Constraint Generalized Predictive Control for Aircraft Pitch Autopilot

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Abstract—This paper presents the design of pitch channel autopilot of aircraft using Generalized Predictive Control with input constraint (CGPC). Due to inconsistent behavior of pitch plane dynamics, estimation errors in vehicle parameters and variation of flight conditions, a robust controller is required to meet the stability constraints. For this purpose CGPC is proposed, which is a predictive control technique that explicitly utilizes the transfer function based mathematical model, to predict the plant output over a certain prediction horizon. CGPC generates a set of future control signals in each sampling interval to optimize the control effort of the controlled system by minimizing a quadratic cost function. It uses receding horizon principle in which only the first control signal from a set of future control signals is implemented. The design process also considers the constraints for amplitude and rate of the input control signal. The efficiency of proposed control scheme for pitch autopilot has been verified by the simulations conducted using MATLAB. The presented simulation results show the excellent performance and capabilities of designed controller which effectively tracks the desired trajectory.

Keywords—Generalized Predictive Control, Flight Control, optimization.

1 INTRODUCTION

Constraint Generalized Predictive Control (CGPC) using receding horizon principle has found important application in industrial level [1]. It is because of the fact that most industrial processes are inherently slow in dynamics and this compensates the relatively large amount of computation required by CGPC for optimization of the repetitive procedure. And, if the constraints of the system are properly considered, CGPC is more efficient and safer for these applications. But, in the past decade, with the invention of fast computing machines and efficient algorithms, it is now possible to successfully implement CGPC on applications with fast dynamics such as a civil aircraft [2, 3].

CGPC imitate human behavior as we select control actions that will produce best predicted output over some degree of horizon and as new observations become available we regularly update our decisions. The objective of CGPC is to compute the future control sequence so that the predicted outputs are driven closer to the reference. The control strategy explicitly utilize the mathematical model, to predict the future process output to possible future control sequence in real time over some limited time horizon. It uses receding horizon principle in which the control sequence is computed for some finite time horizon but only the current control is implemented, the time horizon are shifted forward to one step and the whole process is repeated. This leads to an optimization problem in which a cost function of the tracking error and manipulated variables are minimized [4, 5].

In this paper, the application of CGPC for the control of longitudinal dynamics of an aircraft is presented. Furthermore, the constraints of the system such as the servo bandwidth and deflection limitation are also taken into consideration. The paper also highlights the effect of various tuning parameters of CGPC such as prediction horizon, control horizon and control weighting factor on the controller performance. Another strong feature of CGPC is its robustness against plant parameter variation and external disturbances, this aspect is also discussed in this paper.

The paper is organized in the following manner. Section 2 highlights the mathematical modeling of the aircraft, section 3 describes design methodology of Constrained CGPC, section 4 demonstrates the application of CGPC to the aircraft pitch channel and simulation results of the proposed controller, section 5 and 6 illustrates the robustness analysis and comparison respectively, finally, section 7 concludes the paper.

2 LONGITUDINAL DYNAMICS OF AIRCRAFT

This section describes longitudinal dynamics equations of motion of an aircraft [6].

$$\left(\frac{mU}{Sq}\,'\dot{u}-C_{x_u}\,'u\right)+\left(-\frac{c}{2U}C_{x_{\dot\alpha}}\,'\dot\alpha-C_{x_\alpha}\,'\alpha\right)+\left(-\frac{c}{2U}C_{x_q}\dot\theta-C_w(\cos\Theta)\theta\right)=C_{F_{x_a}}$$

$$-(C_{z_u}\,'u)+\left[\left(\frac{mU}{Sq}-\frac{c}{2U}C_{z_{\dot\alpha}}\right)'\dot\alpha-C_{z_\alpha}\,'\alpha\right]+\left[\left(-\frac{mU}{Sq}-\frac{c}{2U}C_{z_q}\right)\dot\theta-C_w(\sin\Theta)\theta\right]=C_{F_{z_a}}$$

$$-(C_{m_u}\,'u)+\left(-\frac{c}{2U}C_{m_{\dot\alpha}}\,'\dot\alpha-C_{m_\alpha}\,'\alpha\right)+\left(-\frac{I_y}{Sqc}\ddot\theta-\frac{c}{2U}C_{m_q}\dot\theta\right)=C_{m_a}$$

