

Stochastic Volatility and Black – Scholes Model Evidence of Amman Stock Exchange

MOHAMMAD. M. ALALAYA

Associate Prof in Economic Methods Al-Hussein Bin Talal University,
Collage of Administrative Management and Economics, Ma'an, Jordan.
Email: dralaya@yahoo.com

SULIMAN ALKHATAB

Professer in Marketing, Al-Hussein Bin Talal University,
Collage of Administrative Management and Economics, Ma'an, Jordan.

AHMAD ALMUHTASEB

Assistant Prof in Marketing, Al-Hussein Bin Talal University,
Collage of Administrative Management and Economics, Ma'an, Jordan.

JIHAD ALAFARJAT

Assistant Prof in Management, Al-Hussein Bin Talal University,
Collage of Administrative Management and Economics, Ma'an, Jordan.

Abstract

This paper makes an attempt to decompose the Black – Scholes into components in Garch option model, and to examine the path of dependence in the terminal stock price distribution of Amman Stock Exchange (ASE), such as Black – Scholes', the leverage effect in this paper which represents the result of analysis is important to determine the direction of the model bias, a time varying risk, may give fruitful help in explaining the under pricing of traded stock sheers and traded options in ASE. Generally, this study considers various pricing bias related to warrant of strike prices, and observes time to time maturity. The Garch option price does not seem overly sensitive to (a, B1) parameters, or to the time risk premium variance persistence parameter, $\Omega = a1+B1$, heaving on the magnitude of Black –Scholes' bias of the result of analysis, where the conditional variance bias does not improve in accuracy to justify the model to ASE data.

Key Words: ASE, Black-Scholes, Garch model, Option Prices, Stochastic volatility, Strike Prices.

Introduction

We would like to clarify some terms which were used in this study, such as:

- Option: the right to buy or sell a particular good at an exercise price, on or before a specified date, but which is not an obligation.
- Exercise price or strike price: a price that indicates the fixed price at which a good may be bought or sold.
- In the money: gains from an immediate exercise, its positive intrinsic value.
- At the money: a situation whereby the share price is equal to the exercise price.

Objectives of the Study

This paper aims at investigating the ASE stock and strike prices, and its other sub – objectives are:

- Incorporate modification of the Black – Scholes’ options, pricing model can used in this regard.
- Identification in empirical testing of original Black –Scholes’ equation, with respect to the market value on the basis of stochastic volatility of the index of ASE.

Research Questions

The main question raised was: ‘are the prices paid to investors in ASE market ~~were~~ fair prices, and should these prices be valued by Black – Scholes model evaluation’? Other questions which were raised include:

- ‘Is Black –Scholes model reasonably accurate for in –the money pricing of stock shares in ASE’?
- ‘Is Black –Scholes model appropriate for pricing the traded warrants on the ASE’?
- ‘Can an investor who has little grasp and little understanding of the nature of option warrants really make good profit’?

Brief Notes on ASM

ASM (Amman stock market) or Amman stock exchange (ASE), as some would prefer to call it, is characterized by low turnover ratio, low liquidity, low transparency, and the non existence of market decision makers, like any other emerging market. The turnover ratio of our data for the period under investigation is 17.53%, and the average daily turnover is 0.9593 %, these ratios are considered to be very small, and to be one of the major problems that might affect trading activities. The average daily turnover is made possible by individual investors and institutions, and the government. The ownership structure and the ASE have both witnessed an increase in the number of listed companies all through the years, which gives an indication of economic growth in Jordan, and also a sense of stability during the periods of 2003 - 2010 in ASE.

The trading volume increased from year to year during the period of study, and the results of visibility of ASE became more superior to those of the other stock markets in middle east region, it experienced accelerated growth especially during the last 6 years, due to the stability it enjoyed from the Arab shares such as, from Iraqi ~~an~~ investors, also, Jordan government represented the board of international accounting standard. Some indicators of ASE includes the fact that it was established 1976, and it is an emerging stock market, the capitalization is \$9.765m, and the changes that it experienced from1999-2001 is rated as 8.4%, and 27.3%, from 1996-2010, where the capitalization ratio to GNI is 58.9%, the turnover is \$13.54m, and the turnover (liquidity) is 18.7%. (M. Alaya).

Previous Studies on the Subject

Whaley used the striking price, time to expiration, variance biases and CBOE data for the period between1975 – 1978. He demonstrated empirically that the striking prices and time to expiration could be virtually eliminated; he concluded that dividend induced probability could be integrated into American call option pricing.

Brown has observed a random behavior of option pricing, he noticed that the motion of pollen floating in water did not follow any distinct pattern, this observation and its subsequent theory is known as ‘Brown motion’ which propelled some mathematicians into the creation of stochastic calculus.

Fischer Black and Myron Scholes employed these tools in their research to develop an effective and reliable model to price derivative securities known as option. The Black-Scholes formula was used to price European call and to put options based on asset price on dynamic stochastic model, which depends on stochastic differential equations; therefore, we can solve this model by geometric Brown motion, where the certain two real parameters of volatility drift. The use of statistical tests to solve the problem associated with calibrating stochastic dynamical system (Fischer Black and Myron Scholes, 1973).

We can define option as: a security which gives one the right to buy or sell an asset, subject to a certain conditions, and within specified period of time. A simple kind of option is one that gives the right to buy a single share of common stock; this can be referred to as call option (Black-Scholes, 1973). They noticed that when there is a higher price of the stock, there would be greater value of the option, and when the stock price is greater than the exercise price, then, the option is most sure to be exercised, also, if the expiration date is very near, the value of option will be equal to stock price.

