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Inflation Welfare Cost Analysis for Pakistan: An ARDL Approach

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Abstract

Substantial changes in price inflation rate have serious economic consequences on the welfare of consumers in an economy but the issue has not, thoroughly, been reckoned in Pakistan. The study aims at assessing the nature and extent of welfare cost of inflation (WCI), employing well-established theory of money demand for the purpose. To address the issue more deeply, high frequency biannual series of gross domestic product (GDP) have been constructed and used benefitting from earlier works, a la Arby (2008). Biannual Divisia monetary aggregate (DMA) M2 series has, specifically, been constructed along with simple sum monetary aggregate (SSMA) M2 to establish the adequacy of DMA over SSMA in measuring the WCI in the long run based on versatile money demand specifications estimated using autoregressive distributed lag model (ARDL) framework. Short-term interest rates are regressed on money income ratios for Pakistan as implied by the theory. Based on information criteria, it is shown that DMAs rendered better fit and reliable estimates and log-log specification better approximate money demand than theirs counterparts. It is found that a decrease in inflation rate from 15 to 5 percent amounts to an annual welfare gain in the range of rupees 105 to 118 billion with log-log specification entailing one of the DMA or SSMA. These estimates of WCI are actually much less than the true inflation costs borne by an economy as moral inflation costs such as corruption, suicide attempts due to joblessness, political instability, currency devaluations etc. cannot be accounted for due no concrete measurability for these attributes. The study concludes that price stability on lower prices inflation rate must be the prime goal in monetary policy, and if the government spends a few billion rupees on price stability, it will gain too much not only in lowering the rampant corruption but also in gain political, moral, and financial stability. The study recommends the construction, publication, and use of high frequency DMAs and GDP series by the SBP that will opens many avenues for further improvements in evaluating WCI.

Keywords: Welfare cost of inflation (WCI), Divisia monetary aggregate (DMA), simple sum monetary aggregate (SSMA), log-log form, autoregressive distributed lag (ARDL), Pakistan.

1. Introduction

Substantial changes in price inflation rate have serious economic consequences on the welfare of consumers in an economy but the issue has not, thoroughly, been addressed in Pakistan. In an economy, inflation is referred to a rise in the general price level of goods and services over a span of time. Price inflation is measured by the inflation rate, which is defined as the percentage change in a general price index over a time period. GDP-Implicit Price Deflator (GDP-IPD) measures the level of the price of all the goods and services included in GDP. A rise in inflation rate lowers the purchasing power of the currency. Accordingly, real value of the unit of account and medium of exchange in an economy is lowered causing diverse influences of inflation. Specifically, as a unit of account, money is the most useful if economic agents envisage and compute in nominal terms. Furthermore, stable prices improve the fluency of the system of prices such that variations in relative prices can be distinguished without being perplexed by variations in the general price level. Economic agents get marginal utility from holding cash balances is manifested by theory of Money demand. In many monetary models with potent role of money, welfare is maximized at rate of inflation, which is sufficiently low, so that the nominal interest rate is zero (Wolman, 1997). If the money market distortions are considered, it should be equivalent to offsetting the real rate of interest. "The optimum quantity of money that will be attained by a rate of price deflation, which makes the nominal rate of interest equal to zero", stated by Friedman (1969, 34) and is attributed to his name 'the Friedman rule'.

Under the nominality principle, the credit, wage contracts, tax laws etc. rules that define monetary payments are characteristically build upon nominal rather than real terms. Unanticipated and anticipated inflation, each will be capable of generating real imbalances as the real sense of a nominal contract will change with the change in price level. In principle, these imbalances can be evaded by time to time adjustment of the rules of payment, which involves menu costs and shoe-leather. At zero nominal rate of interest realizes the resource allocation efficiency in perfectly competitive environment at which inflation caused distortions are minimum. Therefore, the inflation rate that achieves the zero nominal interest rate is the optimum inflation rate. Higher than the optimum inflation level, the nominal interest rate surpasses zero to deprive off Pareto optimality of allocations thus dropping social welfare, which incurs welfare cost. If these costs were large and known to policy makers, they would certainly opt for price stability at low inflation rate in an economy.

The problem was how to estimate these costs as money neutrality posed a major obstacle in the way. For a long period, the old saying of money neutrality hampered this intense analysis. It was the Friedman rule that paved the way for profound analysis setting aside the classical dictum monetary neutrality. Martin Bailey (1956) was the first to suggest that holding money has an opportunity cost given by some interest forgone such that at zero nominal interest rate (optimum inflation level) the cost is zero, but as the as the rate of inflation surpassed the optimum level, the cost occurs as inflation reduces welfare. Bailey has suggested that this cost could be measured by computing the area beneath an inverse money demand curve from zero to some positive interest rate, called Bailey's triangle. This tringle determines the inefficiency of holding money whenever nominal interest rates are positive. The main objectives of the study are:

- Set up and identify best a money demand function for measuring welfare cost of inflation based on two types of aggregates: SSMAs and DMAs.
- From the two functional forms; the log-log and the semi-log debated in the regularly literature on WCI.
- From the four models strike out bad fit models or otherwise strike out bad aggregate or the bad functional form based on standard criterions.
- From the best selected models measure the welfare cost for three scenarios at 5%, 10%, and 15% rates of inflation in billions of rupees using ARDL approach to cointegration.
- To provide a policy implication for SBP regarding construction and dissemination of DMAs and GDP series at higher frequency for evaluation of WCI.
- To provide a policy implication for SBP regarding the use of monetary aggregates as 'nominal anchor' vs resort to inflation targeting.

To address these objectives, literature is reviewed in the next section, in section three methodology is outlined, section four comprise of results and discussion while conclusions and policy implications are presented in fifth section and finally references are given at the end.

2. Review of Literature

The simple-sum monetary aggregation assumes equality of all financial assets thereby allocating unity weight over time which is not the true rationale for the underlying phenomenon (Drake and Mills, 2002, 2005; Drake and Fleissig, 2004). So, SSMA is based on balance sheet or accounting principles. The SSMA is only justified if all constituents are perfect substitutes (Barnett, 1984). Yu and Tsui (2000) conclude that SSMAs cannot confirm the assumption of perfect substitution due to recent financial developments that resulted in varieties of interest-bearing assets. Each of these assets performs different degrees of monetary services, so these should be weighted by degree of their 'services render as money' in obtaining an appropriate monetary aggregate as pointed out by Friedman and Schwartz (1963). Further developments was devoted to inclusion of the feasibility of alternative weighting schemes, but a weighing which later became a standard called Divisia monetary aggregate was pioneered and provided by Barnett (1978). DMAs excel SSMAs in two aspects. First, when the demand specifications for the various assets are recognized, closed-form solutions can be reached at a lower algebraic price i.e. only line integrals to be solved. Second, in case the demand functions for the various assets are unknown, or expensive to get, approaching a welfare measure by use of DMAs permits the investigator to rank different sets of opportunity costs straight from market observed data of interest rates and SSMAs (Cysne, 2000).

It is emphasized that the components of a monetary aggregate be weakly separable. Several researchers—Jones et al. (2005), Drake and Fleissig (2006), and Elger et al. (2008) among others— have successfully applied tests to confirm whether specific components of financial assets were weakly separable. As DMI is based on theories of approximation and revealed preference —straightforwardly derived from the problem of utility maximization assuming weak separability and linear homogeneity— it is able to trace the unknown inherent aggregator function precisely. Finally, as the DMAs are derived from index number theory, it offer specification and parameter free estimation of aggregator functions, consequently are independent of the one's choice of specification.

The growing body of confirmations from empirical investigations across the world support the use of DMA for better estimation of welfare cost. In a detailed study, carried out by Serletis and Virk (2006), examined the inflation welfare effects with help of the three different monetary summation methods SSMAs, DMAs, and Rotemberg's currency equivalent aggregates to provide a comparison among the aggregation procedures, at four distinctive orders of monetary summations. They concluded that DMAs, which envisaged differences in opportunity costs with precisely quantifying the financial services provided by the interest bearing constituents, resolved a reduced welfare cost as compared to SSMAs and the currency equivalent aggregates.

For Pakistan, Tariq and Matthew (1997) estimated, after testing for cointegration, an efficient SEECM of money demand (as weak exogeneity was significant for real income and opportunity cost variables) using DMA and SSMA M_2 and M_1 for the period 1974:Q4–1992:Q4, for almost 73 quarterly time points. Yet, the efficient SEECM estimation of the short-run dynamics of all the specifications of money demand remained doubtful due to the existence of GC from money to income, which negated strong exogeneity of income. Little evidence of the superiority of DMAs for the period was found. The stability of demand for money function was not explored. Another study estimated dynamics of money (M_2) demand by using cointegrated SEECM for the period from 1960 to 1999, i.e. 40 annual time points for Pakistan, it disclosed that bond yield, interest rates, and call money rates were the key factors affecting money demand behaviour in the long run, but inflation rate emerged as the most important one. While inflation rate and real income were main determinants in the short-run. It was established that the inflation rate fluctuations were insignificant to M_2 growth (Qayyum, 2005). Sarwar et al. (2010) estimated both long and short run money demand and validated the stability only of Divisia M_2, using ARDL approach to cointegration based on time series of real GDP, financial innovation, DMAs constructed for M_0, M_1, M_2 with their price duals for time span of 1972-2007 for Pakistan. They found the use of DMAs was more realistic, these generated more information contents, and that SBP should replace SSMAs with DMAs. Omer (2010) examined steadiness of money velocity on which depends the stability of demand functions, which is a pre-requisite for ongoing monetary targeting strategy of central bank in Pakistan. He did not find support for those arguing that the central bank should abandon the monetary targeting monetary policy strategy against inflation targeting. Hence, he validated the use of monetary aggregates as 'nominal anchor'. The results, based on the annual data starting from 1975 to 2006, using ARDL approach to cointegration, further confirmed that relationship of all three velocities was stable with their determinants.

However, all earlier mentioned studies lacked capturing structural changes in the cointegrating vector. Given the monetary policy and political instability that occurred in

Pakistan during the study period, it would be reasonable to allow for the presence of structural changes that might have affected the money demand. Some of the studies entailing step dummies are Dahmardeh and Izadi (2011) for Iran and Rao and Kumar (2011) for the US, among others.

In macroeconomics, by definition, the WCI relates to the analyses of aggregate changes in social welfare of economic agents caused by changes in general price level. Lucas (2000) has exposed that Bailey's framework for measurement of WCI can be judged as a close estimate to the GE framework. Lucas (2000) assesses precise measurements of the utility gains via the CS for log-log and log-linear forms. For the hundred years of interest rates series for US, his assessments neared the exact CV with the both forms. Lucas' calculation was two times the computations of Bailey's and others due to use of a log-log form, demonstrating a better fit to the data. The Lucas's log-log form assessed larger welfare gains due to the fact that cash holdings stock expands for all time as the rate of interest tends to zero as opposed to the log-linear form that presumes a limited cash balances stock is held when the nominal interest reduces to zero. Lucas favours a log-log demand function because it better fits the American data. Applying the growth of the revenues as product of real money M/P and ρ i.e. $(M/P)\times(\rho)$, the revenues R $[=\rho A\rho^{-\eta}=A\rho^{(1-\eta)}]$ constantly grows with inflation. The money-demand data that extends from 1900 to 1994, he computes the inflation welfare cost using an interest rate of six percent as about 1.2% of GDP. He himself did not trust his exercise because if the specifications of the money demand functions as consequence of the theoretical models are not excellent approximations of the true shape of money demand in the range of near zero interest rates, the correct welfare cost of inflation may differ drastically from the 1.2 % figure (Lucas, 2000).

After the Lucas, a new spur in this area of research occurred and long lists economists estimated it for different economies though earlier studies mainly focused on US economy. Among others these costs are measured by Ireland (2009), Calza and Zaghini (2011), Silva (2012), Chiu and Molico (2010) for the US; Yavari and Mehrnoosh (2005) for Iran; Chen and Ma (2007) for China; Gupta and Uwilingiye (2008, 2009a, 2009b, 2010) for South Africa; Yan (2009) for ASEAN-5 and Canada; Serletis and Yavari (2005) for Italy; Serletis and Yavari (2007) for Euro zone; Yavari and Serletis (2010) for Latin America; Boel and Camera (2011) for twenty-three defferent OECD countries. Pakistan economy, plagued with moderate to high inflation rate, certainly requires a thorough treatment for the assessment with regard money demand specification and used of monetary aggregation. New and interesting estimates are expected to be generated by the kind study that can provide new sense and insights for policy implications.

3. Data and Methodology

3.1 Deriving the DMAs

In DMAs the components included for the aggregation must be weighted in terms of the monetary services they offer. Thus, currency and zero interest deposits and get the maximum weight as these amounts to largest user costs and vice versa (Binner et al., 2010). Hence, the growth rate of DMA equals the summation of the spending portion weighted growth rates of its constituents. Measures of these spending portions are dependent upon the quantifications of the opportunity cost of the financial assets, which equals the magnitude of deviation between the financial assets' own return rates and the return rate on a standard asset termed as "benchmark" asset—an asset not offering any financial services in the present period.