(1)

Where, $U=U_o$, $q=\frac{\rho U_o^{\,2}}{2}$, $'u=\frac{u}{U_o}$, $'\alpha=\frac{w}{U_o}$ and $'\dot\alpha=\frac{\dot w}{U_o}$

Equation (1) is derived by assuming that the aircraft is a rigid body with constant mass. The earth is taken as inertial reference. Origin of earth axis system is at the center of gravity of aircraft. The aircraft is flying straight with altitude of 40000 ft having constant velocity of 600 ft/sec.

Since the aircraft is moving with constant forward speed, the forces in X direction are neglected. More over the coefficients $C_{z_{\dot{\alpha}}}$ and C_{z_q} are also neglected because the change in drag due to pitching rate and rate of angle of attack on horizontal stabilizer is negligible as compared to the drag of the rest of aircraft. With these assumptions and by taking Laplace Transform the equation (1) is rewritten as [6]:

$$\left(\frac{mU}{Sq}s - C_{z_{\dot{\alpha}}}\right)\alpha(s) + \left(-\frac{mU}{Sq}s - C_w(\sin\Theta)\right)\theta(s) = C_{z_{\delta_e}}\delta_e(s)$$

$$\left(-\frac{c}{2U}C_{m_{\dot{\alpha}}}s - C_{m_\alpha}\right)\alpha(s) + \left(\frac{I_y}{Sqc}s^2 - \frac{c}{2U}C_{m_q}s\right)\theta(s) = C_{m_{\delta_e}}\delta_e(s) \tag{2}$$

The values of all the parameters and stability derivatives for this aircraft are listed in table 1 [6]:

TABLE 1 - PARAMETERS AND STABILITY DERIVATIVES

Parameters and Stability Derivatives	Values
Mass (m)	5800 *slugs*
Forward velocity (U)	600 *ft / sec*
Reference area (S)	2400 *sq ft*
Dynamic pressure (q)	105.1 *lb / sq ft*
Gravity coefficient (C_w)	−0.74
Local flight path angle (Θ)	0
Mean chord (c)	20.2 *ft*
Inertia (I_y)	2.62×10^6 *slug ft*^{2}
Slope of the normal force (C_{z_α})	−4.46
Static longitudinal stability (C_{m_α})	−0.619
Downwash lag on moment ($C_{m_{\dot{\alpha}}}$)	−3.27
Damping in pitch (C_{m_q})	−11.4
Slope of elevator normal force ($C_{z_{\delta_e}}$)	−0.246
Moment of elevator ($C_{m_{\delta_e}}$)	−0.710

After substituting these values in equation (2) we get:

$$(13.78s+4.46)\alpha(s) - 13.78s\theta(s) = -0.246\delta_e(s)$$

$$(0.0552s+0.619)\alpha(s) + (0.514s^2+0.192s)\theta(s) = -0.710\delta_e(s) \tag{3}$$

The transfer function for δ_e input and θ output using determinants [6] is given by:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-1.39(s+0.306)}{s(s^2+0.805s+1.325} \tag{4}$$

In discrete domain (z) the transfer function is written as:

$$\frac{\theta(z)}{\delta_e(z)} = \frac{-3.46e^{-5} - 3.447e^{-5}z^{-1} + 3.445e^{-5}z^{-2} + 3.456e^{-5}z^{-3}}{1 - 2.992z^{-1} + 2.984z^{-2} + 0.992z^{-3}} \tag{5}$$

In the next section we describe the procedure to design the longitudinal autopilot of aircraft using generalized predictive control.

