Testing Information Efficiency using Random Walk Model: Empirical evidence from Karachi stock exchange

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Abstract

This study investigates the weak form of efficiency of Karachi stock exchange using daily, weekly and monthly data for the period of June 2002 to June 2012. This study employs different parametric and non-parametric tests for examining random walks i.e., Jarque-Bera and Kolmogrov-Smirnov (KS) test for normality, autocorrelation and Run test for autocorrelation, Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) for stationarity and multiple variance ratio (MVR) tests. The results of this study indicate that by using all approaches none of the returns (daily, weekly and monthly) are following random walk and it is concluded that Pakistani stock market is not weak form efficient. The investors have an opportunity to get benefit from the predictable behavior of this market.

Keywords: Random walk, weak form of efficiency, efficient market hypothesis

Introduction

Stock market efficiency is one of the most debated topics of modern finance. Modern financial markets have one proposition that these markets are efficient. The term efficiency refers to the association of information with stock prices. In this context, the efficient market hypothesis (EMH) refers to the adjustment of stock prices in a timely manner and is based on the rapid incorporation of relevant information. So, no investor is able to get the abnormal return from any investment (Reilly and Brown, 2011). According to Fama (1970), if a market is efficient it reflects all the available information and accurately pricing all stocks then it is more helpful to allocation of recourses. For the purpose of resource allocation it is more important to observe the behavior of the market. Idea of efficiency envelops different facets and categorize in different contexts of economics and finance. Fama (1970) explains the market efficiency more precisely into three different categories weak

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form, semi-strong form and strong form of efficiency on the basis of available information.

The market is said to be efficient if there is a rapid and quick response from the market (Jones, 2007). In other words, market price of the security is an accurate price of the security that is traded in any market and contains all the information necessary for transaction is always unbiased to the new information coming in. On the other hand there is a possibility that a stock or security does not contain the accurate information and investor may not be able to interpret the information in a better way. That may result in the inefficiency of the market and reject EMH (Aumeboonsuke and Dryver, 2014).

Bachelier (1900) has provided the roots and theoretical framework of EMH, with a debate that random fluctuations persist in commodity prices. Samuelson (1965) opens a new avenue in modern economic literature by expanding the work of Bachelier (1900). From 1965 to 1998, Fama's numerous studies on market efficiency develop a new approach for market efficiency and called father of market efficiency. From 1960's to early 70's most debate conclude that changes in prices are linked with the individual securities or with the market (Fama 1965; Samuelson, 1965 and Sharpe, 1966).

Whereas, in 1980's more focus was on the testing of these theories both theoretically and empirically. Most studies (Fama 1965, 1970, 1991 and Fama and French, 1988) report consistent evidences and are aligned with the hypothesis that security prices reflect all available information and efficient markets are unsuccessful to give anomalous profits. Historical, EMH has been subdivided by Robert in 1967 and then extending from the idea of Robert, Fama introduces the word "efficient market". Following are the categories which are classified by Fama on the basis of available information set.

The objective of this study is to test the weak form of efficiency in Pakistani stock market. Pakistani market is an emerging market and during last few years has reported phenomenal growth. At the same time bubbles and burst have also been observed during the period of study. So, it is need of time to revisit the price behavior. So, this study provides insight to investors for efficient allocation of recourses through active investment management.

Literature Review

Role of information in determining prices is an undoubted fact in financial markets. In existing literature, several empirical and theoretical studies discuss the role of EMH. Fama (1970) has presented the theoretical foundation of EMH. Release of any news induces information to financial markets and the impact of such news depends upon the intensity of the news in both directions (Reilly and Brown, 2011).

Number of studies (Kendall and Hill, 1953; Osbone, 1959; Robert, 1959 and Fama, 1970) reject the EMH in developed economies, that fluctuations in equity prices cannot predictable on the basis of historically available price information and is also called "Random walk Model" (RWM). Whereas, some studies observe that these developed economies have power to predict changes in future prices (Poterba and Summers, 1988; Fama and French, 1988). Despite of developed economies different studies in rest of the world in both emerging and developing markets also provide mix results. Number of studies reports that these markets are weak form efficient (Barnes, 1986; Dickinson and Muragu, 1994; Ojah and Karemera, 1999) and Cheung, Wong and Ho (1993) presents the evidence that these markets are not weak form efficient.

