed to be aligned with NED frame. The local gravity and earth rate can be expressed in the navigation frame as [3],

$$g^n = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \quad (20)$$

$$\omega_{ie}^n = \begin{bmatrix} \omega_{ie}\cos\varphi \\ 0 \\ -\omega_{ie}\sin\varphi \end{bmatrix} \quad (21)$$

Where g and ω_{ie} represent magnitude of gravity and earth rate respectively. φ is local geographical latitude.

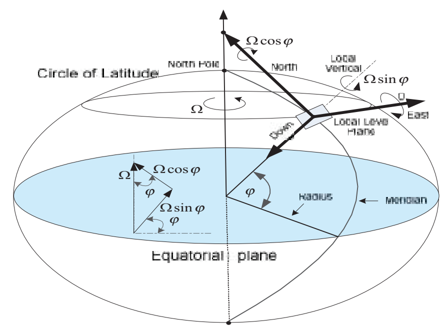

Fig (2) Local level Navigation Frame

As we know that, body and navigation frames are related through DCM.

$$f^b = C_n^b g^n \quad (22)$$

$$\omega^b = C_n^b \omega_{ie}^n \quad (23)$$

$$f^b \times \omega^b = C_n^b \left(g^n \times \omega^n\right) \quad (24)$$

By combining above three relations (22), (23) and (24) then,

$$\begin{bmatrix} g^b \\ \omega^b \\ g^b \times \omega^b \end{bmatrix} = C_n^b \begin{bmatrix} g^n \\ \omega_{ie}^n \\ \left(g^n \times \omega_{ie}^n\right) \end{bmatrix} \quad (25)$$

As C_n^b is an orthogonal matrix .Therefore, $\left(C_n^b\right)^{-1} = \left(C_n^b\right)^T$

$$C_b^n = \begin{bmatrix} \left(g^n\right)^T \\ \left(\omega_{ie}^n\right)^T \\ \left(g^n \times \omega_{ie}^n\right)^T \end{bmatrix}^{-1} \begin{bmatrix} \left(g^b\right)^T \\ \left(\omega^b\right)^T \\ \left(g^b \times \omega^b\right)^T \end{bmatrix} \quad (26)$$

In equation (26),

$$g^n \times \omega_{ie}^n = \begin{bmatrix} 0 \\ -g\omega_{ie}\cos\varphi \\ 0 \end{bmatrix} \quad (27)$$

$$\begin{bmatrix} \left(g^n\right)^T \\ \left(\omega_{ie}^n\right)^T \\ \left(g^n \times \omega_{ie}^n\right)^T \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & -g \\ \omega_{ie}\cos\varphi & 0 & -\omega_{ie}\sin\varphi \\ 0 & -g\omega_{ie}\cos\varphi & 0 \end{bmatrix}^{-1} \quad (28)$$

$$\begin{bmatrix} \left(g^n\right)^T \\ \left(\omega_{ie}^n\right)^T \\ \left(g^n \times \omega_{ie}^n\right)^T \end{bmatrix}^{-1} = \begin{bmatrix} \frac{-\tan\varphi}{g} & \frac{1}{\omega_{ie}\cos\varphi} & 0 \\ 0 & 0 & \frac{-1}{g\omega_{ie}\cos\varphi} \\ \frac{-1}{g} & 0 & 0 \end{bmatrix} \quad (29)$$

It should be remembered that on the stationary condition of SINS, accelerometers measure only gravity effect in body frame on certain elevation and latitude.

$$f^b = \begin{bmatrix} -a_x^b & -a_y^b & -a_z^b \end{bmatrix}^T \quad (30)$$

On the stationary condition of SINS, gyros sense the earth rate in body frame on certain elevation and latitude.

$$\omega^b = \begin{bmatrix} \omega_{iex}^b & \omega_{iey}^b & \omega_{iez}^b \end{bmatrix}^T \quad (31)$$