The Black-Scholes model is still used in option pricing, where the input required for the pricing of a call option is on a non dividend paying stock with a Black-Scholes' formula, and the parameters such as: current stock prices, interest rate, volatility, and time to maturity can be observed in the formula.

Hull and White (1987), provided a throng of power series which was referred to as 'How model,' which when compared, could provide two models with stochastic volatility. Rubinstein (1994) states that this systematic behavior is driven by changes in volatility rate of asset returns, he also proved that the volatility is a deterministic function of asset price and time. Brande et al. (2002), discovers that the implied binomial tree model poises the American style options, while Cox et al. (1979), provided a tree model with constant volatility, some authors performed a no better than an ad-hoc procedure of mourning Black-Scholes' implied volatility, which seems to be a cross section between strike-Prices and maturities. As the volatility increases, the probability that stock price will either rise or fall also increased, which in response, also increased the value of both full and call options. The return of volatility is a major role in option form pricing.

This paper is organized as follows: Section One: which deals with the objective of this paper and the assumptions of Black-Scholes' model, then the hypothesis of this paper. While section two discusses the back ground for understanding black-Scholes' model, and also introduced the reader to the types of economic reasoning which forms part of mathematics back ground. Sections three: contains the pricing equation derivation, section four: discusses how the data are obtained, and gives a general overview of methodology of the analysis, while section five included the results and empirical results of this paper, lastly, we concluded and discussed some pitfalls of the Black-Scholes model.

Assumptions of Black-Scholes Model

The Black-Scholes model is based on the following assumptions

1. That the interest rate is known and is constant through time in a short term period.
2. That the stock pays on dividend or other distributions.
3. That the stock price follows a random walk in continuous time with a variance rate, which is proportional to the square of the stock price.
4. That there are no transactions costs in paying or selling the stocks or options.
5. That there are no penalties to short selling, and that the seller who does not own security will simply accept the price of the security from a buyer (Black & Scholes, 1973).
6. That the underlying asset follows a long normal random walk.
7. That the Arbitrage arguments allow us one to use a risk-neutral valuation approach (Cox-Rubinstein Proof).

Pricing Option

Black-Scholes in their paper derived an option pricing model of which one of the main assumption was that, underlying stock follows a Geometric Brownian motion. They also suggested several ways to drive stock option prices in their paper. In their opinion, many approaches can be used including, martingale's approach which depends on risk-neutral valuation formula such as:

$$\frac{V_{op}}{\eta_{asset}} = E_c \left[\frac{V_{pay}}{\eta_{asset}_t} \right] \quad \dots\dots\dots(1)$$

Where V_{op} : option pay at maturing, $V_{pay} = \eta(K, ST)$ is some deterministic function and V_{op} is option value as of time $t = 0$, η_{asset}_t is a so called neural asset used for relative price $\frac{V_{pay}}{\eta_{asset}_t}$.

Black-Scholes model assumes that there are no arbitrage opportunities, which emphasizes the fact that there are opportunities to gain profits, without any risk being involved. The arbitrageurs themselves should be aware of it by buying more from one stock market, which can cause the price of the stocks to rise, and selling to other stock markets which can cause the price to decline. There are other instances such as buying more stock in New York stock market, and when there is movement to raise up prices, rushing to sell them in London stock market.

The Black-Scholes model for option pricing disagrees with the idea of trading with delta hedging in mind. Their equation is likely to pave the way soon, for an influx of mathematical finance and financial aspects which involves practical aspects of finance. The pricing for Millar can be expressed in Black-Scholes' option as:

$$f(S, T, K, r, \sigma) = Ke^{-rt} N(-d_2) - SN(-d_1) \quad \dots\dots\dots(2)$$

Where, S is the current stock price, T is time until expiration in years, and σ is the annualized volatility of the stock, N is the cumulative standard normal distribution function, r is the current risk-free rate of return. The above function inputs are required for the pricing of a call option on non-dividend paying stock with current stock price strike price, interest rate, volatility and time to maturity. We can observe all these parameters with the following alternative hypotheses which are:

H (1): The market value for 'in the money' does not tally with Black-Scholes' value for 'in the money' warranty, also, for 'out-of-money' warrants.

H (2): The market value for warrants with high variance underlying stocks.

H (3): The market value for near-maturity warrants.

H (4): The market values on all daily prices observations are also volatile time to time.

H (5): Trading in stock takes place continuously and markets are always open;

H (6): The stock pays no dividend on any type of underlying assets or security;

H (7): The stock price follows a Geometric Brownian motion process with μ and σ as a constant.