Kendall (1953) investigates long term and short term movements in US equity market. He finds that movements in these stocks prices are random, no serial correlations, and report no discernible pattern.

Fama (1970) argues that in weak form of efficiency prices must release all historical information as it is revealed by using RWM and Fair game model. He has used serial covariances for all lags and all lagged values of "fair game" with unconditional expectation of and find no evidence of substantial linear dependence. Alexander (1961), Fama (1965), Fama and blume (1966), Levy (1967) and Jensen and Bennington (1970) have found no abnormal returns.Sweeney (1988) investigates Dow 30 with more Mechanical Trading rules than Fama and Blume (1966) and report significant abnormal returns.

Solnik (1973) inspects the RWM in European stock indices of 8 countries with 234 securities by using individual security rather than stock market index. Results of this study show that European markets depict more visible deviation from RWM than the US market. Sharma and Kennedy (1977) and Barnes (1986) report that Bombay stock index and Kuala Lumpur stock market is weak-form efficient. Whereas, Summers (1986) argues that common models take long time horizons and this study has empirically showed that fundamentally prices have slowly crumbling stationary components and short horizon returns have significant importance to account for. Summer (1986) argues that long horizon returns are important for mean reverting price adjustment component and gives a clearer impression.

Lo and MacKinlay (1988) have examined US equity market using 1216 weekly returns for the period of 1962-1985 and sub-periods of 608 weeks on aggregate returns indices (both equally and value weighted). Results of this study reject the null hypothesis for both sample period and sub-periods. Poterba and Summers (1988) examine weak form efficiency of US along with 17 other stock markets. Their findings suggest that in short run intervals there is positive serial correlation, but in long run negative correlation exists. Fama and French (1988) reports that 40% of the variation of long term holding returns in US stock market are predictable from the information of past returns. Ojah and Karemera (1999) investigate random walk of Latin American emerging equity markets. Documented results have suggested that these markets follow random walk and are weak form efficient. In Latin American equity markets, investors cannot get the benefit from historical information.

Abraham, Seyyed and Alsakran (2002) investigate three gulf stock markets by employing the methodology of Beveridge and Nelson (1981). The results of this study suggest that these markets are not following random walk. But, using true indices the results of weak form efficiency and RWM have been changed for Bahrain and Saudi Arabia. Buguk and Brorsen (2003) test weak form efficiency of Istanbul Stock exchange. Results reveal that all indices follow random walk, whereas nonparametric test rejected random walk in some series. Chakraborty (2006) investigates weak form efficiency of KSE for the period of 1996 to 2005 overall and two sub-periods (1996-2000 and 2001-2005) by using daily stock index. This study rejects the hypothesis of variance ratio for overall period but accepted only for second sub-period. Hamid et al. (2010) explore the weak form efficiency for 14 Asia-Pacific stock markets from 2004 to 2009 by using parametric and non-parametric tests. Results reveal that these markets do not follow random walks and market participants having the opportunity of arbitrage profit. Aumeboonsuke (2012) examines six ASEAN stock markets for the period of 2001 to 2012 and results of this study also confirm the results of Hamid et al. (2010).

Data Description and Methodology

Data description

The daily, weekly and monthly closing prices of KSE-100 index are taken to calculate returnsfor the period of 2002 to 2012. Continuous compounding daily, weekly and monthly returns are collected by using natural log of (Pt / P_{t-1})**.** Where,P_t and P_{t-1} are closing prices on Day, Week or Month t and t-1 respectively. Stock index data is collected from Karachi stock exchange, which is reliable source of information.

Methodology

To test weak form of efficiency this study employs number of econometric tests that are previously used by different studies starting from descriptive statistics to MVR test. Following are the different econometric tools to test weak form of efficiency.