Then updating equation (26), from the (29), (30) and (31)

$$C_b^n = \begin{bmatrix} \frac{-\tan\varphi}{g} & \frac{1}{\omega_{ie}\cos\varphi} & 0 \\ 0 & 0 & \frac{-1}{g\omega_{ie}\cos\varphi} \\ \frac{-1}{g} & 0 & 0 \end{bmatrix} \begin{bmatrix} -a_x & -a_y & -a_z \\ \omega_{ie}^x & \omega_{ie}^y & \omega_{ie}^z \\ a_z\omega_y - a_y\omega_z & a_x\omega_z - a_z\omega_x & a_y\omega_x - a_x\omega_y \end{bmatrix} \quad (32)$$

$$C_b^n = \begin{bmatrix} \omega_{ie}^x/(\omega_{ie}\cos(\varphi)) + (f_x^b\sin(\varphi))/(g\cos(\varphi)) & \omega_{ie}^y/(\omega_{ie}\cos(\varphi)) + (f_y^b\sin(\varphi))/(g\cos(\varphi)) & \omega_{ie}^z/(\omega_{ie}\cos(\varphi)) + (f_z^b\sin(\varphi))/(g\cos(\varphi)) \\ (f_y^b\omega_{ie}^z - f_z^b\omega_{ie}^y)/(g\omega_{ie}\cos(\varphi)) & -(f_x^b\omega_{ie}^z - f_z^b\omega_{ie}^x)/(g\omega_{ie}\cos(\varphi)) & (f_x^b\omega_{ie}^y - f_y^b\omega_{ie}^x)/(g\omega_{ie}\cos(\varphi)) \\ f_x^b/g & f_y^b/g & f_z^b/g \end{bmatrix} \quad (33)$$

As C_n^b is an orthogonal matrix. Therefore $\left(C_n^b\right)^{-1} = \left(C_n^b\right)^T$

$$\theta = \sin^{-1}\left(C_n^b(1,3)\right), \qquad \phi = \tan^{-1}\left(\frac{C_n^b(2,3)}{C_n^b(3,3)}\right) \quad (34)$$

The results, obtained from accelerometers and gyros of SINS, at certain elevation and latitude, are shown in Table 4 by using equation 34. It should be noted that azimuth is usually found by optical means therefore it is not discussed here.

TABLE 4
ELEVATION AND ROLL

EL (deg)	ROLL (deg)
68	0
68.0	0
67.9999	0.0001
68.0011	-0.0011
68.0146	-0.0146
68.1524	-0.1523
68.3066	-0.3062
68.4621	-0.4611
68.6172	-0.6154
68.7734	-0.7706

B. METHOD 3

In third method, elements of second and third rows can be re-arranged from gravity and earth rate vectors in both body and navigation frames; such as,

$$\begin{bmatrix} g^b \\ g^b \times \omega_{ie}^b \\ \left(g^b \times \omega_{ie}^b\right) \times g^b \end{bmatrix} = C_n^b \begin{bmatrix} g^n \\ g^n \times \omega_{ie}^n \\ \left(g^n \times \omega_{ie}^n\right) \times g^n \end{bmatrix} \quad (35)$$

In the relation (35), accelerometers measure gravity effect and gyros sense only the earth rate on the desired elevation and latitude of the earth in body frame [3].

$$C_n^b = \begin{bmatrix} g^n \\ g^n \times \omega_{ie}^n \\ \left(g^n \times \omega_{ie}^n\right) \times g^n \end{bmatrix}^{-1} \begin{bmatrix} f^b \\ g^b \times \omega_{ie}^b \\ \left(f^b \times \omega_{ie}^b\right) \times f^b \end{bmatrix} \quad (36)$$

