Satellite Attitude and Orbital Dynamics Simulator

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Abstract— In this paper, the design of the Satellite Attitude and Orbital Dynamics Simulator (SAODS) is described in detail, including working principle and control system. The development of AODS aimed to provide a facility for researchers to quickly examine satellite orbit and develop attitude control algorithms. The attitude dynamics has been modeled in term of quaternion whereas for satellite orbital maneuvers, differential equations in inertial frame are used. Different conservative and non-conservative perturbing forces have been considered in the sinulation model to simulate the behavior of satellite in actual space environment. Models of different actuators and sensors are also the key element of SAODS simulator. The simulation models have been implemented in the MATLAB(SIMULPAK environment. Simulation results are compared and presented in comparison with some commercially available software.

Keywords— Satellite Simulator, Attitude Simulator, Orbit Simulator, SAODS.

1 INTRODUCTION

Modeling and simulation with increased accuracy have become an important research topic in the field of Satellite dynamics and control. Matching the mathematical and simulation models with the real world behavior is highly essential for a satellite mission. Since these missions are so expensive and are operated far from the user, therefore it is not possible to conduct testing in an actual environment. Due to this a very accurate and authentic simulation tool with high reliability simulating the actual space environment is highly required in this industry.

Simulation tools are also essential in attitude and orbit control design, the available tools are too expensive and sometime cannot be used directly because it is not easy to customize these commercial simulators to suit specific needs [1]. These simulation tools are not only useful for the development of control algorithms but they also provide a testing bed to conduct various kind of testing. These tools also provide a simulation environment for the hardware in the loop testing to test the algorithms including physical hardware that has to be used in the real flight.

The prime objective of this paper is to develop a powerful satellite simulation tool that can be used as a baseline to develop, implement and analyze the attitude control algorithms for satellite missions. The developed sensor and actuator models could further simplify and speed the development of AOCS algorithms. Further flexible MATLAB based structure of this simulator facilitates customized design which could suit a number of satellite missions.

For a spacecraft engineer, numerous satellite simulation packages are currently available in the market like AutoCon developed by NASA [2], SATCOS by SAIC [3], FreeFlyer by A.I. Solutions [4], and Satellite ToolKit (STK) by Analytical Graphics Inc [5]. Cost of these simulation environments is very high and also it is not easy to modify the basic structure to suit the specific requirements of a satellite mission. Further attitude and orbital control algorithms are mostly developed in Matlab as Matlab provides lot of built-in features which could speed up the whole developmental path. Only few available satellite simulators can be coupled with Matlab to test these AOCS algorithms but even then it is not easy to handle them and they increase complexity for the designers. SAODS is exclusively developed in Matlab environment to provide a readily available test bed to implement and test the AOCS algorithms. The ease to modify makes it suitable for number of satellite missions including LEO and GEO satellite missions.

The paper is organized in following manner. Section I briefly describe the reference frames used in SAODS, which is followed by the mathematical modeling of orbit motion and perturbations, Section III discusses the dynamic and kinematic modeling of rotational motion with modeled disturbance torques and sensors while some simulation results, their verification and concluding remarks are presented in last three sections.

2 REFERENCE FRAMES

To demonstrate spacecraft position and attitude several coordinate systems are used, which depends on mission circumstances, operating conditions and standards. Following reference coordinate systems are used in SAODS. Earth-Centered Inertial (ECI): ECI is fixed in space with X_i directed from the center of the Earth to the vernal equinox, Z_i is in the Earth's orbita) angular velocity direction, and Yi completes the orthonormal triad to X_i and Z_i . Orbital Frame: It's a noninertial frame in orbit rotates with the body; Z_o originates from center of mass of body to the Earth center of mass (nadir), Y_0 directed reverse to the orbit normal and X_o is perpendicular to Y_{o} and Z_{o} axis. Body Frame: Origin is fixed in the satellite; the axes directions are the inputs from satellite engineers because this frame defines how a satellite is oriented with respect to an external frame [6]. Default body axis is defined as X_b is in velocity direction, Z_b lies in payload (cameras) direction and Y_b is orthonormal to X_b and Z_b .

Variations between time frames are normally very small but space object velocities are extremely high and a minute difference creates large position differences. Hence it is essential to use different time frames in SAODS, following time frames are used in this simulators[7].