3 GENERALIZED PREDICTIVE CONTROL

The process to be controlled is classified by following CARIMA model [7]:

$$a(z)y_k = b(z)u_k + d_k \tag{6}$$

In this model y_k is the pitch angle output, u_k is the input control deflection δ_e and d_k is the unknown disturbance term. Now with $A(z) = a(z)\Delta(z)$, the incremental form of equation can be written, as it allows offset free prediction and also ignore the unknown term:

$$A(z)y_k = b(z)\Delta u_k \tag{7}$$

$$A(z) = 1 + A_1z^{-1} + A_2z^{-2} + \ldots + A_{n+1}z^{-n-1} \tag{8}$$

$$b(z) = b_1z^{-1} + \ldots + b_nz^{-n} \tag{9}$$

$A(z)$, $b(z)$ are the polynomials and $\Delta = 1 - z^{-1}$ is the difference operator. The difference equation with one-step ahead prediction for y_{k+1}, given previous data and current input increment Δu_k can be written as:

$$y_{k+1} = -[A_1,\ldots,A_{n+1}]\,y_{\leftarrow k} + [b_2,\ldots,b_n]\,\Delta u_{\leftarrow k-1} + b_1\Delta u_k \tag{10}$$

For many steps ahead prediction the difference equation is written as:

$$y_{\rightarrow k} = H\Delta u_{\rightarrow k-1} + P\Delta u_{\leftarrow k-1} + Qy_{\leftarrow k} \tag{11}$$

Where $\rightarrow$ represents future values of vector, $\leftarrow$ represents past values of vector and H, P and Q are matrices. For details refer [7].

3.1 Optimization

The optimal control action is acquired by the minimization of quadratic cost function J which is written as:

$$J = \sum_{i=n_w}^{n_y}\left\| r_{k+i} - y_{k+i}\right\|_2^2 + \lambda\sum_{0}^{n_u-1}\left\|\Delta u_{k+i}\right\|_2^2 \tag{12}$$

Where,

$n_w \rightarrow$ Initial horizon,

$n_y \rightarrow$ Prediction horizon and

n_u $\rightarrow$ Control horizon

λ $\rightarrow$ Control weighting

Control increments beyond the control horizon are zero i.e.

$$\Delta u_{k+i} = 0,\ i \geq n_u \tag{13}$$

In more compact form the cost function J can be written as:

$$J = \left\| \underset{\rightarrow}{r} - \underset{\rightarrow}{y} \right\|_2^2 + \lambda \left\| \underset{\rightarrow}{\Delta u} \right\|_2^2 \tag{14}$$

Substitute $\underset{\rightarrow}{y}$ in equation (14) the cost function becomes:

$$J = \left\| \underset{\rightarrow}{r} - H \underset{\rightarrow}{\Delta u} - P \underset{\leftarrow}{\Delta u} - Q \underset{\leftarrow}{\Delta y} \right\|_2^2 + \lambda \left\| \underset{\rightarrow}{\Delta u} \right\|_2^2 \tag{15}$$

The online control law $\underset{\rightarrow}{\Delta u}$ is determined from the minimization of cost function. After expanding, equation (15) can be written as:

$$\min_{\underset{\rightarrow}{\Delta u}} J = \underset{\rightarrow}{\Delta u}^T (H^T H + \lambda I) \underset{\rightarrow}{\Delta u} + 2 \underset{\rightarrow}{\Delta u}^T H^T \left[P \underset{\leftarrow}{\Delta u} + Q \underset{\leftarrow}{y} - \underset{\rightarrow}{r} \right] \tag{16}$$

The unique minimum can be found by setting first derivative of J to zero i.e.

$$\frac{dJ}{d \underset{\rightarrow}{\Delta u}} = 0 \Rightarrow \underset{\rightarrow}{\Delta u} = (H^T H + \lambda I)^{-1} H^T \left[\underset{\rightarrow}{r} - P \underset{\leftarrow}{y} - Q \underset{\leftarrow}{\Delta u} \right] \tag{17}$$

Since CGPC control law is defined by first element of $\underset{\rightarrow}{\Delta u}$ so equation (17) is written as:

$$\Delta u_k = e_1^T (H^T H + \lambda I)^{-1} H^T \left[\underset{\rightarrow}{r} - P \underset{\leftarrow}{y} - Q \underset{\leftarrow}{\Delta u} \right] \tag{18}$$

Where,

$$\Delta u_k = e_1^T \Delta \underset{\rightarrow}{u} \text{ and } e_1^T = [I, 0, 0, .., 0]$$

Fig. 1 will illustrate you briefly the working principle of CGPC.