Normality tests

Normality tests examine the distribution properties of data. These tests compare the data set with normal distribution. Fisher and Jordan (1991) suggest that the distribution of random occurrence should follow the normal distribution pattern. Therefore, if the changes in returns follow normal distribution pattern then these are random. To test normality of the data following test are used.

Jarque-Bera test

Most tests of normality are based on comparing the empirical and theoretical normal cumulative distribution or empirical and theoretical quantiles. Whereas, Jarque-Bera (1982, 1987) test check the normal distribution of skewness and kurtosis and it is a test of goodness of the fit. This test is defined as,

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \quad (1)$$

Where,
n= Number of observations
S= Skewness
K= Kurtosis

Kolmogorov-Smirnov test

Kolmogorov (1933) forms asymptotic distribution in Kolmogorov-Smirnov (KS) test and Smirnov (1948) gives the table of distribution for this test for null hypothesis. KS test is used to compare the empirical and theoretical normal cumulative distribution. This test is defined as,

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{X_i \le x} \quad (2)$$

Where,
Fn = Distribution function
n = Independent and identically distributed random observations
$I_{X_i \le x}$ = Indicator function, equal to 1 otherwise 0

The KS statistic cumulative distribution for given function is

$$D_n = sup_x |F_n(x) - F(x)| \quad (3)$$

Where,
D_x = cumulative distribution function
sup_x = supremum

If sample is taken from a distribution F(x), under the assumption of Glivenko Cantelli theorem then $D_n$converges nearly to 0.

Autocorrelation test

Autocorrelation test is widely used to test the relationship of the series return with its lag value. If there exists, a positive and significant autocorrelation in series then it indicates that trend exist. If there is a negative and significant autocorrelation in series then it shows a reversal in price movement. A return series is called random if there is no autocorrelation exists. Two approaches are used in this study to test autocorrelation.

Parametric autocorrelation coefficient

The autocorrelation coefficient test is used to test the relationship between current and previous period returns. If there is zero autocorrelation coefficient then it assume, that this return series follow random walk. This test is defined as,

$$R_{i,t} = \alpha_i + p_j R_{i,t-k} + \varepsilon_{i,t} \quad (4)$$

Where,

$R_{i,t}$ = stock return for a stock "i" at time t

α_i = constant

$\epsilon_{i,t}$ = random error

k = various time lags

To test autocorrelation two tests are used as detailed below.

Autocorrelation function and Q-Ljung Box test

This test is used to examine the relationship of current returns with its different lag returns. Ljung-Box (1978) test is also used to test the overall randomness on the basis of number of lags, rather testing randomness at each different lag.

$$Q_{Ljung-Box} = n\,(n+2) \sum_{t-1}^{k} \frac{\Psi^2(t)}{n-1} \quad (5)$$

Where,

n = number of usable data points after any differencing operations.

Ψ = accumulated sample autocorrelations up to any specified time lag *t*

Non Parametric Run test

Runs tests are 'a succession of identical symbols which are followed or preceded by different symbols or no symbol at all' (Siegel, 1956). Run test measures the serial independence in return series. It does not require the series returns are normally distributed. The run test is based on the argument that there is a random trend in the price changes (returns changes) than the numbers of expected runs necessarily close to the numbers of actual runs (Runs). It is also noted that test statistic is regarding normally distributed for bigger sample size. The formula for runs tests has been given by Wallis and Roberts (1956) as.

$$Z = \frac{(U - U_{\mu})}{\sigma_{ti}} \quad (6)$$

Where,

$$U_{\mu} = \frac{2m+m_{-}}{m} + 1 \text{ and } \sigma_{ti} = \sqrt{\frac{2m+m_{-}-(2m+m_{-}-m)}{m^{2}(m-1)}} \quad (7)$$

The 'positive returns' (+) are reflected by +m and the entirety of 'negative returns' (-) are reflected by –m concerning to a sample by means of observations 'm', where m= m+ + m_.