- 1. Sidereal Time
- 2. Solar Time
- 3. Universal Time
- 4. Julian Date
- 5. Modified Julian Date



Figure 2 - Orbit Frames

3 **ORBIT MODEL**

J. Kepler [6] gave basic laws to explain the trajectory of satellite object in unperturbed planetary gravitational field. Kepler orbit is not absolutely accurate and the Kepler equations do not gives the exact behavior of the space vehicle [8]. Orbit calculations have been done by using two body force equation in inertial coordinate system [9]. The dynamical and kinematical equations are.

$$\begin{pmatrix} dv_{xi}/dt \\ dv_{yi}/dt \\ dv_{zi}/dt \end{pmatrix} = \begin{pmatrix} -\mu x_i/r^3 \\ -\mu y_i/r^3 \\ -\mu z_i/r^3 \end{pmatrix} + \begin{pmatrix} a_{xi} \\ a_{yi} \\ a_{zi} \end{pmatrix}$$
$$\begin{pmatrix} dx_i/dt \\ dy_i/dt \\ dz_i/dt \end{pmatrix} = \begin{pmatrix} v_{xi} \\ v_{yi} \\ v_{zi} \end{pmatrix}$$

$$r = \sqrt{x_i^2 + y_i^2 + z_i^2}$$

Where a_{xi} , a_{yi} and a_{zi} are the components of perturbing forces

getting more accurate results from simulations, perturbations are included in the model. Perturbations are the deviations from idealized model. In SAODS following perturbations are included.

- 1. J_2 gravity model
- Atmospheric drag
- Solar radiation pressure
- Controlled thrust.

Gravity Field of the Central Body

Due to a non-homogenous or non-spherical earth, the orbiting satellite experience variation in gravitational force, J_2 perturbation model [9] is used in SAODS.

$$a_{gxi} = -\frac{\mu x_i}{r^3} \left(\frac{3}{2} J_2 (\frac{R}{r})^2 \left(1 - 5(\frac{z_i}{r})^2 \right) \right)$$
$$a_{gyi} = -\frac{\mu y_i}{r^3} \left(\frac{3}{2} J_2 (\frac{R}{r})^2 \left(1 - 5(\frac{z_i}{r})^2 \right) \right)$$
$$a_{gzi} = -\frac{\mu z_i}{r^3} \left(\frac{3}{2} J_2 (\frac{R}{r})^2 \left(3 - 5(\frac{z_i}{r})^2 \right) \right)$$

Atmospheric Drag

The effect of drag is more prominent in low earth orbit. Drag perturbation reduces the kinetic energy of the satellite which in turn decreases the altitude [10]. A simple model of drag is

$$a_{Dxi} = -\sigma\rho v_a (v_{xi} + y_i \omega_e)$$

$$a_{Dyi} = -\sigma\rho v_a (v_{yi} - x_i \omega_e)$$

$$a_{Dzi} = -\sigma\rho v_a (v_{zi})$$

Where, C_D is the computed drag coefficient, σ is the ballistic coefficient and S is the cross sectional area of the satellite.

3.1.3 Solar-Radiation Pressure
It's a non- conservative perturbation caused by the impact of
photon on absorbing or reflecting surface [2]. It is very small
force and mainly depends on solar-radiation pressure
$$p_{sr}$$
.

$$a_{srxi} = -p_{sr}kS\cos\sigma_{sr}\cos\Lambda$$
$$a_{sryi} = -p_{sr}kS\cos\sigma_{sr}\cos\varepsilon\sin\Lambda$$
$$a_{srzi} = -p_{sr}kS\cos\sigma_{sr}\sin\varepsilon\sin\Lambda$$

Where, σ_{sr} is the angle between solar ray and normal to surface, *k* is the reflectivity whose value is 0.0 indicates for translucent, 1.0 for completely absorbing body and 2.0 for an absorbing and reflecting body.

3.1.4 Controlled Thrust

Due to perturbations in orbit, the satellite changes its orbit parameters. These parameters are controlled artificially by use of propulsive force generated by onboard thrusters. This orbit maneuver may be impulsive or continuous.



3.2 Equation of Rotational Motion

Rotational dynamics are time variant attitude due to force and torque with respect to another frame. Rotational motion has been simulated by using following equations with the assumption of rigid body and zero products of inertia [6].

$$\begin{pmatrix} d\omega_x/dt \\ d\omega_y/dt \\ d\omega_z/dt \end{pmatrix} = \begin{pmatrix} ((I_y - I_z)\omega_y\omega_z + \sum M_x)/I_x \\ ((I_z - I_x)\omega_z\omega_x + \sum M_y)/I_y \\ ((I_x - I_y)\omega_x\omega_y + \sum M_z)/I_z \end{pmatrix}$$

Where, $\sum M$ is the disturbance torque and ω is the satellite angular velocity about satellite body axis. In the simulation model, the quaternion technique is used to define kinematics of rotational motion. Convenient product rule for consecutive rotations without gimbal lock is the main advantage of quaternion [8].

$$\begin{bmatrix} dq_0 / dt \\ dq_1 / dt \\ dq_2 / dt \\ dq_3 / dt \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

The elements of body to inertial transformation matrix are obtained as

$$L_{lb} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_3q_1 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_3q_1 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

3.3 Disturbance Torques

Satellite in real situation experiences many disturbance torques which may be environmental or artificial [9], following disturbance moment are included in SAODS.