3.2 Constraint Handling:

The ability of online constraint handling is major characteristic of CGPC. Due to servo hardware limitations, constraints are applied on upper and lower limit of the control input magnitude and the rate of change of control input as mentioned below:

$$-10 \leq u \leq 10$$

$$-0.5 \leq \Delta u \leq 0.5$$

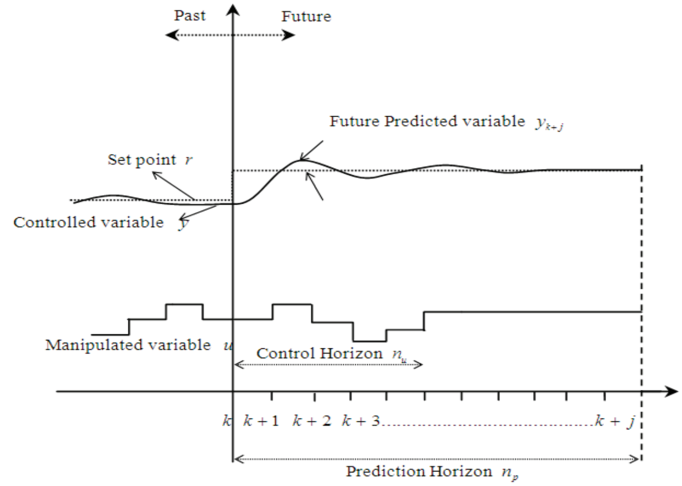

Figure 1 - CGPC working principle

It is our desire that during optimal prediction none of the constraint is violated and satisfied simultaneously. To accomplish this, optimization function is written as follow:

$$\underline{\min_{\underset{\rightarrow}{\Delta u}} J = \underset{\rightarrow}{\Delta u}^T (H^T H + \lambda I) \underset{\rightarrow}{\Delta u} + 2 \underset{\rightarrow}{\Delta u}^T H^T \left[P \underset{\leftarrow}{\Delta u} + Q \underset{\leftarrow}{y} - r \right]}$$

$$\downarrow \tag{19}$$

Subject to Constraints

4 APPLICATION OF CGPC TO AIRCRAFT PITCH AUTOPILOT

In this section we demonstrate how CGPC is tuned quickly and effectively for application in which longitudinal channel of aircraft is controlled as shown in Fig. 2.

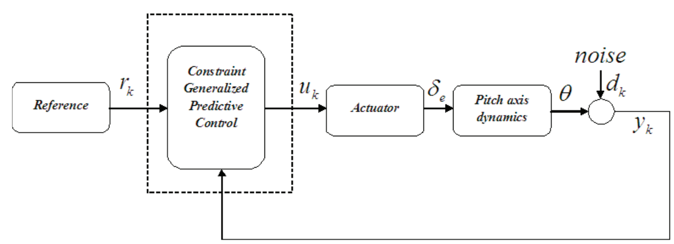

Figure 2 - Complete system block diagram

This section presents the performance of CGPC scheme by various selection of tuning parameters like control horizon n_u, prediction horizon n_p and control weighting factor λ. The control horizon should always be less than the prediction horizon. Table II shows the different case of tuning parameters.

TABLE 2 - PERFORMANCE COMPARISON

Case #	Prediction Horizon (n_p)	Control Horizon (n_u)	Control weighting factor (λ)
Case1	50	2	30
Case2	50	10	30
Case4	50	20	10
Case5	50	20	50
Case6	30	20	10
Case7	70	20	10

Closed loop simulations are carried out for these various cases. In case 1 to 3, closed loop performance for step input profile is improved by increasing control horizon by keeping fixed values of prediction horizon and control weighting factor as shown in Fig. 3-5.