DMAs are founded on the assumption of microeconomic model of the economic agents' decision making who try to maximize their utility subjected to the budget constraints. The total expenditure Y on monetary assets at time t will be:

$$y = \sum_{i=0}^{n} \pi_{it}\, m_{it} \tag{1}$$

The budget constraints is the aggregate of products of π_{it} and $\overline{m}_{it}$, where π_{it} is the user cost (the opportunity costs of holding monetary assets, where the interest rate represents the liquidity of the monetary asset) of monetary asset i at time t. These are termed as the differentials in interest rate between the benchmark asset's and the monetary asset's own return rates (Barnett, 1978);$\overline{m}_{it}$ is the stock of financial asset 'i' in optimum aggregate at time 't'. The expenditure share on financial asset 'i' at time 't' is the total user cost of the optimal financial aggregates divided by the total expenditure:

$$S_{it} = \frac{\pi_{it}\overline{m}_{it}}{Y_t} \tag{2}$$

The DMI has the nice property such that its logged deviations are a weighted means of the logged deviations its constituents. The user costs π_{it} can be measured by:

$$\pi_{it} = \frac{\overline{p}_t(R_{it} - \rho_{it})}{1 + R_{it}} \tag{3}$$

Where R_{it} is the benchmark (is the highest rate of return of a riskless financial asset providing no financial services) rate and ρ_{it} is the rate of return of an asset i at time t and $\overline{p}_t$ is the GDP-IPD. Barnett (1980) advocated for Divisia quantity index to build the DMA as follow:

$$DMA_t = DMA_{t-1} \prod_{i=1}^{n} \left(\frac{\overline{m}_{it}}{\overline{m}_{i,t-1}}\right)^{\bar{S}_{it}} \tag{4}$$

$$\text{Growth rate (DMA)} = \sum_{i=1}^{n} \bar{S}_{it}\, GR(\overline{m_{it}}) \tag{5}$$

$$\text{Where} \quad \bar{S}_{it} = \frac{1}{2}\left(S_{it} + S_{i,t-1}\right) \tag{6}$$

In continuous time, the DMI is exact for any utility function that is linearly homogeneous.

3.2 Money Demand Specification

Generally, money demand relates the quantity of money demanded with a group of dynamic economic indicators. According Lucas (2000), with the assumption that the economic agents do not face money illusion problem, the money demand specification is as follow:

$$M_d/P = f(Y, \rho) \tag{7}$$

Where M_d is the real money demand (aggregated in simple sum, and DMA) i. e. money demanded divided P, the price level GDP-IPD and Y is the real income level (real GDP). Other variable that affect money demand is 'ρ.'

The welfare measures used by both Bailey (1956) and Lucas (2000) were developed for monetary aggregates, which pay no interest, by considering only the M_1 as the pertinent monetary total. But the fact that had been clearly found in literature for last two decades of the past century that there were several interest-bearing deposits providing monetary services contained in M_1 time series aggregates, was ruthlessly ignored (Cysne, 2010)**.** In the vein of Lucas (2000), Serletis and Yavari (2005) and Serletis and Virk (2006), supposing a double log specification for the demand for money as given below.

$$Z = A\rho\eta$$

With ρ as the nominal interest rate, Z= m/y is the ratio of real money stock (Md /P) to real income y, and η as the interest elasticity of real money demand per unit of income. Let us define the WCI as $w(\rho)$; the income recompense required by the economic agent to make him indifferent between two states: to remain in a steady state with a constant rate of interest at $\rho>0$ or another similar stable state with $\rho=0$ (Lucas, 2000; Sidrauski, 1967). Considering a homothetic present time duration utility function, and envisaging a relevant dynamic optimization objective function, WCI can be assessed using the following equation:

$$\log(Md/P)t = \log f(Y, \rho)t + \varepsilon t \qquad (8)$$

3.3 The Consumer Surplus (CS) Approach

Bailey (1956) envisaged the inflation cost as a loss of consumer surplus, which could be obtained from the reduction of ρ from $\rho>0$ to $\rho=0$. The nominal interest rate represents for a consumer a private opportunity cost of holding cash instead of deposit. An implicit assumption here is that the foregone interest rate is justified with the benefits brought by holding currency in terms of transaction-facilitating services. Any rise in ρ (that reflects the rise in inflation rate) induces a corresponding fall in money demand and a decline of the benefits yielded by cash. This flow of productivity is associated with the area under the curve of "liquidity preference function" relating demand for real cash balances, m, to nominal interest rate, ρ. More precisely, the area is calculated via integration under the inverse function of money demand [money demand inversely related to ρ, the opportunity cost of money]. Considering (8) as money demand specification:

$$M/P = f(\rho, y) \text{ or } m = f(\rho, y), \quad \text{where } M/P = m.$$

Let the $f(\rho, y)$ is denoted by $F(\rho, y) = \Phi(\rho)y$, then equation (8) takes the form $m = \Phi(\rho)y$, with m being the stock of real money demanded. M/P as function of ρ interacted with Y.

$$m = \Phi(\rho)y = F(\rho, y), \text{ or } m/y = \Phi(\rho)$$

Writing $z = m/y = \Phi(\rho)$ entails demand for balances of real money per income unit as a function of ρ. Hence,

$$z = \Phi(\rho) = F(\rho),$$

in log-log form with all variables at t, time point. Particularly, based on Bailey's CS method, we can assess $z = F(\rho)$, invert it $\rho = \Phi^{-1}(z) = F^{-1}(z) = \Psi(z)$, and denote function of WCI by $w(\rho)$ so that

$$w(\rho)=\int_{\Phi(R)}^{\Phi(0)} \psi(z)dz = \int_{\Phi(R)}^{\Phi(0)} \Phi^{-1}(z)\,dz = \int_{0}^{R} \Phi(z)\,dz - \rho\,\Phi(\rho) \quad (9)$$

Above, $w(\rho)$ is the WCI given as a portion of income. Since any assessment of the WCI hinges on $\Phi(\rho)$, the function of demand for money employed. Traditional methods (Bailey1956, Friedman 1969) use a log-linear specification for $\Phi(\rho)$ but recent studies (Lucas 2000, Serletis and Yavari 2005, 2007; Serletis and Virk, 2006; Chen and Ma, 2007) employ double-log specification. Specifically, using a log-log money specification, we get

$$\ln \Phi(\rho)=\ln A - \eta \ln \rho, \quad (10)$$

Taking exponentiation of both sides, we get:

$$\Phi(\rho) = A\rho^{-\eta} \quad (11)$$

The WCI specification (9) takes the form:

$$w(\rho) = \int_{\Phi(R)}^{\Phi(0)} \psi(x)\,dx \ \int_{\Phi(\rho)}^{\Phi(0)} \Psi(x)dx = \int_{0}^{\rho} \Phi(x)dx - \rho\,\Phi(\rho) = \left[\frac{A\ x^{-\eta+1}}{-\eta+1}\right]_0^R - \rho A\rho^{-\eta} = \frac{A\ \rho^{-\eta+1}}{-\eta+1} - A\rho^{1-\eta}$$

$$= A\rho^{1-\eta}\left[\frac{1}{-\eta+1} - 1\right] = A\,\rho^{1-\eta}\left[\frac{\eta}{1-\eta}\right] \quad (12)$$

The WCI as a fraction of income is obtainable as follows:

$$w(\rho) = A\left[\frac{\eta}{1-\eta}\right]\rho^{1-\eta}$$

The constant $A>0$, prevailing ρ and $\eta>0$ is the interest elasticity of money demand in absolute terms if the variables are in levels and measure a change in the interest elasticity of money demand in absolute terms, if the variables are differenced. To estimate $w(\rho)$, one has to use an assessment of η and evaluate 'A' so that the ensuing curve touches the geometric means of the observations. But, with a log-linear specification for $\Phi(\rho)$,

$$\ln \Phi(\rho)=\alpha - \xi\rho, \quad (13)$$

where ξ in semi-log is equivalent of η in log-log form. By taking exponentiation of both sides, we get:

$$\Phi(\rho) = Be^{-\xi\rho}, \quad (14)$$

the WCI specification (9) takes the form given below by working on the same lines as in (12)

$$w(\rho) = \int_0^R \Phi(x)dx - \rho\Phi(\rho) = \left[\frac{B\ e^{-\xi x}}{-\xi}\right]_0^R - \rho B e^{-\xi\rho}$$

$$= \left[\frac{B\ e^{-\xi R}}{-\xi} + \frac{B}{\xi}\right] - \rho B e^{-\xi\rho} = \frac{B}{\xi}\left[1 - \frac{e^{-\xi\rho}}{-1} - \xi\rho e^{-\xi\rho}\right]$$

$$= \frac{B}{\xi}\left[1 - (1 + \xi\rho)e^{-\xi\rho}\right] \tag{15}$$

The WCI as a fraction of income is obtainable as follows:

$$\frac{B}{\xi}\left[1 - (1 + \xi\rho)e^{-\xi\rho}\right]$$

3.4 The Compensating Variation (CV) Approach

Lucas (2000) computes the decline in consumption in order to find amount recompense for (and quantify) utility gain from enlarged money balances, thus deriving an exact magnitude for the WCI using the Sidrauski (1967) approach. Starting with the Sidrauski (1967) approach with supposition of a state inhabited by very large number of agents with each having penchants (symbolized as t =0, at any random time) provided by very large (infinity) summation of all individual homothetic utility functions as:

$$u = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t) \tag{16}$$

with homothetic utility function for an individual

$$u(c,m) = \frac{1}{1-\sigma}\left[cf\left(\frac{m}{c}\right)\right]^{1-\sigma} \tag{17}$$

with σ =1. As homothetic utility function entails that the indifference curves slope (termed MRS) is dependent only upon m/c; the ratio of real balances to consumption. In the setting of equation (17), we have

$$\frac{\frac{\partial u(c,m)}{\partial c}}{\frac{\partial u(c,m)}{\partial c}} = \frac{\left[cf\left(\frac{m}{c}\right)\right]^{-\sigma} + \left[f\left(\frac{m}{c}\right) - f'\left(\frac{m}{c}\right) - \left(\frac{m}{c}\right)\right]}{\left[cf\left(\frac{m}{c}\right)\right]^{-\sigma} f'\left(\frac{m}{c}\right)} = \frac{f\left(\frac{m}{c}\right) - f'\left(\frac{m}{c}\right)\left(\frac{m}{c}\right)}{f'\left(\frac{m}{c}\right)}$$

which depends on the m/c ratio. Further, assuming, that every household has endowment only one time unit which having inelastic supply toward market to generate $y_t = (1 + \Gamma)^t y_0$ consumption good units in period t. Due to no storability of the consumption goods, an equilibrating stipulation occurs as

$$c_t = y_t = (1+\Gamma)^t y_0, \tag{18}$$

with Γ considered real growth rate not dependent upon financial policy. A cash flow restriction households faces is not in real terms in time t, given by

$$P_t y_t = P_t c_t + M_{t+1} - M_t + LST_t, \tag{19}$$

With entire lump-sum taxes are indicated by LST_t (or lump sum transfers, if $LST_t < 0$), if we divide (19) by P_t both sides; the real cash flow constraint for household can be expressed as under:

$$y_t = c_t + (1+\Pi_{t+1})m_{t+1} - m_t + lst_t, \tag{20}$$

the inflation factor $(1+\Pi_{t+1}) = P_{t+1}/P_t$ and $lst_t = LST_t/P_t$

Let a constant proportion of transfers to income, LST/y maintains a balanced growth steady path with constant rate of monetary growth $\mu_t = (M_{t+1} - M_t)/M_t$. This situation calls for the constancy of m/y so that the condition $1+\Pi = (1+\mu)/(1+\Gamma)$ satisfies. Suppose the value function $\tilde{v}(m, y)$ be the maxima of dynamic programming, objective function (16) of a household in such an equilibrium with m such that the total economy income touched y. Then Bellman (1957) equation is satisfied by $\tilde{v}(m, y)$

$$\tilde{v}(m, y) = \max_{c}\left\{\frac{1}{1-\sigma}\left[cf\left(\frac{m}{c}\right)\right]^{1-\sigma} + \beta\tilde{v}(m', (1+\Gamma)y)\right\} \tag{21}$$

with m' denotes real money balances, m_{t+1} for subsequent period's obtained in (20). The envisaged homotheticity simplifies $\tilde{v}(m, y)$ into single state variable function $z = m/y$, so a new function of value is defined v(z) as $v(m, y) = v(z)y^{1-\sigma}$. If $\Omega = c/y$ denotes choice variable for a household, then maxima of v(z) denoted as $\tilde{\mathbf{v}}(\mathbf{z})$ holds

$$\tilde{v}(z) = \max_{\Omega}\left\{\frac{1}{1-\sigma}\left[\Omega f\left(\frac{m}{\Omega}\right)\right]^{1-\sigma} + \beta(1+\Gamma)^{1-\sigma} v(z')\right\} \tag{22}$$

where $\acute{z}$ is value of next period's z_{t+1}, defined as:

$$\acute{z} = \frac{\acute{m}}{(1+\Gamma)y} = \frac{y - c + m - lst}{(1+\Pi)(1+\Gamma)\,y} = \frac{1 - \Omega + z - lst/y}{1+\mu}$$