H (8): The assets are completely divisible in nature;

H(9): There are no penalties on short selling of shares and investor also get full use of short-sell Procedure; and Stock price option follows an explicit type of stochastic process called diffusion process; with an exception of volatility in the market, we can express d_1 and d_2 in the () equation as:

$$d_1 = \frac{\log\left(\frac{\delta_t}{K}\right) + \left(r + \frac{^2\sigma}{2}\right)(T - t)}{\sqrt{^2\sigma(T - t)}} \dots\dots\dots(3)$$

$$d_2 = d_1 - \sqrt{^2\sigma(T - t)}$$

$^2\sigma$: is the variance of continuously compounded return. Stochastic processes have many properties that appear in numerous application, such as:

- 1- The distribution of $X_{t_i} + h - X_{t_i}$ depends only on h, then X_t which has stationary increments.
- 2- if $t_1 < t_2 < t_3$, $X_{t_2} - X_{t_1}, X_{t_3} - X_{t_2}$, are independent, that X_t , this gives us a sign that the process is filled with indecent increments.
- 3- As we discussed in this review X_t is said to be with the market property due to the indecent state of variables.
- 4- We can see that the transition probabilities should be binomially distributed as follows:

$$P_j K = f(K, 2N, P_i) = \left(\frac{2N}{K}\right) P_j K (1 - P_j)^{2N - K} \dots\dots\dots(4)$$

Where $K \in (1, 2, \dots, 2N)$ and f is the probability mass

Furthermore; from the functions of the binomial distribution, one can restrict his analysis by looking at stationary market shares.

Hedging Portfolio through Black-Scholes' Equation

There is one important area to be considered from our analysis, and this has to do with driving the Black-Scholes' equation; via a binomial pricing process, which is based on the binomial price model described in the section above. First, we assume one risk-free rate from which we can borrow or lend money, and then we assume that there is only one period left to call option on the stock.

By so doing, the fair value of the option which we have determined it to be by requesting that it be the value that equals it, is expected to be paid off. This can be determined merely by knowing some information about interest rate, underling stock, the strike price, and the range of the values that the stock can take on after a period and the use that possible values of stock could be put to after this period, which is relative to the volatility in a way in the Black-Scholes' equation. Volatility of stock can be deference and made to be the standard deviation of log returns of the stock. Black-Scholes model assumes a constant volatility, and the one way which can be used to estimate it, is to use historical volatility, the estimation of the equation can be given as:

$$\hat{\sigma} = \frac{\sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2}}{\sqrt{t}} \dots\dots\dots(5)$$

where: $(n - 1)$ price data from $(n - 1)$ and u_i is $\ln \frac{\delta_i}{\delta_{i-1}}$, \bar{u} is the sample average of all u_i ,

δ_i is the stock price in period t and T is the total length at each period in the year. T equals 1/252 or

1/365. The numerator represents the log-returns of the stock, and denominator is scaling future to make the estimates to be one of the year's volatility.

(Hull, et al.,) wrote an article which he titled, 'pricing option on assets with stochastic volatilities,' which he used to discover solutions to the problem of pricing in European earl options. They determined that the price depends upon some stock price S , and that it is instantaneous in variance, $V = \sigma^2$.

$$d_s = \phi S d_t + \sigma_s d_w \dots\dots\dots(6)$$

$$\text{Where } d_v = \mu d_{vt} + \delta_v d_2$$

ϕ : for the stock price may depend on S , σ ant t .

U : and the diflusion coefficient.

δ : for the variance which may depend only on σ and t .

The two processes are correlated. Hall and White made some assumptions in the course of their work, such as:

- 1- That the volatility (V) is uncorrelated with the stock price.
- 2- That the volatility U is uncorrelated with aggregate consumption.
- 3- That S and δ are the only two variables which affect the price of derivate f , and therefore, that the risk free rate must be constant or deterministic.

The expected returns of the stock for example are not independent of risk preferences. Risk in stock market are averse to investors in relations to how he would ask for higher expected returns for increasing risk level, and in the risk of sacking an investor who would ask for lower expected returns for decreasing risk levels.

Volatility Models

Moving Average

We can depend on hysterical returns from stock prices if we use the moving Average model, the formula is:

$$u_i = \frac{S_{ti} - S_{i-1}}{S_{i-1}} \dots\dots\dots(7)$$

where u_i percentage changes or compounded return during day i (between the beginning of day $i-1$ and the end of day i)

S_i : the value of assets at the end of day i .

The value of the day before of u_i is $u_i = \ln \left(\frac{S_i}{S_{i-1}} \right)$ the unbiased estimate – of one volatility as:

$$\sigma_n^2 = \frac{1}{m-1} \sum_{i=1}^m U_{n-i} - \bar{U}^2 \dots\dots\dots(8)$$

Stochastic model proposed some properties, such as:

- 1- The implied volatilities have a random behavior in time, but have smooth dependence.

- 2- Implied volatilities are positive, in cases where the nose terms are taken to be wiener processes, or where they are at a log normal random variable.
- 3- Calibration to market implies that volatilities are simply reduced to specifying the initial condition.
- 4- The model allows easy evaluation of any portfolio, the price of any option is simply given by the Black-Scholes' equation.

Option Pricing Model

Black-Scholes in one of their articles, 'the log of normal distribution', said that stock price can never fall by more than 100 percent, but on the other hand, there is a small chance for it to rise more than 100 percent. According to Black-Scholes model in one of their assumptions, the stock price is log normally distributed.

The pricing derivative provides a pay off at one exact time in the future simply because it takes a discount vat, and is risk-free in interest rate (r), recent study findings show that volatility tends to vary over time according to this situation, the assumption of constant volatility is unrealistic. Garth option model assumes that the conditional volatility of stock prices depends on the past pricing errors.

Therefore, one phase of this study is Garth models, and the study is based on historical return from ASE stock prices. Regarding the nature of random variables which are distributed as log-normal, a Markova process is a certain stochastic that may make use of random variable, and may be followed through. On the other hand, the history of variable prices is irrelevant and the only present value is used to predict the future values. Brownian motion is a particular case of Market process with a mean of zero and variance of 1.0 per year (John. C. Hull, 2006).