As C_n^b is an orthogonal matrix. Therefore $\left(C_n^b\right)^{-1} = \left(C_n^b\right)^T$

$$C_b^n = \begin{bmatrix} \left(g^n\right)^T \\ \left(g^n \times \omega_{ie}^n\right)^T \\ \left(\left(g^n \times \omega_{ie}^n\right) \times g^n\right)^T \end{bmatrix}^{-1} \begin{bmatrix} \left(f^b\right)^T \\ \left(g^b \times \omega_{ie}^b\right)^T \\ \left(\left(f^b \times \omega_{ie}^b\right) \times f^b\right)^T \end{bmatrix} \quad (37)$$

$$C_b^n = \begin{bmatrix} 0 & 0 & 1/(g^2\omega_{ie}\cos(\varphi)) \\ 0 & -1/(g\omega_{ie}\cos(\varphi)) & 0 \\ -1/g & 0 & 0 \end{bmatrix} \begin{bmatrix} -f_x^b & -f_y^b & -f_z^b \\ f_z^b\omega_{ie}^y - f_y^b\omega_{ie}^z & f_x^b\omega_{ie}^z - f_z^b\omega_{ie}^x & f_y^b\omega_{ie}^x - f_x^b\omega_{ie}^y \\ f_y^b(f_x^b\omega_{ie}^y - f_y^b\omega_{ie}^x) + f_z^b(f_x^b\omega_{ie}^z - f_z^b\omega_{ie}^x) & f_z^b(f_y^b\omega_{ie}^z - f_z^b\omega_{ie}^y) - f_x^b(f_x^b\omega_{ie}^y - f_y^b\omega_{ie}^x) & -f_x^b(f_x^b\omega_{ie}^z - f_z^b\omega_{ie}^x) - f_y^b(f_y^b\omega_{ie}^z - f_z^b\omega_{ie}^y) \end{bmatrix} \quad (38)$$

After simplification (35) becomes

$$C_b^n = \begin{bmatrix} -(f_y^b(f_x^b\omega_{ie}^y - f_y^b\omega_{ie}^x) + f_z^b(f_x^b\omega_{ie}^z - f_z^b\omega_{ie}^x))/(g^2\omega_{ie}\cos(lat)) & (f_x^b(f_x^b\omega_{ie}^y - f_y^b\omega_{ie}^x) - f_z^b(f_y^b\omega_{ie}^z - f_z^b\omega_{ie}^y))/(g^2\omega_{ie}\cos(lat)) & (f_x^b(f_x^b\omega_{ie}^z - f_z^b\omega_{ie}^x) + f_y^b(f_y^b\omega_{ie}^z - f_z^b\omega_{ie}^y))/(g^2\omega_{ie}\cos(lat)) \\ (f_y^b\omega_{ie}^z - f_z^b\omega_{ie}^y)/(g\omega_{ie}\cos(\varphi)) & -(f_x^b\omega_{ie}^z - f_z^b\omega_{ie}^x)/(g\omega_{ie}\cos(\varphi)) & (f_x^b\omega_{ie}^y - f_y^b\omega_{ie}^x)/(g\omega_{ie}\cos(\varphi)) \\ f_x^b/g & f_y^b/g & f_z^b/g \end{bmatrix} \quad (39)$$

Since due to orthogonal property, $\left(C_n^b\right)^{-1} = \left(C_n^b\right)^T$, Therefore,

$$\theta = \sin^{-1}\left(C_n^b(1,3)\right), \qquad \phi = \tan^{-1}\left(\frac{C_n^b(2,3)}{C_n^b(3,3)}\right) \quad (40)$$

The experimental and simulation results from accelerometers and gyros data are given in Table 4 and Table 5 by using Table 1 and equation 40 respectively.

TABLE 5
EXPERIMENTAL RESULTS FOR ELEVATION AND ROLL

Second Method		Third Method	
El(deg)	Roll(deg)	El(deg)	Roll(deg)
68.057	0.11443	68.059	0.11443

drifts (deg/hour)	biases(m/sec^2)	EL (deg)	ROLL (deg)
0	0	68	0
0.03	1e-5g	68.0	0
0.05	1e-4g	68.0011	-0.0011
0.07	1mg	68.0146	-0.0146
0.09	2mg	68.1524	-0.1523
0.1	3mg	68.3066	-0.3062
0.2	4mg	68.4621	-0.4611
0.3	5mg	68.6172	-0.6154

V. ALIGNMENT ERRORS DUE TO BIASES AND DRIFTS

In this section, initial error analysis of SINS alignment is presented. It is always cumbersome to select the inertial sensors with precised alignment. This paper presents the solution of choice of different grades of inertial sensors for desired initial alignment of SINS [2, 3]. Two methods from section IV are exploited for determining the DCM to calculate elevation and roll for different grades of the sensors as shown in Tables 6, 7 and 8. In this section analysis is based on simulations with zero drifts in Table 6 and zero biases in Table 7 and assumed drifts and biases in Table 8. The results seem to be more reliable and provide self-assured results for initial alignment. Inertial sensors for preference with given specifications may be analysed on the base of these methods.