The FOCs [attained by equating partial derivative of v(z) w. r. t. Ω to zero and evaluating along any equilibrium path $\Omega = 1$] for maximization of the (22), evaluated at c = y (so $\Omega = 1$) are

$$[f(z)]^{-\sigma}\,[f(z) - z\acute{f}(z)] = \acute{v}(\acute{z})/[1+\rho]; \tag{23}$$

And
$$\acute{v}(z) = [f(z)]^{-\sigma}\,\acute{f}(z)\acute{v}(\acute{z})/[1+\rho]; \tag{24}$$

With ρ defined by

$$1/(1+\rho) = [\beta(1+\Gamma)^{1-\sigma}]/(1+\mu) \tag{25}$$

Since z remains constant at balanced path, therefore $\acute{v}(z) = \acute{v}(\acute{z})$. Elimination of $\acute{v}(z)$ and $\acute{v}(\acute{z})$ in (23) and (24) provides

$$\rho = \acute{f}(z)/[f(z) - z\acute{f}(z)]. \tag{26}$$

If the value of z that satisfies (26) is denoted by $\Phi(\rho)$, it is the kind of relation at a stable equilibrium which Lucas termed it as 'money demand function.' Now, define $w(\rho)$ the WCI function of ρ, as the income required to recompense the household to make him/her indifferent in shifting between a steady state ρ constant and ρ constant at zero. Hence, $w(\rho)$ can be obtained by solving the next equality expressed as under:

$$U[(1+w(\rho))\,y, \Phi(\rho)y] = U[y, \Phi(0)y] \tag{27}$$

For (17), the equality in (27) can be reduced by rewriting it as

$$\frac{1}{1-\sigma}\left\{[1+w(\rho)]yf\left[\frac{\Phi(\rho)y}{[1+w(\rho)]\,y}\right]\right\}^{1-\sigma}=\frac{1}{1-\sigma}\left\{yf\left[\frac{\Phi(0)y}{y}\right]\right\}^{1-\sigma}$$

Rearranging, we get.

$$[1+w(\rho)]f\left[\frac{\Phi(\rho)}{[1+w(\rho)]}\right]=f\,[\Phi(0)] \tag{28}$$

The $\Phi(\rho)$ provides the empirical amount of $w(\rho)$. Let $\rho = \Psi(z)$ be the inverse of a given $\Phi(\rho)$ which is put into equation (26) to achieve differential equation of the form

$$\acute{f}(z)=\frac{\Psi(z)}{1+z\Psi(z)}f(z) \tag{29}$$

The derivative of (28) w. r. t. ρ, provided

$$\acute{w}(\rho)f\left[\frac{\Phi(\rho)}{1+w(\rho)}\right]+\acute{f}\left[\frac{\Phi(\rho)}{1+w(\rho)}\right]\left[\acute{\Phi}(\rho)-\frac{\acute{w}(\rho)\Phi(\rho)}{1+w(\rho)}\right]=0 \tag{30}$$

Application of (29) with $z=\frac{\Phi(\rho)}{1+w(\rho)}$ to (30) and reordering gives the differential equation expressed as under:

$$\acute{w}(\rho)=-\Psi\left[\frac{\Phi(\rho)}{1+w(\rho)}\right]\acute{\Phi}(\rho) \tag{31}$$

in the welfare cost function w. For a given $\Phi(\rho)$, the numerical solution of (31) can be obtained for an exact $w(\rho)$ [Possibly (29) can yield a solution to f to retrace the (17)]. Using a double-log specification (13) for $\Phi(\rho)$, equation (31) can now be expressed as under:

$$\acute{w}(\rho)=-\eta\,A\rho^{\eta}\,[1+w(\rho)]^{-1/\eta} \tag{32}$$

This can be solved to get:

$$w(\rho)=\exp\{[\tfrac{-\eta}{\eta+1}]\ln[\frac{-1}{ARe^{\eta\,\ln R}-\frac{\eta}{A(\eta+1)}+\frac{1}{A(\eta+1)}}]\}-1=\exp\{[\tfrac{\eta}{\eta+1}]\ln[\frac{\eta}{A(\eta+1)}+\frac{1}{A(\eta+1)}-ARe^{\eta\,\ln R}]\}-1 \tag{33}$$

$$=\exp\{[\tfrac{\eta}{\eta+1}]\ln[\frac{\eta}{A(\eta+1)}+\frac{1}{A(\eta+1)}-AR^{1+\eta}]\}-1=[\frac{\eta}{A(\eta+1)}+\frac{1}{A(\eta+1)}-AR^{1+\eta}]^{\frac{\eta}{1+\eta}}-1$$

$$=[\frac{\eta+1}{A(\eta+1)}-AR^{1+\eta}]^{\frac{\eta}{1+\eta}}-1\quad w(\rho)=[\frac{\mathbf{1}}{\mathbf{A}}-AR^{1+\eta}]^{\frac{\eta}{1+\eta}}-1 \tag{34}$$

or

$$w(\rho)=-1+[\mathbf{1}-AR^{1-\eta}]^{\frac{\eta}{\eta-1}}$$

$$w(\rho)=-1+[\mathbf{1}-AR^{1-\eta}]^{\frac{\eta}{\eta-1}} \tag{35}$$

$$w(\rho)=1-[\mathbf{1}+AR^{1-\eta}]^{\frac{\eta}{\eta-1}} \tag{36}$$

The (34) easily yields the WCI via CV method whereas (35) and (36) yields CV and Equivalent Variation (EV) bounds of WCI given in (34), (Gupta and Uwilingye, 2008).

3.5 Data and Variables Series Involved

The data series consist of SSMAs M_2, DMA M_2, real GDP, short-term interest rate and GDP-IPD. The 3-month T-bill rate offers good proxy for short-term interest rate. Since the constructed money stock is based on different measures of money. All the data series including those used to construct Divisia M_2 are extracted from SBP (2010) and SBP-MSB (2011).

This study utilizes biannual data, which ranges from 1972I to 2009II for four main variables. The variate S_t denoted the SSMA of broad money (M2). The DMA constructed from SSMA M2 according to the procedure detailed in section 3.1 is denoted by D_t. The general price level, GDP-IPD, denoted by P_t, is extracted from nominal and real biannual series of GDP based at 1999-00 market and constant prices, respectively— current year GDP at current year prices (nominal) divided by Current year GDP at base year prices (real). These biannual GDP series are extracted from GDP spliced series of Federal Bureau of Statistics for 1999-00 prices (GOP, 2010) quarterly ratios are calculated from quarterly estimates provided by Arby (2008) and the half-year ratios estimates were obtained by raising the quarterly figures to half year. For the period, 2005/6 to 2009/10, where Arby's estimates quarterly estimates are not available, estimates of biannual GDP are obtained by employing the average of 2000-01 to 2004-05 provided by Arby (2008) and raising the post 2004-05 quarterly ratios. The real GDP series, based at 1999-00 prices, is denoted by G_t. Three-month short-term interest rate biannual series is denoted by TB_t and its logged values are denoted by LTB_t. The logged money M_2 and real gross income ratio i.e. $\log(S_t/G_t)$, is denoted by Z_t and the logged Divisia money M_2 and real gross income ratio i.e. log (D_t/G_t), is denoted by DZ_t. Thus, final variates to be in the models are DZ_t, TB_t, LTB_t and Z_t.

3.6 Unit Root Tests and Weak Exogeneity

An exogenous variable is the one, whose value is entirely causally does not dependent on other variables in the system. It has been demonstrated that under weak exogeneity single equation estimation remained efficient in a cointegrated system, whereas in case of its failure, then system modelling was ultimate choice despite the super consistency of estimators in I(1) processes. Weak exogeneity is the base for inference in I(1) process as in I(0) process, with no loss of relevant information(Johansen, 1992b). Constant coefficients can be found from a conditional single equation specification albeit a regime shift exists in the reduced structure. If identification problems persist in full systems, building of a structural model, in a single-equation context, for a single variable might be easier (Hendry, 1995). Testing on assumptions of weak exogeneity can even guarantee a single equation error correction model (SEECM).

Before the testing the cointegration of variates, the ADF Generalized Least Squares (ADF-GLS), P-P and KPSS 1992 unit root tests are performed at the first stage, but then realizing breaks, we resorted to unit root tests in presence of structural breaks. To test weak exogeneity the LR statistic is used by imposing certain restrictions on VECM only when the degrees of freedom of the asymptotic χ^2 distribution are positive. Thus, the ECM permits testing for cointegration, and weak exogeneity centred on ECT coefficient's significance (Kremers et al., 1992).

3.7 Cointegration

When structural shift exits in the considered variates, the traditional tests' power to reject the null of unit root is somewhat reduced. Perron (1989) argued that the power is reduced against true alternative of stationarity if an existing break is ignored, so that existence of cointegration among variates remains dubious with regard validity of estimated results and inferences. He proved that inability to permit an existing shift results in a bias lowering the capability to reject a false null. The way out to all these setbacks estimator is to use (ARDL) approach to cointegration pioneered and put forward by Pesaran and Shin [PS]

(1995, 1996, 1999) and introduced by Pesaran, Shin and Smith [PSS] (1996, 2001) and popularized by Pesaran and Pesaran[PP] (2009) through software packages Microfit 5.02 and its earlier versions. In the usual way, the residuals can be tested for stationarity. PSS showed that ARDL approach remains valid regardless of the order of integration of the regressors thus obviating any doubts. It only needs confirmation that the variates must not be I(2). ARDL readily permits to include trends, seasonal dummy, deterministic, or exogenous variates in cointegrating equation, which was not possible with earlier frameworks that suppose all cointegrating variates to be integrated of the same order. Dummy variate is not I(1)— (PS, 1999, PSS, 2001).

Resorting to ARDL did not improve the estimations. All these proceedings led to further probe for structural changes that may be present. According to earlier researchers, the models built upon longer time span older than 1998/99 debt crisis (following which the exchange rate was liberalized significantly and financial policies targeted macroeconomic stability) suffer from parameter instability associated structural and economic regime changes in Pakistan (Bokil and Schimmelpfennig, 2006). Hence, we resorted to dummies to account for effect of abnormal observations to achieve a guarantee that the probability distribution of the disturbance terms is normal, thus validating the estimations and inference. Step dummy D1=1982II-90II is incorporated to capture the impact of structural changes in the money income ratio or short-term interest rate or both. D1 better captures the impact of the era of pre-financial reforms of the 1990s (SBP, 2002). It was also largely an era of a Marshall Law and as well as Pakistan was facing the Afghan crisis, and major change of its exchange rate policy, from fixed to floating exchange rate in 1980.

Dummy variable (D1) from 1980II to 1991II, a span when the LTB and TB have episode of highest magnitudes. The period covered in this study includes the era of pre-financial reforms of the 1990s, crammed with the use of direct tools of financial policy such as directed interest rates, regulated credit and domination of the public sector financial institutions (SBP, 2002). D2 (=1991I-98II) is portraying short-term changes of an era of a very low money income ratio; probably capturing the impact of the 1990s financial reforms. These reforms were put to be executed because of the earlier outcomes of utilization of direct tools of financial policy such as directed interest rates, regulated credit and domination of the public sector financial institutions (SBP, 2002). D3 (=2003I-2009II) captures the span of very low magnitudes of LTB and TB. The span of time witnessed the second phase of the reforms since 1999. As the impact of sanctions due to detonation of 1998 started diluting, SBP carried on the new of reforms from 1999 onwards, which changed the dynamics of the monetary sector substantially (Omer, 2010).

3.7.1 ARDL Approach to Cointegration: Estimation Procedure

For the specific ARDL models involving logged money gross income ratio and short term interest rate and the step dummies D_1's, and D_2's, the following lagged structured specifications were used bearing in mind that that the 1st two are log-log forms and the last two are semi-log forms.). We assume that

- The residuals in ARDL models e_{tj} are iid $(0; \sigma^2)$ white noise.
- The **x** is weekly exogenous in the system.

- The $\mathbf{z}_t$ be a vector of variates $\mathbf{z}_t = (\mathbf{y}_t, \mathbf{x}_t)$, where $\mathbf{y}_t = (DZ_t, Z_t)$, the is vector of regressands and $\mathbf{x}_t = (LTB_t, TB_t, D_{1t}, D_{2t})$, vector of regressors are at most I(1) in each the individual ARDL models(46-49).