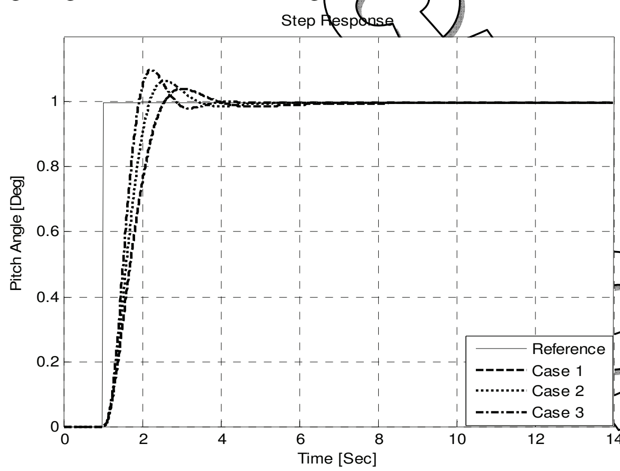

Figure 3 - Step Response

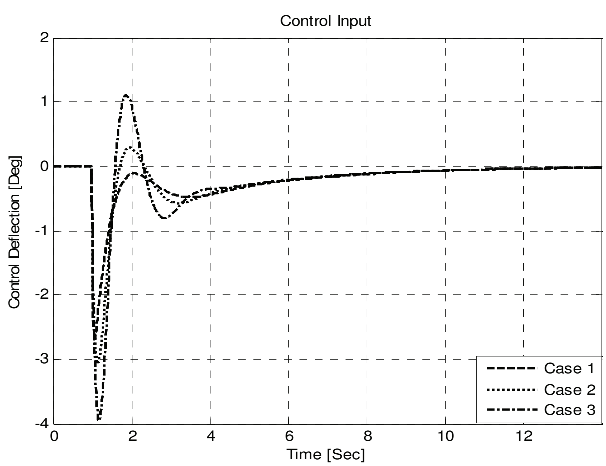

Figure 4 - Control Input Deflection

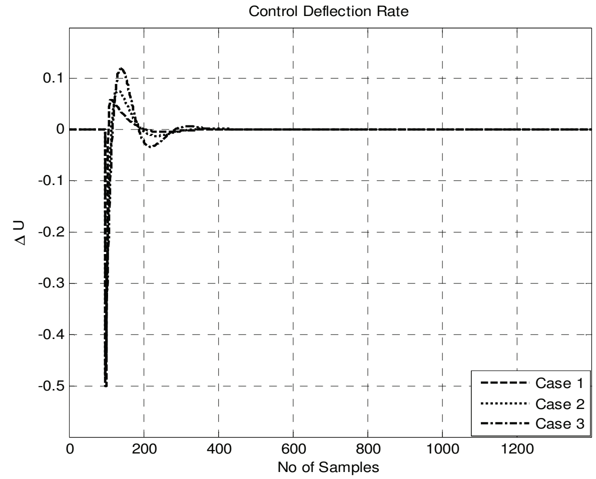

Figure 5 - Control Increments

It has been observed that case 3 provides better closed loop performance. To visualize the effect of control weighting factor, in next two cases (4 and 5) prediction horizon and control horizon remains constant as was in case 3, however control weighting factor is varied.