TABLE 6
ANALYSIS WITH ZERO DRIFTS

drifts(deg/hour)	biases (m/sec2)	El(deg)	Roll(deg)
0	0	65	0
0	1e-5g	64.9992	0.0014
0	1e-4g	64.9924	0.0136
0	1mg	64.9238	0.1353
0	2mg	64.8473	0.2699
0	3mg	64.7707	0.4038
0	4mg	64.6939	0.5372
0	5mg	64.6169	0.6699

TABLE 7
ANALYSIS WITH ZERO BIASIS

drifts (deg/hour)	biases(m/sec^2)	EL(deg)	ROLL (deg)
0	0	65	0
0.03	0	65.0000	0.0000
0.05	0	65.0000	0.0000
0.07	0	65.0000	0.0000
0.09	0	65.0000	0.0000
0.1	0	65.0000	0.0000
0.2	0	65.0000	0.0000
0.3	0	65.0000	0.0000

TABLE 8
ANALYSIS WITH ASSUMED DRIFTS AND BIASES

drifts (deg/hour)	biases(m/sec^2)	EL(deg)	ROLL(deg)
0	0	65	0
0.03	1e-5g	64.9992	0.0014
0.05	1e-4g	64.9924	0.0136
0.07	1mg	64.9238	0.1353
0.09	2mg	64.8473	0.2699
0.1	3mg	64.7707	0.4038
0.2	4mg	64.6939	0.5372
0.3	5mg	64.6169	0.6699

VI. CONCLUSION

Three useful generic methods of initial alignment for SINS are presented in this paper. These methods give the comparative analysis of SINS initial alignment. From the first method, elevation and gamma formulae are derived. In this method elevation and roll are computed by using only accelerometers data as shown in Fig 1 (a). The experimental and simulations results for various grades of inertial sensors are shown in the

Table 3. This table shows the satisfactory accurate initial alignment for elevation and roll of SINS by using only accelerometers data. These results are also compared with the experimental and simulation results of initial alignment for elevation and roll from method 3 and method 4 of section IV. In this method DCM is computed with two different methods by using the information of gravity and earth rates in both body and navigation frames on a stationary position. DCM of both methods seems to be identical except their first row elements. These methods are helpful in design of ground coarse alignment process for strapdown inertial navigation systems. The experimental and simulations results for various grades of inertial sensors are shown in Table 4 and 5. In section V analysis is also performed for sensors assortment for different biases and drifts as shown in Tables 4 and 5. Tables 6, 7 and 8 show the results of alignment with and without the biases and drifts. The results show that stationary accurate alignment can be achieved by using sensors of less drifts and biases. These methods have great privileges over the SINS moving on a rotating disc. The fixed alignment methods require less time of heating and computing and provide the fast results of Initial alignment.

References

[1] Britting, K. R. (1971), Inertial Navigation System Analysis. New York: Wiley Interscience, 1971, ch. 9.

[2] Stieler, B., and Winter, H. (1982), Gyroscopic instruments and their application to flight testing, AGARD, 160, 15 (1982), ch. 7.

[3] Error Analysis of Analytic Coarse Alignment Methods, IEEE Transactions on Aerospace and Electronic Systems vol. 34, no. 1 January 199

[4] Siouris, G. M. (1987), Navigation, inertial, Encyclopedia of Physical Science und Technology,8 (1987), 668-717.

[5] Mortensen, R. E. (1974), Strapdown guidance error analysis, IEEE Transactions on Aerospace and Electronic System, AES-10,4 (July 1974), 451457.

[6] Paul Zarchan, (2004). Strapdown Inertial Navigation Systems, second Edition, AIAA Lexington, Massachusetts, and ch.10.