This method which is known as A wiener process variable (x), tallies with the following method when:

ΔX for two different short intervals of time, Δt are independent. If we put the equation in a simplified approach, so that:

$$\sigma_m^2 = \frac{1}{m} \sum_{i=1}^m U_{n-i}^2 \dots\dots\dots(9)$$

In this method, we should be aware of the data being too old, and being unreliable in predicting the future, then, we should be on the lookout for more data which could lead to better precision. Changes in prices from daily data over "the last 90, 180 or even 252" days are often used.

Garch Model

Many models are used that corresponds to stochastic volatility process characteristic (Engle, 1982), setting unconditional volatility constant model, while allowing the conditional volatility to change over time, this model is known as ARCH model. ARCH model (9) is:

$$\sigma_n^2 = \omega + \sum_{i=1}^9 a_i U_{n-i}^2 \dots\dots\dots(10)$$

where ω is equal $\gamma \cdot V_L$ and is calculated by using the following equation:

$$u_i = \frac{S_i - S_{i-1}}{S_{i-1}} \quad \text{or} \quad u_i = Ln \frac{S_i}{S_{i-1}} \dots\dots\dots(11)$$

and is the long-run average variance rate and γ is the weight assigned to V_i variable 9 is the der of dependency to past returns.

A generalized approach of ArCh model was proposed by (Boterstev, 1986), which was known as Garch model, the Garch model (P, q) can be written as:

$$\sigma_n^2 = \omega + \sum_{i=1}^q a_i U_{n-i}^2 + \sum_{i=1}^p B_i \sigma_{n-i}^2 \dots\dots\dots(12)$$

where: $\omega > 0$. $a_i \geq 0$ and $B \geq 0$.

The changes which happen on (ΔX) during a short period (Δt) is $\Delta X = \sum \sqrt{\Delta t}$ (where \sum disributed normally $\phi(0, 1)$). But it is a long time from (0) to maturity T , $X(T) - X(\sigma)$ is normally distributed with mean zero, standard deviation \sqrt{t} and variance T the wiener process for a variable C add an expected drifts rate μ and volatility, which can be written as: $d_c = d_t + d_x$. The variable C is normally distributed in any time interval T , mean change of C is (μT) , standard deviation $\sigma \sqrt{T}$ and variance $(\sigma^2 T)$.

The stochastic volatility of the underlying asset is a random process than a constant one, with volatility measures which have unexpected changes in the value of financial asset in a certain time period. Since the magnitude of the fluctuation is unknown, volatility is used to measure the risk of certain assets.

Volatility is skewed if we consider options underplaying equity with different strike prices, then, the volatility implied by their market prices should be the same, and the implied volatilities often represents a smile or skew instead of strength line. In this case, the smile is reflecting higher and it implies volatilities for deep in- or out of the money options. When the market prices are used to find the implied volatility model, the stock price follows a stochastic process and according to ArCh model, variables (p, q) are the order of dependency in which the simplest model is Garth (1, 1) model, which can be specified as:

$$\xi_i = \omega + Q \cdot U_{i-1}^2 + B \cdot \sigma_{i-1}^2 \dots\dots\dots(13)$$

and B must sum to one.

(Enle and Bollerstev, 1986) have introduced the I. Garch (integrated Garch), in this model a , and B are equal to 1. The parameters of the model can be estimated using the maximum likelihood method. The likelihood of the (m) observation is as follows:

$$\prod_{i=1}^m \left[\frac{1}{\sqrt{2 \pi \sigma_{vi}}} \exp \left(-\frac{U_i^2}{2 \sigma_{vi}} \right) \right], \text{ vi} = \sigma_i^2 \dots\dots\dots(14)$$

Which is an estimated variance for day I and are the member of observation?

The best parameters are therefore the logarithm of the expression and its equivalent, thus, the log likelihood is expressed as:

$$\sum_{i=1}^m \left[e + \frac{1}{2} \left[-\ln(vi) - \frac{U_i^2}{v} \right] \right] \dots\dots\dots(15)$$

the parameters of Garch (1,1) can be defined as a function of time, it means that the parameters at day (i) are estimated by means of maximum likelihood, and subsequently, when these parameters are used to forecast volatility at day (i) then, we can adapt the equation (16) as:

$$\sigma_i^2 = \omega_i + a_i \cdot U_{i-1}^2 + B \cdot \sigma_i^2 \dots\dots\dots(16)$$

ω , a , b are changing over time v , V_L , U_{i-1}^2 and σ_{i-1}^2 change overtime also, and the model is estimated through dewily maximization of the parameters, and is expected, and known as a geometric Brownian motion C (John, Hull, 2006) $d_v = \phi_v d_t + \zeta_v d_w$ where the d_t and d_w are correlated to be before, and to estimate future volatility on daily basis. The below diagram illustrates this.