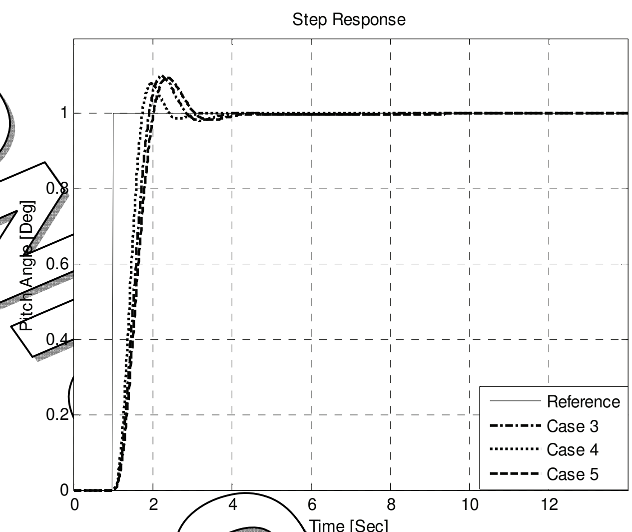

Figure 6 - Step Response Comparison

It is evident from Fig. 6, closed loop performance further improved by reducing the control weighting factor in case 4. In last two cases (6 and 7), optimal values of control horizon and control weighting factor obtained in case 4 are maintained while varying the prediction horizon.

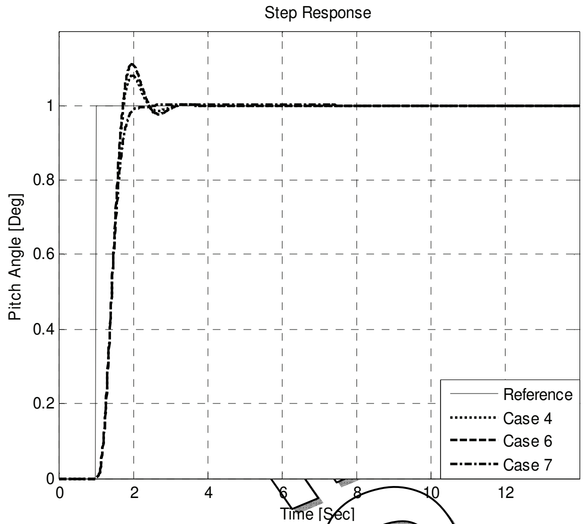

Figure 7 - Step Response Comparison

It is obvious from Fig. 7; by increasing prediction horizon near optimal closed loop response is attained. Based upon the above simulation results, it is concluded that by keeping larger value of prediction horizon and smaller value of control weighting factor, closed loop performance keep improving by increasing control horizon. Hence case 7 is taken as reference for further analysis in the subsequent sections.

5 ROBUSTNESS ANALYSIS

As discussed earlier that the controller is designed to be robust against parametric variations and disturbances. To examine the behavior of proposed scheme, uniformly distributed noise is considered as a disturbance in the output as shown in Fig 8. It is evident from the figure that CGPC can easily handle the disturbances by taking care of all the constraints when affected by noise.

To visualize the response of designed CGPC controller against parametric variations, closed loop simulations are executed for two different cases as shown in Table 3.

TABLE 3 - PARAMETRIC VARIATIONS

Parameter	Nominal values	Case1 (+20%)	Case2 (-20%)
C_{z_α}	-4.46	-5.35	-3.568
C_{m_α}	-0.6149	-0.74	-0.495
C_{m_q}	-11.4	-13.68	-9.12