ARDL models with $P_0 = 3$ i.e.

$$DZ_t=\alpha_{01}+\sum_{i=1}^{3}\alpha_{i1}DZ_{t-i}+\sum_{i=1}^{3}\beta_{i1}LTBt_{t-i}+\sum_{i=1}^{2}\gamma_{i1}Di_t+\sum_{i=1}^{3}b_{i1}\Delta DZ_{t-i}+\sum_{i=1}^{3}c_{i1}\Delta LTBt_{t-i}+\sum_{i=1}^{2}d_{i1}\Delta Di_t+e_{t1} \quad (46)$$

$$Z_t=\alpha_{02}+\sum_{i=1}^{3}\alpha_{i2}Z_{t-i}+\sum_{i=1}^{3}\beta_{i2}LTBt_{t-i}+\sum_{i=1}^{2}\gamma_{i2}Di_t+\sum_{i=1}^{3}b_{i2}\Delta Z_{t-i}+\sum_{i=1}^{3}c_{i2}\Delta LTBt_{t-i}+\sum_{i=1}^{2}d_{i2}\Delta Di_t+e_{t2} \quad (47)$$

$$DZ_t=\alpha_{03}+\sum_{i=1}^{3}\alpha_{i3}DZ_{t-i}+\sum_{i=1}^{3}\beta_{i3}TBt_{t-i}+\sum_{i=1}^{2}\gamma_{i3}Di_t+\sum_{i=1}^{3}b_{i3}\Delta DZ_{t-i}+\sum_{i=1}^{3}c_{i3}\Delta TBt_{t-i}+\sum_{i=1}^{2}d_{i3}\Delta Di_t+e_{t3} \quad (48)$$

$$Z_t=\alpha_{04}+\sum_{i=1}^{3}\alpha_{i4}Z_{t-i}+\sum_{i=1}^{3}\beta_{i4}TBt_{t-i}+\sum_{i=1}^{2}\gamma_{i4}Di_t+\sum_{i=1}^{3}b_{i4}\Delta Z_{t-i}+\sum_{i=1}^{3}c_{i4}\Delta TBt_{t-i}+\sum_{i=1}^{2}d_{i4}\Delta Di_t+e_{t4} \quad (49)$$

Where the first two are log-log and last two are semi-log specifications. The D_i's are dummy variables. Where α's, β's and γ's are parameters and e_i's are the error terms and independent variables include LTB, TB and D_i's. The dependent variables are DZ and Z.

For the above (46-49) equations, the long run solutions are given by:

$$DZ_t = \alpha_1 + \eta_1 LTB_t + \sum_{i=1}^{2}\zeta_{i1}Di_t + \varepsilon_{t1} \quad (50)$$

$$Z_t = \alpha_2 + \eta_2 LTB_t + \sum_{i=1}^{2}\zeta_{i2}Di_t + \varepsilon_{t2} \quad (51)$$

$$DZ_t = \alpha_3 + \xi_1 TB_t + \sum_{i=1}^{2}\zeta_{i3}Di_t + \varepsilon_{t3} \quad (52)$$

$$Z_t = \alpha_4 + \xi_2 TB_t + \sum_{i=1}^{2}\zeta_{i4}Di_t + \varepsilon_{t4} \quad (53)$$

More explicitly for the above equations, the unrestricted error correction version of the ARDL model is given by equations 54-57 which are error correction representation of equations 46-49. The first part of the equations 54-57 gives the cointegrated relationship, while the second part gives the short-run dynamics, εt's are residuals iid~(0, σ2). These equations points to fact that money income ratio growth, in terms of ratio of DMA and SSMA of M2 and real gross income, are likely to be substantially affected by its past values so that it entails another set of disturbances or shocks. Therefore, equations 54-57 are modified to capture these economic episodes and absorb them via ECT to revert to long run equilibrium. Hence,:

$$\Delta DZ_t = a_{01}+\sum_{i=1}^{3}b_{i1}\Delta DZ_{t-i}+\sum_{i=1}^{3}c_{i1}\Delta LTBt_{t-i}+\sum_{i=1}^{2}d_{i1}\Delta Di_t+ECM_1 \quad (ECM_1=DZ_t-\alpha_1-\eta_1 LTB_t-\sum_{i=1}^{2}\zeta_{i1}Di_t) \quad (54)$$

$$\Delta Z_t = a_{02}+\sum_{i=1}^{3}b_{i2}\Delta Z_{t-i}+\sum_{i=1}^{3}c_{i2}\Delta LTBt_{t-i}+\sum_{i=1}^{2}d_{i2}\Delta Di_t+ECM_2 \quad (ECM_2=Z_t-\alpha_2-\eta_2 LTB_t-\sum_{i=1}^{2}\zeta_{i2}Di_t) \quad (55)$$

$$\Delta DZt = a03+\sum_{i=1}^{3}b_{i3}\Delta DZ_{t-i}+\sum_{i=1}^{3}c_{i3}\Delta TBt_{t-i}+\sum_{i=1}^{2}d_{i3}\Delta Di_t+ECM3 \quad (ECM3=DZt-\alpha3-\xi_1 TBt-\sum_{i=1}^{2}\zeta_{i3}Di_t) \quad (56)$$

$$\Delta Zt = a04+\sum_{i=1}^{3}b_{i4}\Delta Z_{t-i}+\sum_{i=1}^{3}c_{i4}\Delta TBt_{t-i}+\sum_{i=1}^{2}d_{i4}\Delta Di_t+ECM4 \quad (ECM4=Zt-\alpha4-\xi_2 TBt+\sum_{i=1}^{2}\zeta_{i4}Di_t) \quad (57)$$

If the cointegration is established, then start estimating the coefficients of the long and short run solutions of equation 46-49 to reach stage of testing their significance in Equations 50-53 and 54-57 by imposing restrictions on the long-run parameters.

$$H_0: \alpha_i = \eta_i \text{ or } \xi_i = \zeta_{ij} = 0; \quad H_1: \alpha_i \neq = \eta_i \text{ or } \xi_i = \neq \zeta_{ij} \neq 0,$$

The asymptotic 'F' distributions are not standard under the null, regardless of the variates are purely I(0) or I(1), or mutually cointegrated. Bounds testing approach uses W and F test-statistics to confirm cointegration among the variates. In the two sets of Critical Value Bounds (CVBs) are generated by PSS(2001), one set assumes all variates to be I(1), referred as upper CVBs and the other assumes all variates to be I(0), referred as the lower CVBs. Thus, PP (2009: Tables B.1 and B.2) offers two sets of asymptotic CVBs for encompassing all feasible cases—dependent on 'k' and the conditional ECM entails an intercept, or trend— that can arise. Microfit 5.2 by PP automatically computes the CVBs using stochastic simulations following a procedure similar to provided in PP (2009). These simulated CVBs are close to the ones CVBs provided, but have the advantage that unlike the tabulated values these automatically computed CVBs continue to be applicable even if shift dummy variates are included amongst the deterministic variates (PP, 2009). If the computed W and F test-statistics are above the upper CVBs, the null is discarded. In this study, we have comparatively small sample of size around T=70, the relevant CVBs here are those reported by Narayan (2004). These are obtained using small sample size between t=30 and t=80 generated by the same procedure used by PSS (2001). These small sample CVBs are roughly 15% higher at t=80. There are no worries, if our test statistics emerge almost 15% or more larger in magnitude than nearest large sample CVBs calculated in the Microfit 5.02. ECT, the coefficient of ECM_{t-1} measures the speed of adjustment from shorter time span towards the longer one. It should be strong and significant with negative sign to augment the existence of cointegration (Bannerjee et al., 1998).

3.8 Diagnostic Tests

The reported estimated model is robust, valid for reliable inferences and interpretations if it passes through diagnostic tests like Breusch-Godfrey (B-G) LM, Jacque-Bera (J-B), regression specification error test (RESET) and autoregressive conditional heteroscedasticity (ARCH) test. All the tests signify that model is free from hazards of serial correlation, abnormality, misspecification, and heteroscedasticity.

3.9 Stability: Testing for Parameter Stability

Unstable coefficients imply unreliable results. To test for parameter constancy or to test for long-run parameter stability are the CUSUM and the CUSUMSQ tests with the null of coefficients' stability are suggested by Brown et al. (1975). Both are based on recursive residuals of the estimated ECMs. If the plots of the CUSUM and CUSUMSQ remain within the 5% CVBs, then the null of coefficients' constancy cannot be rejected.

3.10 Testing in presence of Levels Breaks

When looking at the graph of the DZ_t, LTB_t, $TB_{t,}$ and, Z_t, we find clear indications irregular humps and downs, which are clear-cut signs of some impulses over small regions of historigrams of the series. So, the structural breaks, a common feature of the macroeconomic data, are manifested here. When the level shifts are manifested in time series, unit root and cointegration tests disregarding level shifts, produce a distorted inference concerning the unit root as well as cointegrating rank. Rare situations may occur when disregarding level shifts do not distort the tests' inference concerning the unit root and cointegration rank.

3.10.1Unit Root Test with Break in *levels*

When level shift are manifested in the DGP, it must be regarded in testing unit root since the inference of usual test may be distorted if the level shift disregarded. Saikkonen and Lütkepohl (2002) and Lanne et al. (2002), after adding a shift function, have suggested a new unit root test. The critical values can be seen in Lanne et al. (2002).

3.10.2 Cointegration with Structural Breaks

If structural breaks are pre-specified then Johansen et al. (2000) test works only up to two level breaks and render adjustment to the usual trace test to permit double levels breaks. The double levels breaks Johansen trace tests' critical values and the p-values can be achieved by solving the relevant response surface in accordance with Doornik (1998) if there are no breaks in line with Johansen et al. (2000) if there are up to two breaks.

3.11 Nonlinear Unit Root Kapetanios, Shin, and Snell (KSS)-Test

Cointegration DZ_t, LTB_t, TB_t, and, Z_t, emerge to be difficult and cumbersome task with involvement of dummies to capture regime effects. These proceedings led to the further investigations regarding the form of cointegration to occur. KSS detects the presence nonlinear nonstationarity by judging between a linear nonstationary process and a nonlinear overall stationary exponential smooth transition autoregressive (ESTAR) process described below:

$$\Delta Y_t = \gamma Y_{t-1}\left[1 - e^{-\theta Y_{t-1}^2}\right] + \varepsilon_t, \quad \theta > 0 \qquad (58)$$

Where Y_t is vector of variables considered, $\varepsilon_t \sim$ iid $(0, \sigma^2)$ and $\theta \geq 0$ is known as the ESTAR processes transition parameter indicating the transition speed. KSS tests the null of $\theta = 0$, against $\theta > 0$. If the is null of $\theta = 0$ true, Y_t entail intrinsically a linear unit root process, but its rejection decides nonlinear stationarity of ESTAR process. However, under the null of $\theta = 0$, the loaded parameter γ is unidentified. Therefore, under the null of $\theta = 0$, a first-order Taylor series approximation for the expression $\left[1 - e^{-\theta Y_{t-1}^2}\right]$ was used in the KSS test. Therefore, to approximate (58), the following auxiliary regression is run:

$$\Delta Y_t = \xi + \delta Y_{t-1}^3 + \sum_{i=1}^{k} b_i \Delta Y_{t-i} + \varepsilon_t, \; t=1,2,.,T \qquad (59)$$

The null and the alternative are transformed into $\delta = 0$ against $\delta<0$ for the (59) showing linear and nonlinear ESTAR stationarity, respectively. Therefore, the t-statistic for $\delta = 0$ against $\delta<0$ could be obtained as:

$$\mathbf{t_{NL}} = \frac{\hat{\delta}}{s.e.(\hat{\delta})} \qquad (60)$$

Under the unit root null of $\theta = 0$, the t_{NL} statistic defined by (60) has the following asymptotic distribution:

$$t_{NL} \Rightarrow \frac{\{\frac{1}{4}W(1)^4 - \frac{3}{2}\int_0^1 W(r)^2 dr\}}{\sqrt{\int W(r)^6 dr}}.$$

Here the standard Brownian motion W(r) is defined on r ∈ [0 1]. Against the null and under the alternative of $\theta > 0$ through the ESTAR model in (58), the t_{NL} statistic is consistent. For the simulated CVs for different K reader is referred to Table.1 as of p.363, Kapetanios et al. (2003).

4. Results and Discussion

In the descriptive statistics, DZ_t has lower mean and bigger S.D. value than Z_t while LTB_t has smaller mean and S.D. than TB_t showing a clear benefit of logarithm of the variable. While the Skewness and Kurtosis of DZ_t and TB_t are more close to normal distribution than their counterparts Z_t and LTB_t with DZ_t excelling in all the four variables in this regard, and is expected to behave better in the estimation process. As for as correlation coefficients between the related variables are concerned, these are in range of .15 to .30, not substantial due to presence of regime shifts. Among the related variables, TB_t has 5 times more spread than LTB_t as well as the maximum spread in all four variates and surprisingly, DZ_t has more spread than Z_t. The detailed descriptive statistics are provided in table 1.