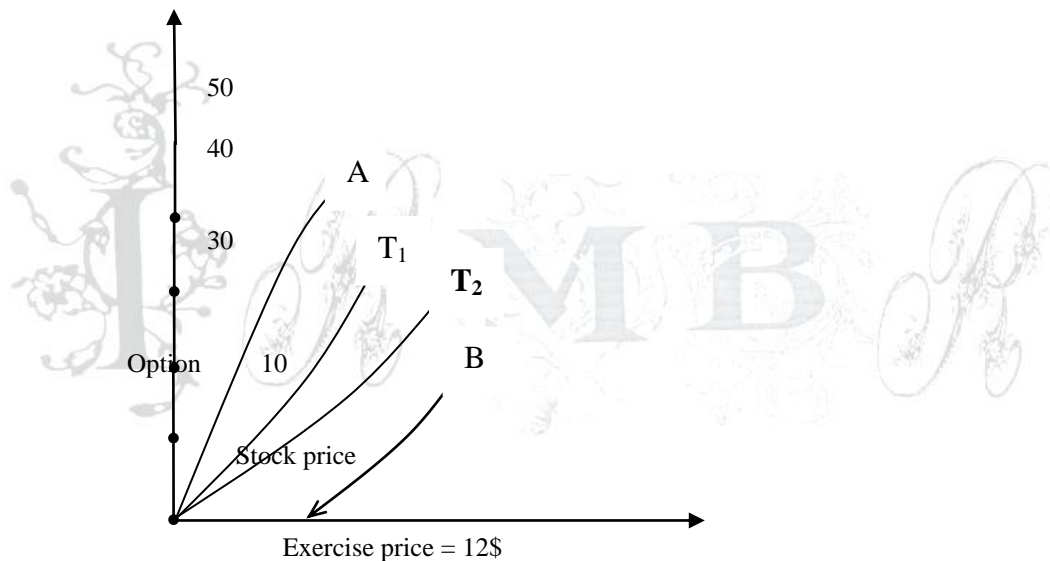


Figure (1): the value of options in ASE.

The curve representing the value of an option concaves upward, and we can notice that the option will be more volatile than the stock, and that the relative volatility of the option is not constant, the volatility depends on both the stock price and on maturity (Fisher Black and Myron Scholes, 1973).

Data Collection and Methodology

I was using the ASE stock prices database for the period from January 2010-2014 as historical data and 3 months data of 2014 to compare other analysis with data about volume of traded shares, price of option, price of underlying stock, strike price, expiration data and a time stamp that reveals the time. The real data studied were from 2014, the daily value of ASE index is seen in the figure above, and the price of the options of this index, specifically, when we considered the daily closing value of the index and the bid

prices of these options during the period of the study with positive and negative prices, and bid prices, traded on the day corresponding to the price considered.

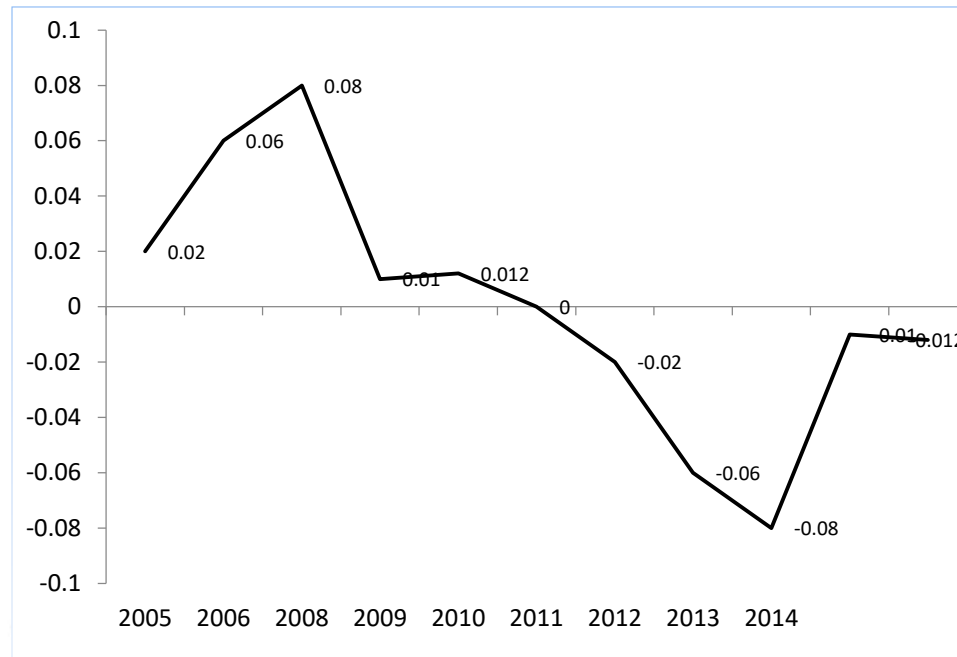


Diagram (2): Daily Return of ASE Index from 2005-2014

There are two methods from this analysis, and these are, the huge volume of data at the first step, here I have used the (OLS), and then applied volatility by Garch (1, 1). A summary of the results for historical volatility is shown in table (1).

Table (1): Descriptive static for historical volatility results N: 425

	Minimum	Maximum	Mean	St/deviation
Volatility	0.02	1.98	8609	1.7976
Residual	6.73	15.489	3.164	0.9682
* Square residual	0.03	1.6313	4.728	3.6443
* Square relative residual	0.00	6.883	3.669	1.4865

* Squared relative deviation:

$$M(\hat{P}) = \frac{(\hat{P} - P)}{P} \quad (\hat{P}); S \text{ is the price of Black-Scholes' equation when we plug in our estimated}$$

volatility, and P is the actual stock price. According to the result of table (1), the volatility is not very meaningful, since the summary here is aggregated across all stock, and the stock have different volatilities. We see that due to historical volatility, the highest of the stocks is over evaluated by (15.429) \$, and the lowest one stock option is over evaluated by (1.64) \$. These results when persistent in their upward bias will show on a plot of volatility versus time, and the relative square residual will reveal an average which is overpricing by 31%, and which allows for 17% under pricing. The (OLS) of the analysis are shown on the following tables, table (2) shows the model summary.