Unit root tests

This test is applied to check whether financial time series is stationary or non-stationary. It is a necessary condition to apply this test for the confirmation of RWM. Means and variance must be constant over time, if a data is stationary (Gujarati, 2008). To test unit root for a time series this study has used two tests i.e. (i) ADF test and (ii) Phillips- Perron test.

Augmented Dickey-fuller test

Dickey and fuller (1979) test assumes that variance of the time series is constant and error term is independent. A simple autoregressive model, AR (1) is,

$$y_t = \rho y_{t-1} + u_t \quad (8)$$

Where,

y_t = variable of interest for time period index t

ρ = coefficient

u_t = error term

The auto-regression model is as under:

$$\Delta Y_t = \alpha_0 + \alpha_1 + (\rho - 1)Y_{t-1} + \sum \varphi_{i\,\Delta X^{n}_{i=1}t-1} + \varepsilon_t \quad (9)$$

Where,

Y = Natural logarithm

T = Linear time trend term

ρ, φ = Parameters

Δ = Operator for first-difference

ε_t = error term.

Phillips Perron test

Phillips and Perron (1988) have provided a substitute (non-parametric) technique for serial correlationfor unit root. This test assumes that error term is not independent and isheterogeneously distributed. The PP testis instituted on the subsequent regression with same critical values used for ADF:

$$\Delta Y_t = \lambda_0 + \lambda_1 Y_{t-1} + \lambda_2 T + \sum_{i=1}^{n} \psi_i \Delta Y_{t-i} + \epsilon_t \quad (10)$$

Where,

Y_t = Given time series

T = Time
λ and ψ = Parameters
Δ = Operator for first-difference
ε_t = Error term

Multiple Variance ratio test
Chow and Denning (1993) propose MVR test, to examine the heteroskedasticity and autocorrelation in the financial series under the assumption of varying distribution. The variance ratio model is symbolized by:

$$VR(q) = \frac{\sigma^2(q)}{\sigma^2(1)} \quad (11)$$

Where,
$\sigma^2(q)$= 1/qth variance
$\sigma^2(1)$ = First differences variance
VR (q) =1 for null hypothesis

Lo and MacKinlay (1988) propose two tests, first Z(q) and the other is Z*(q)under the null hypothesis of 'Homoskedastic increase random walk' and 'Heteroskedastic increase random walk'. Z(q) test for Homoskedastic assumption is as under:

$$Z(q) = \frac{\{VR(q)-1\}}{\sigma_0(q)} \quad (12)$$

Where,

$$\sigma_o(q) = \left[\frac{\{2(2q-1)(q-1)\}}{3q\,(nq)}\right]^{1/2} \quad (13)$$

$Z^*(q)$ test statistic for Heteroskedastic assumption is;

$$\frac{Z*(q)=\{VR(q)-1\}}{\sigma\sigma\,(q)} \quad (14)$$

Where,

$$\sigma_o(q) = \left[4\sum_{k=1}^{q=1}\left(1-\frac{k}{q}\right)^2 \delta_k\right]^{1/2} \quad (15)$$

and

$$\delta_k = \frac{\sum_{i=k+1}^{mq}(p_k - p_{k-1} - q\hat{u})^2(p_{i-k} - p_{i-k-1} - q\hat{u})^2}{[\sum_{i=1}^{mq}(p_k - p_{k-1} - q\hat{u})^2]} \quad (16)$$

MVR test of Chow and Denning's (1993) compares multiple comparisons of different set of variance ratio estimates by generating a procedure for various assessments with unity.

Random walk null hypothesis is rejected if any one H_0i is rejected. The spirit of the multiple variance ratios (MVR) projected by Chow and Denning's (1993) is stood on the result:

$$PR\{\max(1Z(q_1)1 \ldots\ldots (1Z(q_m)1) \leq SMM(\alpha; m; T)) \geq 1-\alpha\}$$

Under the homoskedasticity the refusal of the random walk is due to either the presence of autocorrelation in the series of stock prices and/or due to heteroskedasticity. Therefore, it is the confirmation of autocorrelation in the series of stock returns if there is refusal of 'heteroskedastic random walk'.