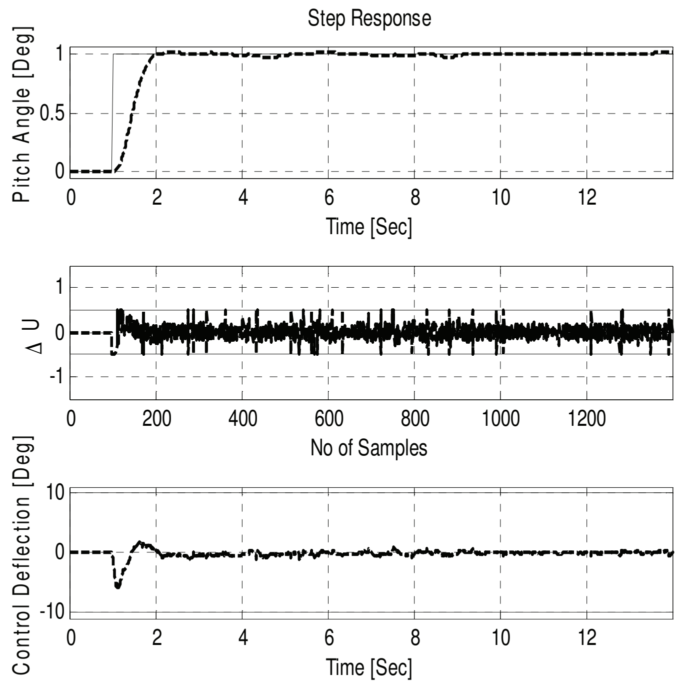

Figure 8 - Robust Performance against Noise

Above table shows ±20% parametric variation in aerodynamic coefficients from the nominal values. Fig. 9 shows that there is no significant deterioration in the step response which reflects the robustness of proposed control scheme.

6 COMPARISON

In this section, a comparison of the proposed control scheme with a conventional PID controller is presented. The PID controller is designed using the frequency response methods.

It is evident from Fig. 10 that GPC provide more efficient transient and steady state response as compared to the PID controller.

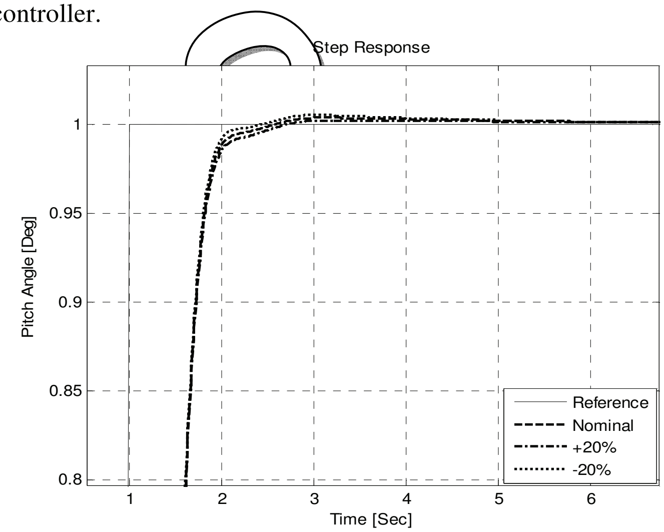

Figure 9 - Robust Performance against Parametric Variations

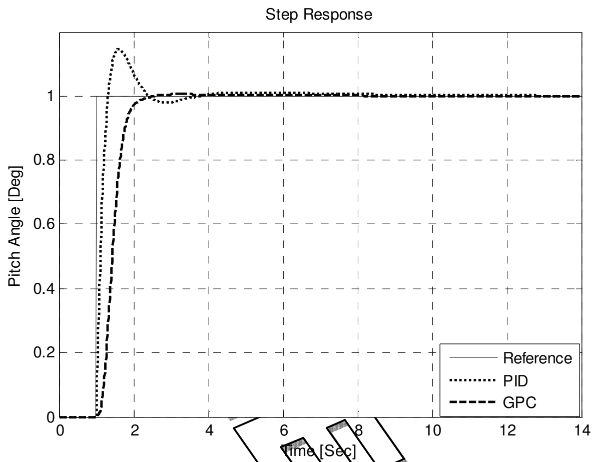

Figure 10 - Comparison of proposed scheme with PID

TABLE 4 - STEP RESPONSE CHARACTERISTICS

Parameter	**PID**	**GPC**
Rise Time (sec)	0.2	0.55
Overshoot (%)	15	0.5
Settling Time (sec)	2.0	1.5

CONCLUSION

In this paper a constraint generalized predictive control methodology is presented for the longitudinal dynamics of a civil aircraft. The simulation results show that the proposed controller efficiently fulfils the design requirements by efficiently tracking the given step inputs while staying within the system constraints. And that the proposed controller is robust against plant parameter variation and external disturbances. Comparative study shows that the performance of the proposed scheme is more accurate than PID controller. The future scope of work in this project is to implement the proposed control scheme on the nonlinear model of the aircraft and carry out the 6 degree of freedom simulations. In conjunction with this, the model of actuators and sensors will also be incorporated to make the simulation model more also be incorporated to make the simulation model more practical.

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