Table (2) Model Summary

Model	R	R ²	Adjusted R ²	St/Error of estimate	Durbin-Watson
1	0.161	0.259	0.213	1.8652	0.367

Dependent Variable Square Relative residuals

Table (3): Coefficient of OLS

Model Constant	Un standardized Coefficients		Stand Coefficient Beta	t-test	Prob. level
	B	Std. / Error			
Time	3.071	0.299		9.735	0.000
	0.019	0.001	0.263	2.594	0.57

Dependent Variable: Square relative residuals

The effect of the length of time which has been chosen on the dependents variable is significant at the 5% level, since the coefficient is still positive. This can be seen as, increasing the time used in handling data, and increasing the pricing bias, so that the short time window could be chosen to minimize prancing bias. The effect of in: Black-Scholes' equation predicts a linear function of the volatility, so, that relative residual becomes predicted price again in the linear function model. Lastly, it seems to be a common s pattern. Some authors said that there are big observable spikes in volatility which are due to sudden changes in the stock price.

Therefore, at the time which was chosen for us to ascertain our work, we regress the squared relative residual on the chosen time, and by so doing, we noticed that several carnation problem appeared s in the table results, which we have not taken into account. This means that the time windows that have been chosen were over lapping. We can concluded that there is a positive auto correlation present in this analysis, this means that values of reeducated volatilities are correlated to each and we also conclude that in the absence of unusual spikes, best estimator of historical volatility are presented in our analysis.

The Maximum Likelihood Estimation of Garch (1, 1) Model

In table (3), the estimated result is presented in the log of Garch (1, 1) model of ASE index.

Table (3): Results of Garch (1, 1) of ASE index

V _L	ω	γ	a	B	a + B
6.532E-04	1.263E-04	0.296	0.0367	0.6932	0.7299

Due to the results of table (3), parameters α is 0.0367, B is 0.6932 and γ is 0.296. This means that past returns give us a sign that provided some information to estimate the present and future volatility of ASE. Adaptive Garch model results are represented in table (1), for ASE stock return which appears in Appendix tables the weights of α , B and γ begin by (0.0276), (0.892) and (0.2144) respectively, and ended by (0.00375), (0.7934) and (0.00731). Corresponding γ evolution of parameters is given in figure (3), for Amman stock exchange daily returns. During the study period, the sum of α , B and γ is always around one, and from figure (3) we can notice that the perimeter is going up.

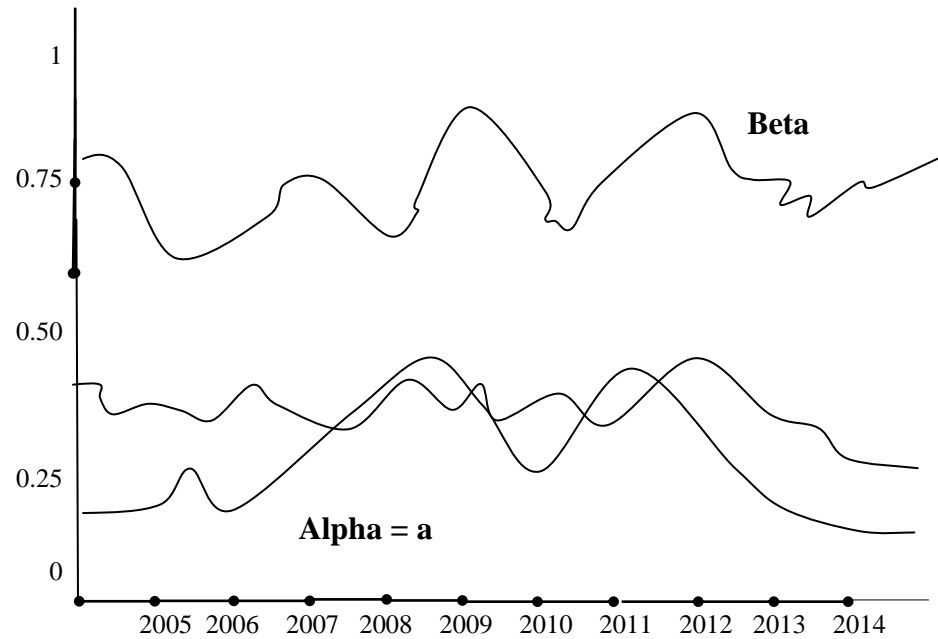
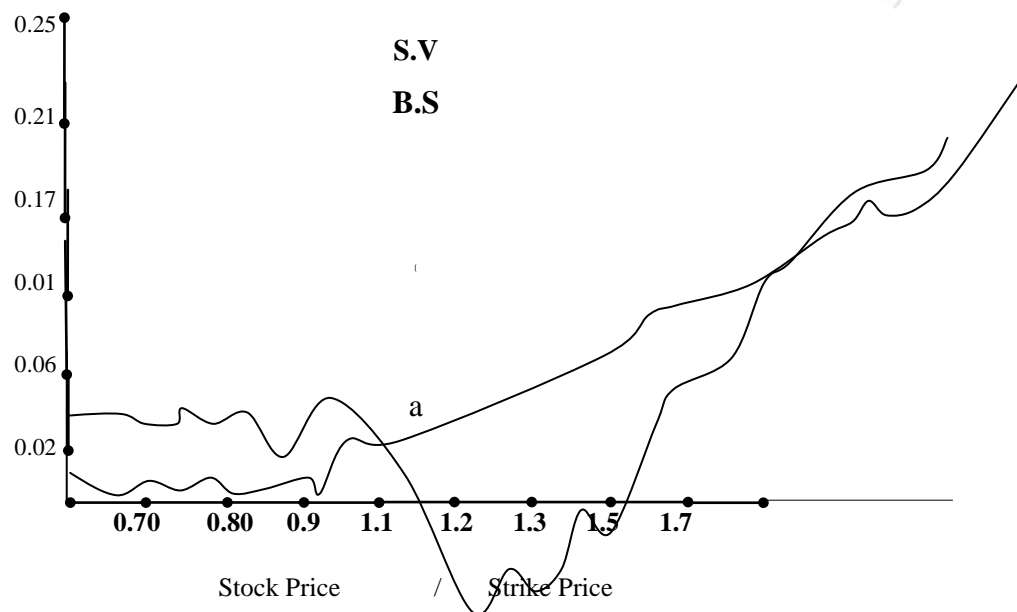