Data Analysis and Empirical results

The statistical behavior of financial time series of daily returns, weekly returns and monthly returns for the period of 2002 to 2012 is presented in Table 1.

Table 1: Descriptive statistics for the period of 2002-2012

Statistic	Monthly Returns	Weekly Returns	Daily Returns
Mean	0.017176	0.003947	0.000827
Median	0.020202	0.008453	0.001342
Standard Deviation	0.082646	0.035050	0.014401
Kurtosis	10.60601	8.228005	5.306997
Skewness	-1.712556	-1.407613	-0.387269
Minimum	-0.448796	-0.200976	-0.060418
Maximum	0.202276	0.109173	0.088254

Table 1 reports the descriptive statistics for the Karachi stock market returns. Descriptive statistics shows that the average daily returns are 0.08 % and the Average standards deviation is 1.4401 %. While the average weekly returns are 0.3947 % and the standards deviation is 3.505 %. Likewise, the average monthly returns are 1.7176 % and the standard deviation for the monthly returns is 8.2646 %. Descriptive statistics results show that all returns are negatively skewed for sample period, which clearly specifies that large negative returns (minimum extreme values) are dominant than higher positive returns (maximum extreme values). The values of the kurtosis for all returns series are greater than 3 which suggest that all return series are leptokurtic means data is peaks than the normal distribution.

Normality Tests

Jarque-Bera test

The results of Jarque-Bera tests are reported in the table 2 given below. Moreover, table 2 reports both calculated values critical values.

Table 2: Jarque-Bera test

	Monthly Returns	Weekly Returns	Daily Returns
JB (Observed	345.0143	758. 0371	610.4747

Value)			
JB (Critical Value)	5.991	5.991	5.991
p-value	0.0000[a]	0.0000[a]	0.0000[a]

Note[a]: *Indicates that null hypothesis of normality assumption is rejected at 1% significance level*

The observed value in the daily data is greater than the critical value. Similarly in case of both weekly and monthly data the observed values of Jarque-Bera are higher than that of critical values. The results of all returns series rejected the normality assumption.

Kolmogorov-Smirnov (KS) test

In order to identify the difference of the underlying probability distribution from a hypothesized distribution the Kolmogorov-Smirnov (KS) test is used. Results of the KS test are presented in the table 3.

Table 3: One-Sample Kolmogorov-Smirnov Test

		Daily Returns	Weekly Returns	Monthly Returns
N		2474	516	119
Normal Parameters[a,b]	Mean	.000826	.0039471	.01717565
	Std. Deviation	.0144011	.03504968	.08264589
Most Extreme Differences	Absolute	.095	.125	.120
	Positive	.061	.077	.077
	Negative	-.095	-.125	-.120
Kolmogorov-Smirnov Z		4.737	2.837	1.312
Asymp. Sig. (2-tailed)		.000*	.000*	.064

[a]. *Test distribution is Normal.*
[b]. *Calculated from data.*
**indicates 1% significance level*

Results shows that the p-value for the monthly returns series (p-value = 0.064) is greater than critical value which means monthly data is not normally distributed, but at 90% level of confidence it is normally distributed. Whereas, weekly and daily returns series has (p-value = 0.000) that directs to the rejection of the normality of data.

Autocorrelation function test and Q test

The autocorrelation coefficient function is calculated up to 10 lags and statistical significance is reported.

Table 4: Autocorrelation and Q-statics returns

Lags	1	2	3	4	5	6	7	8	9	10
Daily Returns										
AC	0.12	0.03	0.06	0.04	0.01	0.01	0.03	0.00	0.05	0.05

Q-Stat	38.02	39.85	48.45	52.38	52.84	53.18	55.62	55.65	61.81	68.97
Prob	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
Weekly Returns										
AC	0.16	0.07	0.14	0.08	-0.06	-0.06	-0.01	-0.02	0.00	0.01
Q-Stat	12.88	15.47	25.74	29.07	31.01	33.00	33.03	33.25	33.25	33.28
Prob	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
Monthly Returns										
AC	0.17	-0.07	-0.04	0.09	0.11	0.05	0.04	-0.02	0.04	-0.06
Q-Stat	3.54	4.20	4.44	5.42	6.97	7.27	7.42	7.50	7.72	8.24
Prob	0.06	0.12	0.22	0.25	0.22	0.30	0.39	0.48	0.56	0.61

** indicates 1% significance level*

The results of Table 4 indicate that there exists auto correlation. So, it can be said that daily and weekly returns Pakistani equity market does not follow random walk. But for monthly returns the null hypothesis is accepted and no correlation exists for any lag.