Table 1: Descriptives and Correlations between the variables

Descriptive	DZ	Z	LTB	TB	**Corr.**	DZ	Z	LTB	TB
Median	-0.035	-0.222	1.956	7.070	DZ	1.00	0.86	-0.18	-0.15
Mode	-0.148	-0.158	1.716	5.560	Z	0.86	1.00	-0.28	-0.26
Mean	-0.043	-0.236	1.840	6.782	LTB	-0.15	-0.26	1.00	0.95
S.E.	0.018	0.012	0.051	0.241	TB	-0.18	-0.28	0.95	1.00
S. D.	0.158	0.104	0.447	2.100	**Descrip.**	DZ	Z	LTB	TB
Kurtosis	-0.066	1.430	6.075	0.345	Min	-0.42	-0.58	-0.010	0.990
Skew.	-0.166	-0.981	-2.25	-0.81	Max	0.27	-0.04	2.31	10.06
Sum	-3.290	-17.91	139.9	515.4	Range	0.695	0.538	2.32	9.07

The historigrams of the individual DZ_t, LTB_t, TB_t, and Z_t depicts negative correlation among variates with existence of regimes reforms in form of humps and downs as manifestation in response to monetary policies amounting to a negative relationship among the variates plagued with uneven level shifts breaks which can be seen in figure 1. The scatter plots of the pairs of concerned variables further elaborate these negative relationships among the pairs of variables that are shown with linear regression line in figure 2. For the DZ_t-LTB_t relationship depicted in panel I of the diagram, the points are more closer to the line and scattered sparsely on the line that amounts to a clear and vivid well-behaved relationship, except those four outliers.

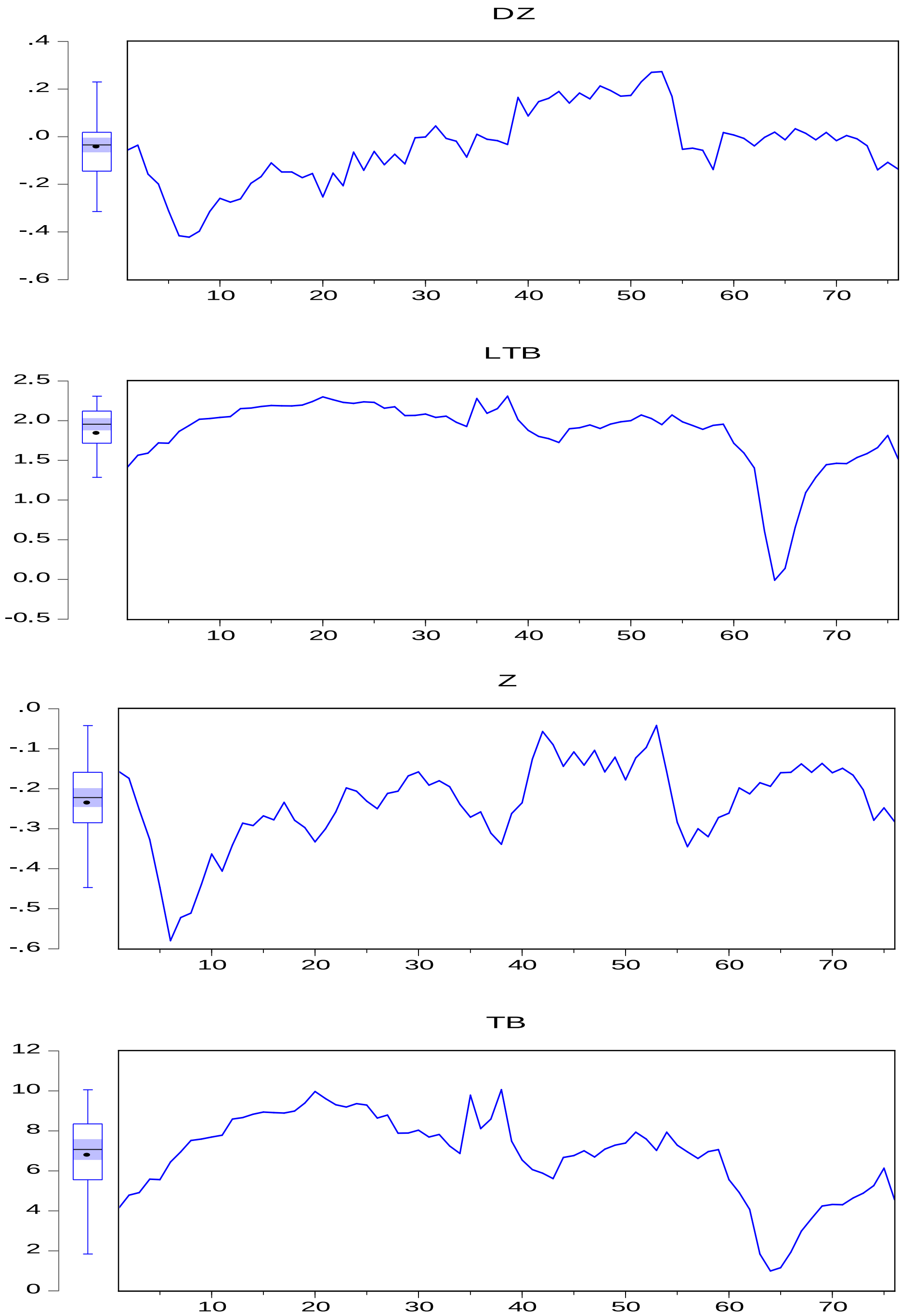
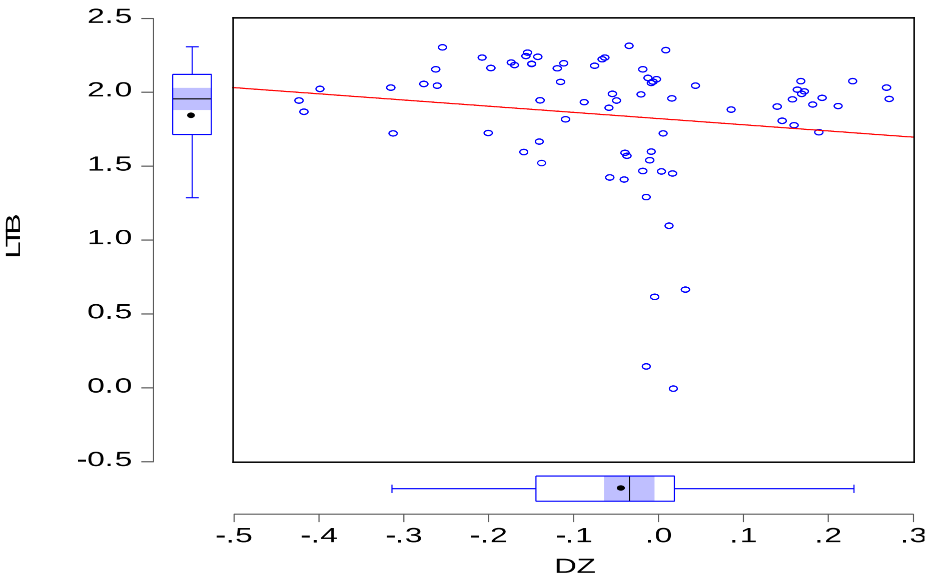
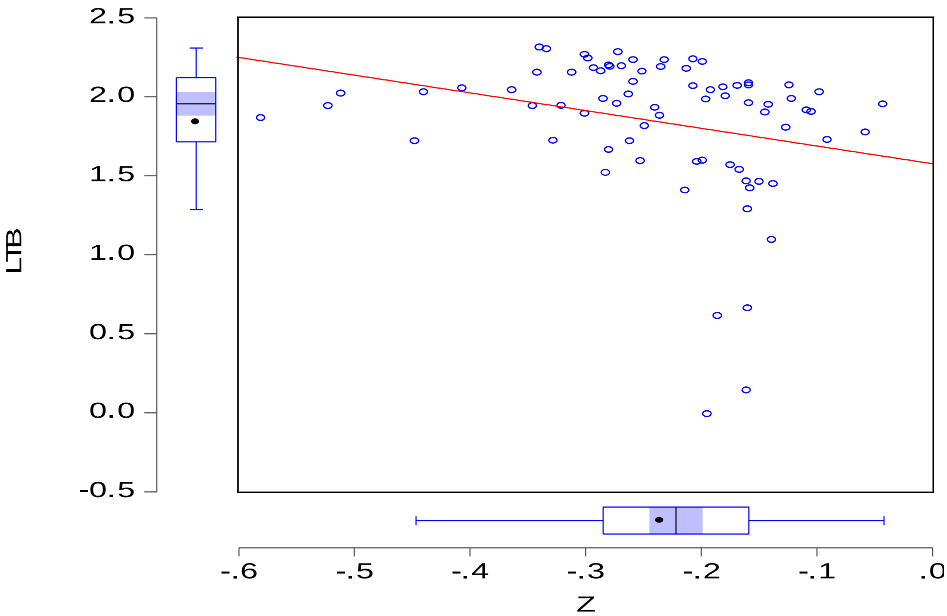


Figure 1: Graphs of the Varieties Involved


2.5
2.0
1.5
1.0
0.5
0.0
-0.5
LTB
-.5
-.4
-.3
-.2
-.1
.0
.1
.2
.3
DZ


2.5
2.0
1.5
1.0
0.5
0.0
-0.5
LTB
-.6
-.5
-.4
-.3
-.2
-.1
.0
Z

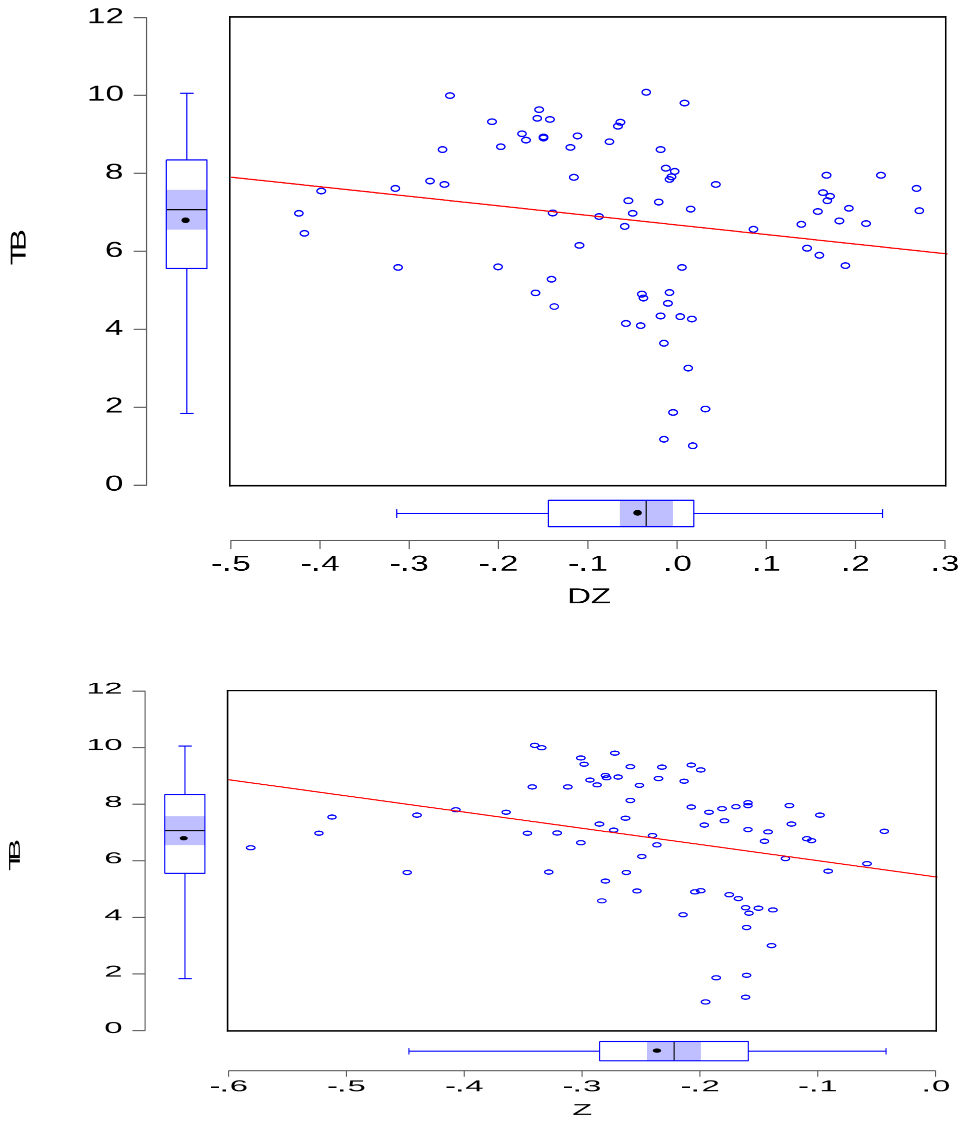


Figure 2: Scatter Plots of the Pairs of Variables Involved

For the Z_t-LTB_t relationship depicted in panel II of the diagram, the scatter points are though closer to the line but not sparsely scattered on the line that amounts to a bit less strong relationship than the first one. In the DZ_t-TB_t and Z_t-TB_t relationships depicted in panel III and IV of the diagram, the points are more stretched across the line and scattered less sparsely on the line, which amounts to rather less clear relationship. Obviously, the relationship between DZt-LTBt and Zt-LTBt is manifested more clearly than relationship between DZ_t-TB_t and Z_t-TB_t.