- 1. Aerodynamic Torque
- 2. Magnetic Dipole
- 3. Gravity-gradient Torque
- 4. Control Torque
- 1. Aerodynamic Torque

If the cross-section is not distributed evenly then Satellite experience a moment from aerodynamic forces, this environmental disturbance moment is dominant below 400 km [8].

$$M_d = F_d(CP - CM)$$

Where, *CP* is the aerodynamic center, *CM* is the center of mass and F_d is the drag force on the spacecraft.

Spacecraft normally contain several magnetic materials and electronic wirings, which produce magnetic fields when they carry electric current this magnetic field interact with earth magnetic field and produce a disturbance dipole moment.

$$M_m = m \times B$$

Where m is magnetic dipole from different sources in satellite and B is magnetic flux density of earth [8].

3. Gravity Gradient Torque

This moment has much importance when passive attitude control is used. Gravity gradient is caused by asymmetry of the body; least inertia axis tends to align with the earth gravity field direction. It is dominant in low-orbit satellites [6].

$$M_{Gx} = \frac{3\mu}{2r^{3}}(I_{z} - I_{y})\sin(2\phi)\cos^{2}(\theta)$$
$$M_{Gy} = \frac{3\mu}{2r^{3}}(I_{z} - I_{x})\sin(2\theta)\cos^{2}(\phi)$$
$$M_{Gz} = \frac{3\mu}{2r^{3}}(I_{x} - I_{y})\sin(2\theta)\sin(\phi)$$
$$\frac{4}{4}$$
Control Torque

For satellite attitude control, an artificial disturbance torque is produced by different means. Three attitude control actuation mechanisms are used in SAODS.

1. Thuster
2. Reaction Wheels
3. Magneto-torquer
Thrusters are modeled by the following equations

$$M_t = r \times F_t$$

Where, F_t is the thrust force produce by a thruster and r is the distance between thruster and center of mass. The momentum wheel spins to transfer angular momentum to the whole system and provide momentum bias continuous and smooth attitude control mechanism. It also provides inertial stabilization to the three-axis stabilized spacecraft about one of its axes. Three and four wheel based configurations are modeled in SAODS. Magnetic torquers are used widely in the attitude control of satellite [6]. Their objective is to produce magnetic dipole moments for different applications like attitude maneuver, active damping in gravity gradient stabilization and magnetic unloading of momentum exchange devices. The moment generation method by magnetic torquer is similar as discussed in magnetic dipole section. Following sensors are modeled in SAODS.

- 1. Coarse Earth Sensor
- 2. Coarse Sun Sensor
- 3. Star tracker
- 4. Magnetometer
- 5. Gyro 6. GPS 4 SIMULATION

Simulation results are presented for a Low Earth Orbit (LEO) with semi major axis of 12000 km, eccentricity of 0.4 and 90° inclination angle. The satellite with mass of 100 kg and the moments of inertia are I_x =4.5 kg m², T_y =4.5 kg m² and I_z =4.5 kg m². Three reaction where are used for three axis stabilization, orbit epoch date is Jan 2010 and universal time is 12:00:00.

5 VERIFICATION

A verification process is performed to ensure the client the authenticity and accuracy of results from the SAODS. Verification examines that the output of the SAODS simulations are equivalent to accepted and verified software package like STK and FreeFlyer. It guarantees that the results are accurate and reliable. STK is selected as the baseline satellite simulation software package to evaluate SAODS



Figure 3 - ECI Position



Figure 4 - ECI Velocity





Figure 8 - Earth Magnetic Field Intensity in ECF Frame

Parameters	SAODS	STK	FreeFlyer
$X_i(km)$	813.29734	813.292899	813.29198
$Y_i(km)$	9186.36187	9186.35002	9186.3504
$Z_i(km)$	13406.4991	13406.4781	13406.477
V_{xi} (km/s)	-3.3286960	-3.328702	-3.328702
V_{yi} (km/s)	1.370732	1.370722	1.3707218
V_{zi} (km/s)	-1.678007	-1.678029	-1.678029

TABLE 1 - PERFORMANCE COMPARISON (SIMULATION TIME= 24 Hrs)

CONCLUSION

The satellite attitude and orbit dynamics simulator model with its accessories is developed and presented. The SAODS provides a platform to develop, implement and test attitude and orbit control algorithms. Verification with a commercially available software package illustrates the accuracy of the implemented mathematical equations. The simulator model is implemented in the MATLAB/SIMULINK environment which provides flexibility to modify to suit various kinds of satellite missions. This simulator could further be used for the hardware in the loop simulations by including real hardware in the loop.

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