Non-Parametric Run test

Run test measures the serial independence in return series whether succeeding price changes have certain trend or these series are autonomous to each other.

Table 5: Runs Test for daily weekly and monthly changes

	Monthly Returns	Weekly Returns	Daily Returns
Test Value[a]	.01717565	.003947137	.00082670
Cases < Test Value	57	216	1189
Cases >= Test Value	62	300	1285
Total Cases	119	516	2474
Number of Runs	50	210	1152
Z	-1.917	-3.817	-3.389
Asymp. Sig. (2-tailed)	.055	.000**	.001**

Note:[a]. Mean

Z- Statistics is ≥1.96 then we cannot be accepted null hypothesis at 5% significance level

*** indicates 5% significance level*

Table 5 shows that the monthly returns are insignificant and p-value is greater than critical value (.055>0.05), which means no autocorrelation exists in monthly returns. The p-value for the daily and weekly returns is less than 0.05 which rejects the null hypothesis of randomness implying that there is an autocorrelation in daily and weekly returns. Similarly, in case of daily, weekly and monthly returns the experimental numbers of runs do not drop within the studied interval at, so the null hypothesis of randomness can be discarded, entailing that few species of autocorrelation exists in the daily returns and weekly returns.

Unit Root Test

In order to understand whether the presentation of the index of the stock market is stationary or not, the ADF and PP tests are used. Table 6 presents the results of Augmented Dickey Fuller test at level and 1st difference for all return series.

Table 6: Augmented Dickey Fuller test at level on KSE-100 Index

ADF test statistic	*Monthly Returns*	*Weekly Returns*	*Daily Returns*
Level	-0.818	-0.915	-0.808
1st difference	-9.973*	-21.291*	-46.080*
Critical value a 5%	-2.884	-2.866	-2.862
Critical value a 1%	-2.579	-2.569	-2.567

* *indicates 1% significance level*

All the reported values in Table 6 show that ADF test statistic results at level are less than critical or tabulated values. But all the reported values show that data is stationary at first difference. Hence, data is non-stationary at level.

Phillips Perron Test

Phillips Perron test which is an alternative test is used that permit the error conflicts to be weakly reliant and heterogeneously distributed. Table 7 presents the results of PP test statistic test at level and 1st difference for all return series.

Table 7: Phillips Perron test at level on KSE-100 Index

PP test statistic	*Monthly Returns*	*Weekly Returns*	*Daily Returns*
Level	-0.923	-0.976	-0.862
1st difference	-9.945*	-21.398*	-46.309*
Critical value a 5%	-2.884	-2.866	-2.862
Critical value a 1%	-2.579	-2.569	-2.567

** indicates 1% significance level*

All the reported values show that PP test statistic results are greater than critical or tabulated values. Hence, data is stationary at level.

Multiple Variance Ratio Tests

MVR tests are employed with the assumption of heteroscesdicity as well as with the assumption of homoscesdicity. First the null and the alternative hypothesis for MVR tests under the assumptions are presented in table 8.