More specifically, the linear relationship of DZ_t -LTB_t seems the best and Z_t-TB_t looks the worst as well as with obvious presence of a few outliers in all the four relationships. However, from a view of all four scatter plots, it is well expected that DZ_t-LTB_t and Z_t-LTB_t would show a stronger relationship than DZ_t-TB_t and Z_t-TB_t, and that is log-log from would perform better than semi-log form. Further to the above, when all four relationships depicted in the figure 2 are compared, it is likely that DZ_t-LTB_t would perform the best among all.

4.1 Weak Exogeneity χ2 -Test

For the all pairs of related variables DZ_t-LTB_t, Z_t-LTB_t, DZ_t-TB_t, and Z_t-TB_t, the existence of long-run weak exogeneity of LTB_t, and TB_t is examined by χ^2 tests. In other words, we evaluate whether the terms of the speed of adjustment w.r.t. LTB_t, and TB_t in the VECMs is statistically insignificant. This is done by performing the LR tests (Johansen, 1992a) using a null hypothesis of $a_{21} = 0$, founded on the constraints imposed on the 'a' parameters of the a usual VAR model. The results of the test are presented in Table 2.

Table 2: LR test of Weak Exogeneity χ2 -Test in VECM

Null	**χ2 tests**	**[P-Val]**
H0: (LTBt) Weakly exogenous, i.e. to(DZt)	χ2 (1)=0.28	[0.59]
H0: (LTBt) Weakly exogenous, i.e. to (Zt)	χ2 (1)=0.15	[0.70]
H0: (TBt) Weakly exogenous, i.e. to (DZt)	χ2 (1)=0.19	[0.66]
H0: (TBt) Weakly exogenous, i.e. to (Zt)	χ2 (1)=0.01	[0.91]

That is, unidirectional causality exists between DZ_t-LTB_t and Z_t-LTB_t and between DZ_t-TB_t and Z_t-TB_t. Finally, neither model violates weak exogeneity conditions.

4.2 Unit Root-Tests

The unit root null hypothesis in ADF-GLS for the variables in levels and for DZ_t, Z_t, and TB_t, 5% and 1% level cannot generally be rejected. Specifically, ADF-GLS and P-P tests' unit root null for Z_t at levels emerged significant at 10% level of significance with constant and with ADF-GLS for LTB_t at 5% with constant and trend, though KPSS and P-P never supported this stance with constant yet it got little support from KPSS with constant and trend. For all the variables at the first differences, the unit root null is easily denied, thus obviously supporting almost all the variables as I(1). Hence, with no unanimous decision about the order of integration, leading to the plausibility ARDL which only requires that the order of integration must not be two or higher, which is confirmed here. The results are not reported here due to brevity.

4.3 Unit Root Test with Structural Break

As these proceedings shed doubts regarding the true order of integration and led further to probe the unit roots in the manifestation of shift breaks. We have used single structural break unit root test and found strong evidence of shift break in vicinities of 19981, 1991 and 1998. The results are tabulated in Table 3.

Table 3: Results of Unit Root Tests with a shift break

Variates with Dummies Used	Targeted Breaks	Sample Range	Lags 1st dif	C-V
DZ (D1=82I-90I, D2=90II-98II)	1981II	[1977 II, 1986 II], T = 19	2	-3.32 **
DZ (D1=82I-90I, D2=90II-98II)	1991 I	[1977 II, 1999 II], T = 45	2	-2.94 **
DZ (D1=82I-90I, D2=90II-98II)	1998II	[1991 II, 2009 II], T = 37	2	-2.96 **
Z (D1=81II-88II,D2=91II-98II)	1980 I	[1973 II, 2009 II], T = 73	2	-3.87***
Z (D1=81II-88II,D2=91II-98II)	1992II	[1973 II, 2009 II], T = 73	2	-3.65***
Z (D1=80II-88II,D2=90II-99I)	1998II	[1973 II, 2009 II], T = 73	2	-2.70*

4.4 Cointegration with Structural Breaks

The single structural break unit root tests confirm the presence of structural breaks, so, we decided to probe cointegration among variates further in the manifestation of structural breaks. *The results of c*ointegration with two structural breaks are *presented in* tables 4, 5. The time span for estimations post 1974I era thus avoiding another famous worldwide structural break of 1974I oil shock As we tested cointegration with only two structural breaks, accordingly, both log-log and the semi-log form entailing DMA and SSMA involve two separate step dummies D_1 and D_2 t*o capture the contributions of shift breaks in the process of cointegration,*

Table 4: Results of Johansen Trace Two Breaks Cointegration Test

Variates	Break Dates	Unrestricted Dummy	Sample Range	Lags	Dim	r0	LR
DZ-LTB (D_1=82I-90I, D_2=90II-98II)	1982I,1998II	D[1982 I] D[1982 II] D[1998 II] D[1999 I]	[1973 I, 2009 II] T = 74	2	2	0	29.0*
						1	11.3
Z-LTB (D_1=81II-88II,D_2=91II-98II)	1981I,1998II	D[1981 I] D[1981 II] D[1998 II] D[1999 I]	[1977 II, 2009II] T = 65	2	2	0	35.8***
						1	11.8
DZ-TB (D_1=81II-90I, D_2=90II-98II)	1981I,1998II	D[1981 I] D[1981 II] D[1998 II] D[1999 I]	[1975 I, 2009 II] T = 70			0	28.7*
						1	10.0
Z-TB (D_1=80II-88II,D_2=90II-99I)	1981I,1998II	D[1981 I] D[1981 II] D[1998 II] D[1999 I]	[1977 II, 2009II] T = 65	2	2	0	35.1***
						1	13.0

Table 5: Results of Johansen Trace Two Breaks Cointegration Test

Variates	Breaks Dates	Unrestricted Dummy	Sample Range	Lags	Dim	r0	LR
DZ-LTB (D_1=82I-90I, D_2=90II-98II)	1990II,1998II	D[1990 II] D[1991 I] D[1998 II] D[1999 I]	[1976 II, 2009II] T = 65	2	2	0	32.5**
						1	8.5
Z-LTB (D_1=81II-88II, D_2=91II-98II)	1990II,1998II	D[1990 II] D[1991 I] D[1998 II] D[1999 I]	[1976 II, 2009II] T = 65	2	2	0	44.4***
						1	13.8
DZ-TB (D_1=81II-90I, D_2=90II-98II)	1990II,1998II	D[1990 II] D[1991 I] D[1997 II] D[1998 I]	[1976 II, 2009II] T = 67	2	2	0	41.2***
						1	7.6
Z-TB (D_1=80II-88II, D_2=90II-99I)	1990II,1998II	D[1990 II] D[1991 I] D[1998 II] D[1999 I]	[1976 II, 2009II] T = 65	2	2	0	39.4***
						1	12.6

*, ** and *** indicate denial of the null at 10%, 5% and 1% rejection level, respectively.

These two dummies for 80’s and 90’s shift breaks in all four combinations of two functional forms and two-aggregation procedure. We tested two sets of level shift breaks one in vicinities of 19981and 1998 and the other in vicinities of 1991 and 1998 employing Johansen trace two breaks cointegration test including intercept. We found a bit weaker evidence of a single cointegrating vector entailing the former but stronger evidence for the latter sets of level shift breaks.

4.5 The ARDL Estimates of the DZ-LTB, D-LTB, DZ-TB, and Z-TB Models

Having been convinced by the fact that there are level shift breaks in vicinities of 1981, 1991, and 1998, we incorporated two dummies in each model with little alterations of time span in order to get more stronger effects of the dummies and the cointegrated ARDL models. Since, the the ARDL procedure is ensured as legimate even in the presence of shift breasks, accordingly, it is performed with a p_0=3. The results of ARDL model estimates are presented in table 6. For log-log form, our results are valid as they clear the range of small sample bias— detected and reported by Narayan (2004) — since our calculated ‘F’ and ‘W’ test statistics are more than 50% larger the than the CVs provided by PSS, hence these are significant at Narayan’s small sample CVBs also.

In all ARDL models, almost all the regressors have significant coefficients with p-values less than 0.05 and the signs are consistent with economic theory. The adjusted R^2s’ lie in the range of 0.80 to 0.90, indicating that nearly 80% to 90 % of the average variation is explained by the regression equations, thus implying a better fit. The DW test statistic value nears two in all equations indicating that absence of first order autocorrelation. Coefficients of regressors LTB and TB are strongly significant. The coefficients of dummy variable showing short-term policy shocks are all significant. The values of F and W test statistics are highly significant ensuring a valid cointegration even in presence the dummy variables. Hence, the included dummies are able to capture the true effects of short-term economic changes, as the coefficients of these dummies are significant with signs as expected. The values selection criteria AIC (as AIC performs better in small samples) and SBC for log-log form are smaller than their simple sum counterparts, hence, we decided to rely more on the results from log-log form and not rely on semi-log form estimated with ARDL framework. However, the results of estimated semi-log with diagnostic tests are not reported in table for brevity.

4.6 Diagnostic Tests

The results of diagnostic misspecification tests are presented in lower part of the table 4.1. a. Diagnostic tests all do not reject the nulls of no serial correlation, normality, no heteroscedasticity, and correct specification at p-values more than 0.05. Hence, each model possesses correct specification and its residuals are free from serial correlation, abnormality, and heteroscedasticity.

Table 6: Estimated Log-Log Form Results And The Diagnostic Tests

Model DZ-LTB of order (2, 0)) for [1976II 2009II]		**Model Z-LTB of order (1, 2) for [1976II 2009II]**	
Variate	COFF.[S.E.]	Variate	COFF.[S.E.]
C	0.047* [0.027]	C	-0.035 [0.021]
DZ(-1)	0.222** [0.118]	Z(-1)	0.456***[0.085]
DZ(-2)	0.259***[0.097]	LTB	-0.062** [0.029]
LTB	-0.056***[0.017]	LTB(-1)	0.066 [0.047]
D1=82I-90I	0.049***[0.017]	LTB(-2)	-0.064** [0.031]
D2=90II-98II	0.159***[0.026]	D1=81II-88II	0.045***[0.014]
		D2=91II-98II	0.087***[0.017]
DIAGNOSTIC MISSPECIFICATION TESTS			
TEST STATISTIC		Model DZ-LTB	Model Z-LTB
F-statistic [P-Val]		17.6** [7. 1 8.1]	20.6** [6.8 7.7]
W-statistic[P-Val]		35.3**[14.2 16.2]	41.2** [13.6 15.5]
R2		0.86	0.80
$\bar{R}^2$		0.85	0.78
S.E. of Regression		0.05	0.04
F-Stat. [P-Val]		F5,58) 71.7***	F(6,60) 40.3***
AIC		98.46	122.76
SBC		91.98	115.04
DW-statistic		1.84	1.91
SC-test	LM-Ver.	CHSQ(2)= 1.50[.47]	CHSQ(1)=0. 57[.75]
	F-Ver.	F(2,56)=0.67[.51]	F(2,58) =0.25[.78]
FF-test	LM-Ver.	CHSQ(1)= .004[.98]	CHSQ(1)=1.07[.30]
	F-Ver.	F(1,57)= .004[.98]	F(1,59) =0.95[.33]
JB-test	LM-Ver.	CHSQ(2)= 0.41[.98]	CHSQ(2)= 1.19[.55]
	F-Ver.	Not applicable	Not applicable
Arch-test	LM-Ver.	CHSQ(1)= 0.43[.52]	CHSQ(1) .33[.86]
	F-Ver.	F(1,62)= 0.41[.53]	F(1,65) =0.03[.86]

Table 7: Estimated short-run log-log models -VECMs results

DZ –LTB Model of (2, 0) for period [1976II 2009II]		**Z-LTB Model of (1, 2) for period [1976II 2009II]**	
Variate	COFF.[S.E.]	Variate	COFF.[S.E.]
dDZ1	-0.259** [0.097]	dLTB	-0.062** [0.029]
dLTB	-0.057***[0.017]	dLTB1	0.064** [0.031]
dD1=82I-90I	0.159***[0.026]	dD1=81II-88II	0.045***[0.014]
dD2=90II-98II	0.049***[0.017]	dD2=91II-98II	0.087***[0.017]
ECM(-1)	-0.518***[0.083]	ECM(-1)	-0.544***[0.085]
DIAGNOSTIC MISSPECIFICATION TESTS			
STATISTIC		DZ –LTB	Z-LTB
R2		0.488	0. 420
$\bar{R}^2$		0.444	0.361
S.E. of Reg.		0.050	0.037
F-Stat.		F(5,58) 11.07***	F(5,61) 8.67***

*, ** and *** indicate denial of the null at 10%, 5% and 1% rejection level, respectively

The results of estimated short-run log-log models are presented in table 7. The ECTs have a correct signs with substantial magnitudes and their values are less than one in both log-log and semi-log forms. In all the specifications, the magnitude of speed of adjustment implying that 52% to 54% errors due to shocks are adjusted per half-year period. The ECTs in log-log model entailing Zt is a little larger in magnitude than that of models involving DZt. The results of estimated short-run semi-log models are not reported for brevity.