Figure (3): Evolution of the Estimated Parameters α , B and γ for the ASE.

Results of Volatility

It is either that the Garch option price does not seem to be overly sensitive to (α and B) parameters, or the unit risk premium (λ) variances persistence parameter, $\Omega = \alpha + B$, is in magnitude of the Black – Scholes model bias, where the unconditional variance bias is not an important accuracy to justify the model to ASE data. Figure (4) declares the option price difference 2 times.



Option prices are different 2 times; in this figure (4), the comparison is between the stochastic volatility and the Black –Scholes, and the diagram shows the difference from one point to another. This happened due to subtracting of the Black –Scholes’ price from the two models; consider the option price as dummy or default.

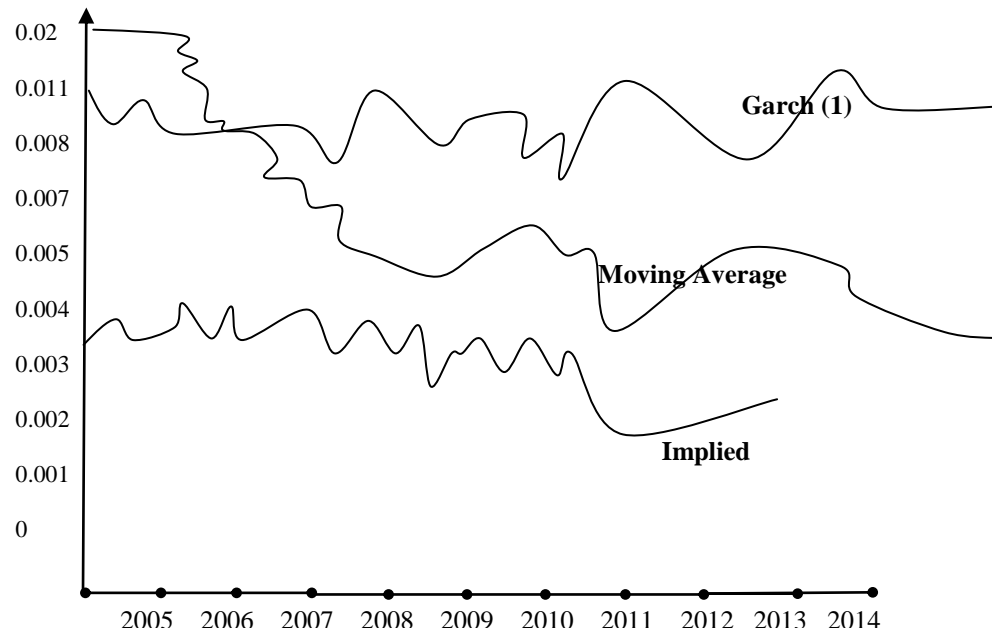


Figure (5): Estimated Volatility of the ASE Index by 3 Models

Figure (5) illustrated the comparison between models with a slight volatility, but the volatility becomes more obvious, Black – Scholes’ overprice is near to the money call option of the stock, while it produces a lower price & for a deep out-of –the money stock price of moving average model, the pattern for implied volatility difference is very small and the reason behind this may be that the implied model generates a volatility, due to the figures, we can conclude that Black –Scholes’ volatility is less than other models’.

Conclusive Remarks

This paper uses historical volatility to estimate true volatility, which gives us the true analysis of the results, and this is a significant bias towards overpricing the option price when we use Black-Scholes’ equation. There are many reasons why we attempted to use the model of volatility as a random process. One of these reasons is that it could simply represent the estimation uncertainty, secondly, it could come up as a fraction from transaction costs, the third reason is that it has a thick (heavy tailed) returns distribution, and the fourth reason which could either be considered a lesser reason, is that it is related to an extended model, and must also have to specify what data it is, that it needs to be calibrated from. We can summarize the calibration procedure as follows:

The fit – near the money implies the volatilities for several maturities as in ASE, so, straight line in the composite variable is called Log – moneyness -to – maturity ratio (LMMR). The estimate of the equation gives us the slope and the intercept (a), since $LMMR = 0$ when stock price equals strike price, where b is exactly, at-the money implied volatility. Also, we can estimate the historical volatility from stock price returns.