Table 8: Results of multiple variance ratio tests (Heteroscedasticity)

	q	2	4	8	12	24	30	60
Daily	VR (q)	0.556	0.275	0.144	0.093	0.051	0.038	0.020
Returns	Z*(q)	-13.910*	-12.702*	-10.035*	-8.639*	-6.449*	-5.891*	-4.403*
Weekly	VR (q)	0.554	0.276	0.155	0.103	0.053	0.040	0.023
Returns	Z*(q)	-5.559*	-5.205*	-4.099*	-3.600*	-2.903*	-2.708*	-2.083*
Monthly	VR (q)	0.658	0.287	0.168	0.107	0.067	0.061	0.047
Returns	Z*(q)	-2.245*	-2.834*	-2.407*	-2.145*	-1.693	-1.556	-1.191

* *indicates 5% significance level*

In the table 8 results clearly shown that daily and weekly returns Z^* (q) statistic is significant for all periods. In case of monthly stock returns it is shown that for monthly returns Z^* (q) statistics is significant for q=2, q=4, q=8, q=12 periods. This significance of the variance ration showed that the null hypothesis of the random walk i.e. monthly stock returns follow random walk is rejected for all periods (q) under heteroscesdicity.

Results of multiple variance ratio tests (Homoscedasticity)

MVR tests are employed with the assumption of heteroscesdicity as well as with the assumption of homoscesdicity. First the null and the alternative hypothesis for MVR tests under the assumptions are presented in table 8. The results of null and the alternative hypothesis for MVR tests under the assumptions of homoscesdicity are fare presented in table 8.

Table 9: Results of multiple variance ratio tests (Homoscedasticity)

	q	2	4	8	12	24	30	60
Daily	VR (q)	0.556	0.275	0.144	0.093	0.051	0.038	0.020
Returns	Z(q)	-22.099*	-19.282*	-14.392*	-12.029*	-8.609*	-7.755*	-5.518*
Weekly	VR (q)	0.554	0.276	0.155	0.103	0.053	0.040	0.023
Returns	Z(q)	-10.127*	-8.782*	-6.483*	-5.432*	-3.923*	-3.535*	-2.511*
Monthly	VR (q)	0.658	0.287	0.168	0.107	0.067	0.061	0.047
Returns	Z(q)	-3.710*	-4.143*	-3.057*	-2.586*	-1.850	-1.654	-1.172

* *indicates 5% significance level*

In the above table 9 the results show that Z^* (q) statistics in daily and weekly is significant for all periods. This significance of the variance ratio showed that the null hypothesis of the random walk i.e. daily and weekly stock returns follow random walk is rejected under homoscesdicity. In case of monthly returns the standardized VR (Variance ratio) test statistics for Z^* (q) is significant for q=2, q=4, q=8, q=12, periods. This significance of the variance ration showed that the null hypothesis of the random walk i.e. monthly stock returns follow random walk is rejected for all periods (q) under homoscesdicity. The results of the powerful variance ratio test statistics are also described in the form of graph under the assumption of homoscesdicity.

Conclusion

This study examines the weak from of efficiency in Pakistani stock market. The purpose of this study is to test random walk model for daily, weekly and monthly returns. If the changes in series follow normal distribution pattern then it is called random. To test normality of data Jarque-Bera and KS test is used and result reveals that daily, weekly and monthly returns are not normally distributed. Therefore, results suggest that there is a predictability element for returns. A return series is called random if there is no autocorrelation exists. Then autocorrelation and Run test is used for autocorrelation coefficient. The results of autocorrelation functions and Q-Ljung box statistics reject the null hypothesis and confirm that there exist autocorrelation in daily and weekly returns. We can say for daily and weekly returns Pakistani market does not follow random walk. But for monthly returns the null hypothesis is accepted and no correlation exists for any lag. Run test also confirms the same results of autocorrelation for daily, weekly and monthly returns series. This study also tests stationarity of the financial time series by using unit root tests. A necessary condition for random walk is that a financial time series should be non-stationary. ADF and Phollips-Perronare used for unit root, both tests report that daily, weekly and monthly returns are stationary at level. Finally, with both assumptions of heteroscesdicity as well as homoscesdicity MVR test is used. The results of MVR test reveal that series does not follow random walk. These results are consistent with Kamal and Rehman (2006), Hassan et. al. (2007) and Hamid et. al. (2010). Therefore it is concluded that investors have an opportunity to get benefit from the predictable behavior of this market.

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