The results of estimated long-run solutions are presented in table 8. It is noticeable that there are some differences in the results with regard to the methods of aggregation and functional forms in the economy of Pakistan. The long-run money per unit of income elasticities of interest rate are low near 0.11 but the money per unit of income semi elasticities of interest rate are almost 0.03 for model both models entailing DMA and SSMA.

Table 8: Results of Long-Run Solution of Log-Log Models

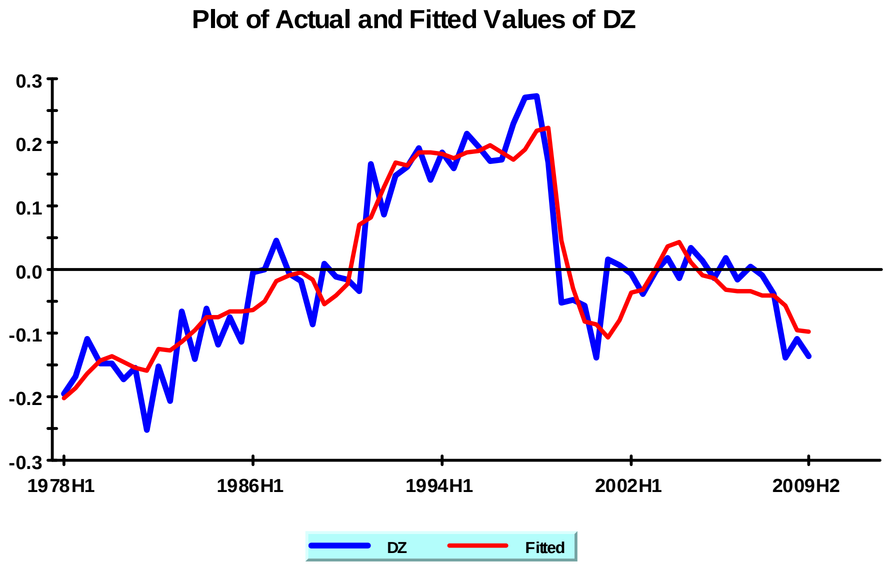
DZ –LTB Model of (2, 0) for period [1976II 2009II]		Z-LTB Model of (1, 2) for period [1976II 2009II]	
Variate	COFF.[S.E.]	Variate	COFF.[S.E.]
C	0.09* [0.050]	C	-0.06** [0.038]
LTB	-0.11***[0.029]	LTB	-0.11***[0.021]
D1=82I-90I	0.31***[0.033]	D1=81II-88II	0.08***[0.023]
D2=90II-98II	0.09***[0.035]	D2=91II-98II	0.16***[0.022]

*, ** and *** indicate denial of the null at 10%, 5% and 1% rejection level, respectively.

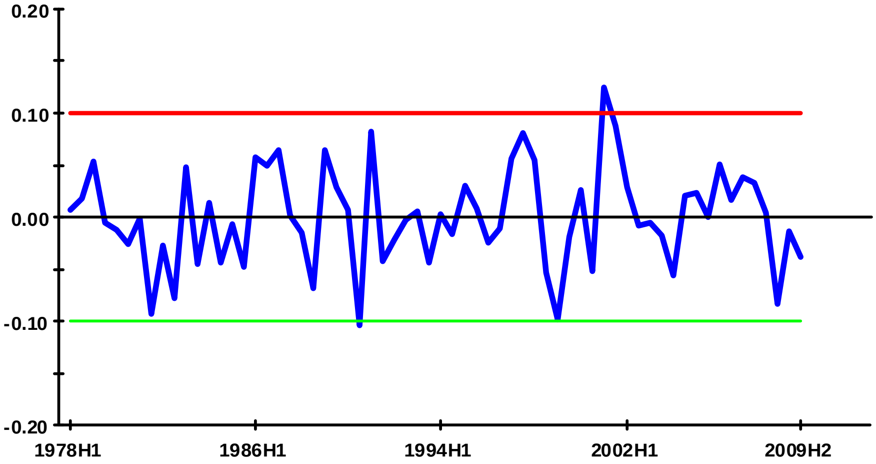
As these much lower elasticities underrates WCI estimates. Based on selection criteria, we decided to rely only on estimates obtained by log-log form. However, the results of estimated long-run solution of semi-log models are not reported for brevity.

4.7 Model Stability: Testing for Parameter Stability

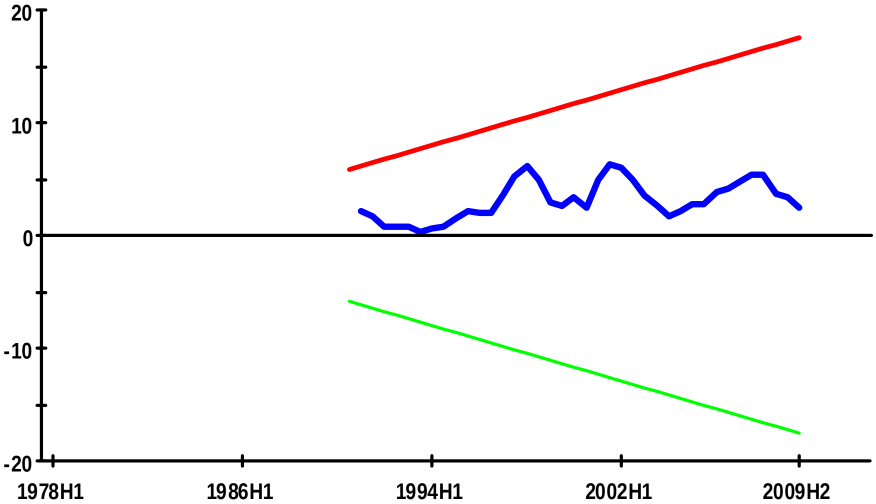
The CUSUM and CUSUMSQ tests are used to check the parameter stability for all four (for two forms log-log and semi-log and two variants of aggregations) models. It is found that at least the models are stable and hence validating our WCI estimates that are mainly based on last structure (from 1999H1 onwards) of the variables. The results of parameter stability can be seen in figures 3 and 4 for log-log form. However, the results stability for semi-log form not reported for brevity.



Plot of Residuals and Two Standard Error Bands



Plot of Cumulative Sum of Recursive Residuals



The straight lines represent critical bounds at 5% significance level

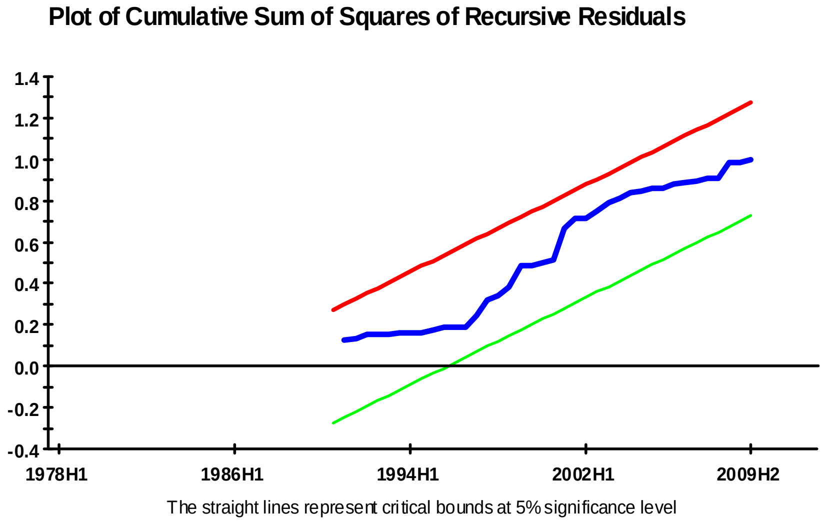
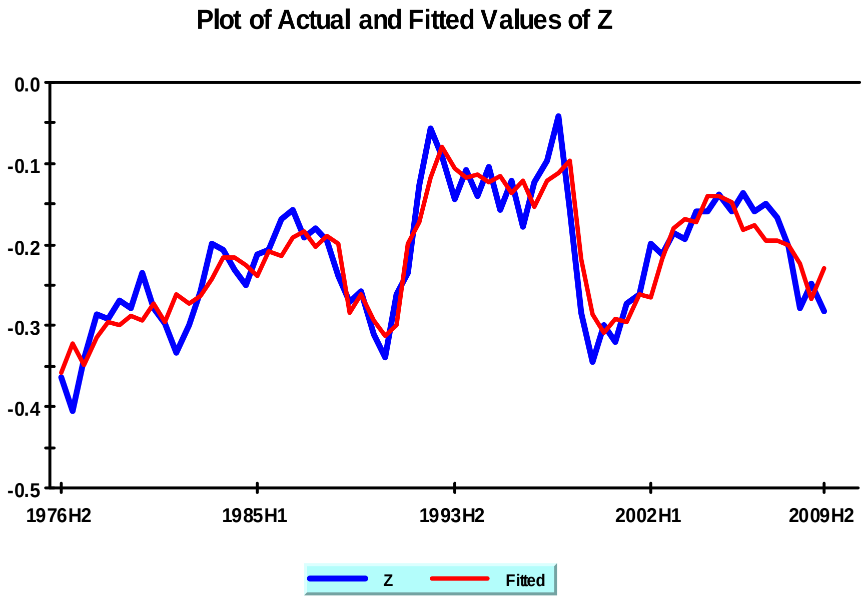
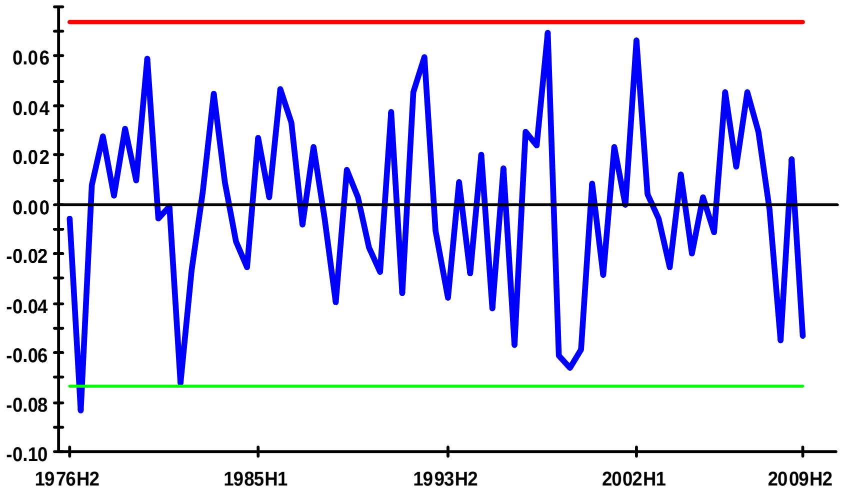


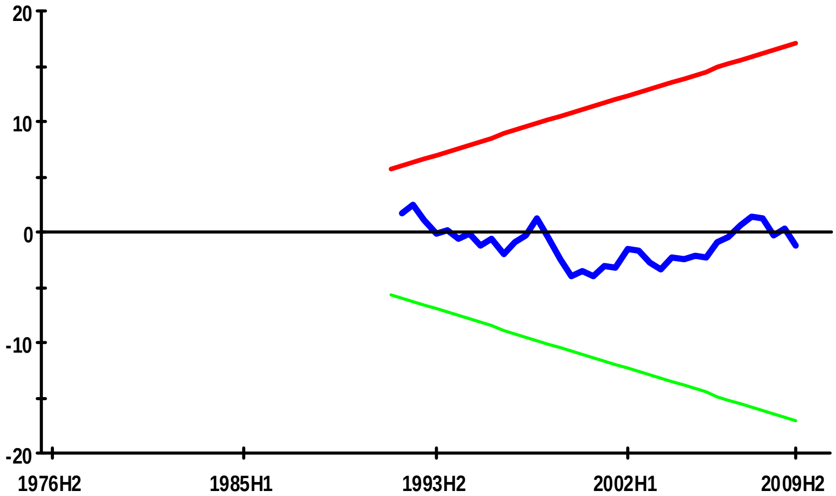
Figure 3: Plot of DZ-LTB model, residuals, CUSUM and CUSUMSQ