References

- Amman stock Exchange , Amman – Jordan, several monthly report from 2000- 2015.
- Berkowitz, J. "Forecasting Option Values With False Models." Bauer College of Business University of Houston Working Paper; (June 26) 2003.
- Black, F.; Scholes, M. "The Pricing Of Options And Corporate Liabilities." *Journal of Political Economy*, 81 (May - June), 637 – 659; 1973.
- Brandt, M. W.; Wu, T. "Cross-Sectional Tests Of Deterministic Volatility Functions.' *Journal of Empirical Finance* 9: 525-550; 2002.
- Bouchand, J.P., Potters, M. (2003), Theory of financial risk and derivative pricing: from Statistical Physics to Risk Management, Second edition, United Kingdom, Cambridge University press, 293-305.
- Central bank of Jordan, Amman, several annual reports from 1999 –2015.
- Cox, J. C.; Ross, S. A.; Rubinstein, M. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7: 229-264; 1979.
- Eberlein, E. (2007). Jump-type Levy processes. In T. G. Andersen, R. A. Davis, J.-P. Krei_, and T. Mikosch (Eds.), Handbook of Financial Time Series. Springer. (forth coming).
- Fisher Black .Myron Scholes (1973): "The pricing of options and corporate liabilities" The journal of political economy , Vol 81,No 3. Pp. 637 – 654.
- John Hull and Alan White, *The Pricing of Options on Assets with Stochastic Volatilities*, The Journal of Finance, Vol. 42, No. 2 (June, 1987), pp. 281-300.
- John C. Hull, *Options, Futures, and Other Derivatives*, 6th Edition, Pearson Education, New Jersey, 2006. pp 265, 269, 293.
- Jochen Wilhelm (2008) " option prices with stochastic interest Black -Scholes and Ho / LEE unifld University Passua, Herausgeber ,Betriebswirtschaftlich Reihe ISSN : 1435-3539..
- Hong Boon Kyun (2004) : " Empirical study of Black –Scholes warrant pricing model on the stock exchange of Malaysia " master thesis ,unpublished thesis.
- Heru.Sataputera (2003): " Black –Scholes option pricing using three volatility models: moving average ,Garch (1,1) and Adaptive Garch " Bachelor thesis ,Erasmus university, Netherlands.
- Khan, M.U., Gupta, A., Siraj, S., Ravichandran, N. (2012), Derivation and Suggested Modification In Black-Scholes Option pricing Model, IME Journal, Vol 6(1), pp. 19-26.
- Khan, M.U., Gupta, A., Siraj, S., Ravichandran, N. (2012), The Overview of Financial Derivative and Its Products, International Journal of Finance & Marketing, Vol 2(3), pp.57-72.
- Liu, B. (2010), Uncertainty theory: A Branch of Mathematics for Modeling Human Uncertainty, Second edition, Springer-Verlag Berlin, Heidelberg, 115-125.
- Lorella.Fatone, Fransesca Mariani, Maria Cristina, and Francesco Zirilli (2012): "The use of statistical tests to calibrate the Black options with uncertain volatility " The journal of probability and statistic , unpublished paper.
- M.Chaudhury, Jason, Z. Wei (1996) " Acomparative study of garch (1.1) and Black-Scholes option prices " unpublished paper.
- Matache, A.-M., T. v. Petersdor_, and C. Schwab (2004). Fast deterministic pricing of options on L_ey driven assets. M2AN Math. Model. Numer. Anal. 38, 37-72.
- Matache, A.-M., C. Schwab, and T. P. Wihler (2005). Fast numerical solution of parabolic integro-differential equations with applications in _nance. SIAM J. Sci. Comput. 27, 369-393
- Merton, R. C. (1973)." Theory of rational option pricing". Bell J. Econ. Manag. Sci.Vol 4, pp. 141-183.
- Merton, R. C. (1976). Option pricing with discontinuous returns. Bell J. Financ. Econ.Vol 3, pp.145.-166.
- Miltersen, K.R., Schwartz, E.S (1998).," Pricing of Options on Commodity Futures with Stochastic Term Structures of Convenience Yields and Interest Rates", in: Journal of Financial and Quantitative Analysis, 33 (1998), 33-59.
- Michiel Kalaverzos. and Micheal Wennerno (2007) " stochastic volatility models in option pricing " , Master thesis in applied mathematics ,Malardalen university, Sweden.

- Peter Cross ,(2006) : " parameter estimation for Black –Scholes equation ", advisor dr.Jialing Dia.Ura ,spring 2006.
- Protter, P. (2004). Stochastic Integration and Differential Equations (3rd ed.). Springer.
- Papapantoleon, A. (2007). Applications of semi martingales and Levy processes in duality and valuation. Ph. D. thesis, University of Freiburg..
- Rubinstein, M. "Implied Binomial Trees" *The Journal of Finance* 49: 771-818; 1994. Tavella, D. (2002), Quantitative Method in Derivatives Pricing: An Introduction to Computational Finance, John Wiley and Sons Printing press, 36-39.
- Raible, S. (2000). Levy processes in _nance: theory, numerics, and empir- ical facts. Ph. D. thesis, University of Freiburg .
- Zhu, J. (2009), Application of Fouries Transformation to Smile Modelling: Theory and Implementation, New York, Second Edition, Heidelberg Dordvecht London, Springer publication, 5-22.