Plot of Residuals and Two Standard Error Bands



Plot of Cumulative Sum of Recursive Residuals



The straight lines represent critical bounds at 5% significance level

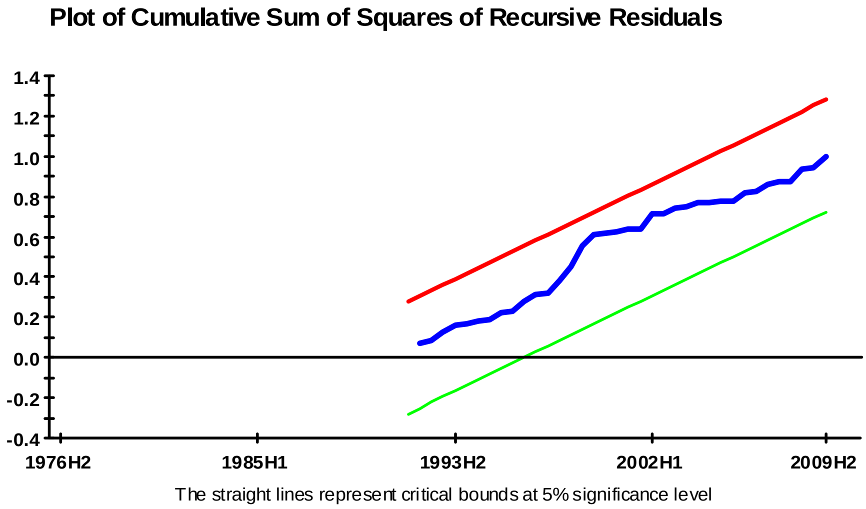


Figure 4: Plot of Z in Z-LTB model, residuals, CUSUM and CUSUMSQ

4.8 Results, and Discussions of Welfare Cost of Inflation

Empirically, the money demand semi-log specification implies money demand interest-elasticity be a growing function of the level of ρ_t, the short-term nominal interest rate, thus lowering the held real balances stock of agents at increased opportunity cost of money. Hence, in case opportunity cost is not very high, the coefficient of money demand interest-elasticity is expected low in magnitude, and thus frictions may occur in lowering the held real balances stock of agents. This relation contradicts the DMRS implicitly implied by the Inada conditions. Further, when opportunity cost of money is decreased and stabilized at moderate levels, the semi-log form entails increasingly hard substitution between real balances and other assets. That is perhaps the reason that explains why semi-log form underrates the WCI estimates. Contrastingly, the log-log form provides a constant money demand interest elasticity, which renders a degree of substitution between real balances and other assets independent of the level of the ρ_t. With log-log form entailing DMAs, for 5% nominal interest rate the WCI is 0.9 % of the GDP and as nominal interest rate rises to 10, 15, and 20% the WCI rises to 1.67, 2.40, and 3.1% 0f the GDP, respectively. However, with log-log form entailing SSMAs, for 5% nominal interest rate the WCI is 0.81 % of the GDP and as nominal interest rate rises to 10, 15, and 20% the WCI rises to 1.50, 2.15, and 2.78% 0f the GDP, respectively. The detailed results are presented in table 9, and discussed below:

Table 9: Estimated WCI (% Of GDP) Using ARDL Model.

ρt	DZ=1.09-0.11LTB		
%	W(ρ) as % of GDP	CV=LB	EV=UB
1	0.21	0.21	0.22
5	0.90	0.86	0.94
10	1.67	1.55	1.82
15	2.40	2.16	2.71
20	3.10	2.72	3.66

ρt	Z =0.942-0.11LTB		
%	W(ρ) as % of GDP	CV=LB	EV=UB
1	0.19	0.19	0.19
5	0.81	0.78	0.84
10	1.50	1.41	1.61
15	2.15	1.96	2.39
20	2.78	2.48	3.20

1. Marty (1999) has argued that the double-log specification performs better during the period of moderate inflation, but is unlikely to perform better in times of hyperinflation. Lucas (2000) also uses a log-log form as it performs better on the US time series, as these series do not contain hyperinflation regions or interest rates near zero. This resembles prevailing stance of the Pakistan economy. Hence, our WCI estimates from log-log form are more relevant to the economic situation in Pakistan.
2. The cointegration is achieved by incorporating dummy variables defining different regime shifts. All the models have almost similar model fits but the results of WCI for long run semi-elasticities ξ are very small as low as .03, which might be due to manifestation of nonlinear unit root, though many dummy variables are attempted to improve these coefficients. Further, the semi-log form provide more than 30 times lower estimate of WCI as compared to log-log form. The values of selection criteria SBC for log-log form are smaller than semi-log form their simple sum counterparts; hence, it is decided not to rely on semi-log as it underrates the true inflation cost. In many countries semi-log form did not work well see, e. g., Bali, T. G. (2000) for US, Chen and Ma (2007) for China, and Gupta and Uwilingiye (2008) for South Africa among others.
3. The selected estimated money demand functions proved to be stable which is consistent with other estimated stable money demand functions for Pakistan by Sarwer et al.(2010) and Qayyum(2005) among others.
4. Our results are similar to Jones et al. (2004), who found in their theoretical model, the estimates of WCI are always much lower in the models with mixed monies—Non-interest bearing monies and interest bearing deposits—than in the models with only non-interest bearing currency (as assumed in SSMA). However, our results are in contrast to Serletis and Virk (2006), who found smaller costs with DMAs. When Pakistan is considered a small economy, our larger cost are in similarity with estimates of Serletis and Yavari (2005, 2007) who have estimated larger costs for smaller economies and smaller costs for big economies.
5. Considering the better fits of log-log form for the estimations of WCI with ARDL approach, we have calculated CV (also known as lower bound of WCI) and EV (also known as upper bound of WCI). These CVs as infimum and EVs as supremum of the WCI, consistently, contain the estimated WCI from ARDL approach.

6. Like Ireland (2009), the defined real rate of return= nominal interest rate - inflation rate, and we use the average of nominal interest rate of last 21 quarters as the prevailing rate. The estimate of WCI, using the average value of real interest rate is ρ_t = 7.433 for last 21 semi-years that corresponds to, 5, 10, 15 and 20% rates inflation [5+7.433=12.43] would provide nominal interest rate 12.43, 17.43, 22.43, and 27.43 only. For range of 5% to 20% inflation rate, it provides small figures in the range of 1.82% and 4.11% of the GDP with log-log specification for both methods of aggregation see table 10.

Table 10: Estimated WCI % of GDP using ARDL model

% Inflation Rate	DMA-EST.	DMA -CV	DMA -EV	SSMA-EST.	SSMA -CV	SSMA-EV
5	2.03	1.86	2.25	1.82	1.68	1.99
10	2.74	2.44	3.17	2.46	2.22	2.78
15	3.44	2.98	4.14	3.08	2.71	3.60
20	4.11	3.48	5.17	3.68	3.17	4.47
(Based on the average value of interest rate ρ_t for last 21 semi-years)						

7. For 5% to 20% inflation rate, the estimated annual WCI is in the rage of 1.82 % to 4.11 % of the real GDP that amounts to the range of 103.24 to 233.33 billion rupees per annum for real GDP of 2010, which is substantial range of WCI. With DMA, log-log form, per annum WCI for 5, 10, and 15% inflation rates amounts to 115.10, 155.65, and 194.96 billion rupees, respectively.

8. For any annual figure of GDP, say, for example, for 2010 real GDP figure of 5670.768 billion rupees the annual loss for 10% inflation rate adds up to a yearly total for lower and upper bound as 138.43 billion rupees and 179.56 billion rupees respectively for DMAs, and 125.79and 157.68 billion rupees entailing SSMAs, see table 11.

Table 11: Estimated WCI (billion Rs. per annum) in Log-Log the Form

% Inflation Rate	DMA-EST.	DMA -CV	DMA -EV	SSMA-EST.	SSMA -CV	SSMA-EV
5	115.10	105.33	127.52	103.24	95.50	112.80
10	155.65	138.43	179.56	139.48	125.79	157.68
15	194.96	168.84	234.50	174.58	153.76	204.37
20	233.33	197.08	293.17	208.81	179.86	253.42
(Based on the average value of interest rate ρ_t for last 21 semi-years)						

9. Based on the average value of nominal interest rate ρ_t for last 21 semi-years, reducing inflation from 15% to 5% would provide a welfare gain of 79.86 billion rupees with log-log DMA model and somewhat less with SSMAs model amounting to welfare gain of 71.34 billion rupees per annum. When the reduction is sought from 20% inflation rate to a moderate 5% inflation rate, these welfare gain figures rise to 118.23 and 105.57 billion rupees for log-log model entailing DMAs and SSMAs, respectively.

4.9 Nonlinear ESTAR Unit Root Test Results

It is felt necessary to further probe the DGP to render cointegration to occur easily. The results of the test are presented in table 12 below. At levels, the null is rejected for all the involved variables and even rejected for DZt and Zt at first difference, however, for LTBt and TBt at first difference it could not be rejected at five percent and one percent levels of significance. It follows that DZt and Zt are variables are more complicated and contains a nonlinear ESTAR unit root. This definitely posed problem in cointegration to occur in linear specification, though the challenge is met by resorting to dummy variables. This evidence of nonlinear cointegration between variants of money income ratios and interest rates requires a separate thesis and may be thought of agenda for future research. The CVs for 1%, 5% and 10% levels are 2.82, 2.22, and 1.92, respectively. Simulated CVs refer to table 1 in Kapetanios et al. (2003, P.364). The selected lag order based on significance of δ, which is indicated by the numbers in parentheses. Note that Rejection of null of linearity implies acceptance of nonlinearity

Table 12 KSS-Test Results

Variable	KSS statistic at level	KSS statistic at 1st Difference
DZt	15.35(2)***	-2.52 (3)**
Zt	9.23(4)***	-2.54(2)**
LTBt	23.36(1)***	-1.83(1)
TBt	18.70(3)***	-2.12(4)*

***, **, and * indicates significance levels at 1, 5, and 10 %, respectively.

5. Conclusions and Policy Implications

5.1 Findings and Policy Implications of Welfare Cost of Inflation

1- Generally, the models with DMA provide larger estimate of WCI as compared to models incorporating SSMA, which might be due to distortions created by inflation rate greater than the opportunity cost of holding cash balances, in general.

2- Given the lower value of information criteria AIC and SBC, among the four (*DZ-LTB, D-LTB, DZ-TB, and Z-TB models*) DZ-LTB model is the best one whose estimated costs can be the most relied upon. Though log-log form both models *DZ-LTB and D-LTB had lower values of* AIC and SBC *information criteria than the models with their simple-sum counterparts, log-log form obviously better fits money income data of the economy of Pakistan* according to Marty's (1999) argument.

3- For 10% inflation rate the welfare estimated cost is 155.65 and 139.48 billion rupees with log-log specification for both of the aggregates, respectively. For 15% inflation rate the welfare estimated cost is 194.96 and 174.58 billion rupees with log-log specification for both of the aggregates, respectively. For 20% inflation rate the welfare estimated cost is 233.33 and 208.81 billion rupees with log-log specification for both of the aggregates, respectively. Thus, large increase in price level could inflict huge inflation cost in the economy; while maintaining lower the price level paybacks to the economy.

4- Reducing inflation rate from 15% to 5% provides a welfare gain 79.86 and 71.34 billion rupees annually with log-log specification using DMA and SSMA respectively, under ARDL approach to cointegration when the average value of interest rate R_t for last twenty one half-years is used as proxy for future interest rate.

5- The above mentioned are substantial costs to society for price inflation even excluding moral costs mentioned earlier. Hence, price inflation must be stopped at lower levels. A natural question arises: to what threshold it should be brought down.

The answer is not difficult considering the recent research scenarios on inflation as it is sometime considered positively influencing the economy but only in low magnitude. For these reasons, it well-known fact that many developed countries try to maintain it around 2 to 3 %.

6- So, inflation must be tamed to limits where its positive influences offsets its bad influences i.e. to the limit where it is contributing more to growth then than its cost to society. Likewise it surely requires careful empirical reckoning of the magnitude of its contribution, as done in this study, in growth at different inflation rates. Comparing carefully estimated welfare costs and contribution in growth an optimum rate of inflation can found where its contributions less costs are some positive quantity.

7- *As the DZ-LTB model proved best on the basis of information criteria, it* comes out that DMA's better describe money income ratio behavior properly. Construction of DMAs and GDP series takes a lot of time at lower frequencies and hampers the researchers for further work. Hence the study recommends the calculation, use, publication and dissemination of, at least quarterly, Divisia monetary aggregates and indices series by SBP in its routine and research works

8- It revealed that DZ and Z variables are more complicated and contains a nonlinear ESTAR unit root. These definitely posed problems in cointegration to occur in linear specification, but the challenge is met by resorting to dummy variables. This evidence of nonlinear cointegration between variants of money income ratios and interest rates may be thought of agenda for future research.

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