

A Visualization of Fisher's Least Significant Difference Test

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Abstract

When a hypothesis of the equality of several means is tested by using ANOVA and it is rejected for a given level of significance, Least Significant Difference (LSD) test is most commonly applied to know which mean(s) is(are) significantly different from which. This procedure usually becomes lengthy. To overcome this situation, a new visualization of LSD test is proposed in this article. This graphical form provides a quicker and clearer picture of the least significant difference test. It is an objective approach when the difference between means is approximately equal to the LSD. Short listing of experimental treatments becomes quite easy by this visualization.

Keywords: Visualization, ANOVA, Fisher's least significant difference test.

1. Introduction

Two population means are compared by a two sample t-test. But for the comparison of more than two means pair-wise t-test cannot be used because (i) it is time consuming and the procedure gets complicated and (ii) a great increase in the overall level of significance. Fisher introduced Analysis of Variance (ANOVA) technique for the comparison of several populations means simultaneously. When hypothesis of equality of several means cannot be accepted in an analysis of variance for a given level of significance, then the experimenter is interested to know which means are significantly different from which. For this purpose some post hoc tests or multiple comparison tests were proposed. Fisher's Least Significant Difference (LSD) test is one of the post hoc tests. Fisher proposed this simplest and widely used LSD test in 1935. This method is based on the smallest difference among the two means which is considered to be

significant at a particular level of significance. All pair-wise absolute differences are compared with LSD and decision of significant pairs is made. Carmer and Swanson (1973) compared a number of multiple comparison tests on the basis of Monte Carlo simulation studies. They reported that LSD method is an effective test for the detection of true differences in means if it is applied when F-test in the ANOVA is significant at a particular level of significance. If there are k samples, difference between means for all $[k(k-1)/2]$ possible pairs is compared with the value of LSD. This procedure becomes lengthy for large k ; therefore, visualization of LSD is the best solution.

2. Visual Extension of the LSD by Iqbal And Clarke (2003)

When number of treatments k is large then LSD method becomes lengthy, time consuming and laborious. One solution to the problem was given by Iqbal and Clarke (2003) by introducing Visual Least Significant Difference (VLSD). Their method proceeds as follows. A line representing positions of ordered means is drawn horizontally. Another horizontal line is drawn above this line to represent LSD. Both the lines are drawn on the same scale. A visual comparison between difference in means and length of LSD is made. If the distance between two treatments is greater than the LSD line's distance then it is concluded that these two treatment means are significantly different. This is also discussed in Iqbal (1999).

This graph shown in Figure-1 is taken from Iqbal and Clarke (2003), where they used a dataset for cotton crop with 10 treatments which was taken from an experiment on cotton that was repeated for 14 consecutive seasons in a row from 1982 to 1995 on the Randomized Complete Block Design with three replications. In the example the VLSD method is applied on a response Seed Cotton Yield for last 4 years i.e. for the years 1992, 1993, 1994 and 1995. Here it is concluded that in case of year 1992, treatment 1 and 2 are not significantly different from each other but significantly different from 4, 5, 6, 7, 8, 9 and 10. Treatments 4 and 10 are not significantly different from each other but they are from rest of the treatments which are 1, 2, 3, 5, 6, 7, 8 and 9. Also treatments 3, 5, 6, 7, 8, and 9 are not significantly different from each other.

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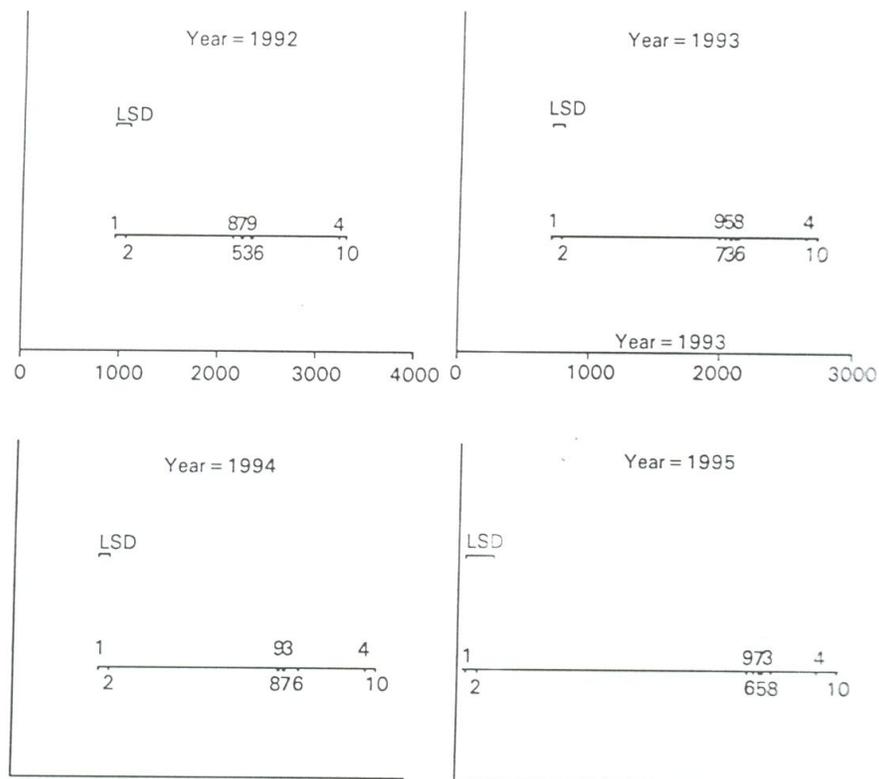


Figure: 1

3. A New Visualization Method For LSD Test

In this section, a new visualization method for LSD Test is proposed which gives a clearer and quick picture of the LSD. The value of LSD is calculated in the usual manner. The outlines of the procedure are as under.

- (i) All means (arranged in ascending order) are plotted at equal distances using thin lines.
- (ii) A thick line representing LSD is augmented at the top of each thin line of means. Both thin lines of means and thick line of LSD are drawn on the same scale.
- (iii) A horizontal dashed line is drawn at the top of each of the augmented lines (mean + LSD).
- (iv) A mean is significantly different from all those means if the horizontal dashed line of the mean cuts the thin line of any other mean.

The procedure is explained with the following examples.

Example: 1. Data is taken from Montgomery (1996), pp102.

$$\bar{Y}_1=9.8, \bar{Y}_2=15.4, \bar{Y}_3=16.6, \bar{Y}_4=21.6, \bar{Y}_5=10.8,$$

$$\alpha =0.05, \text{ LSD}= 3.75$$

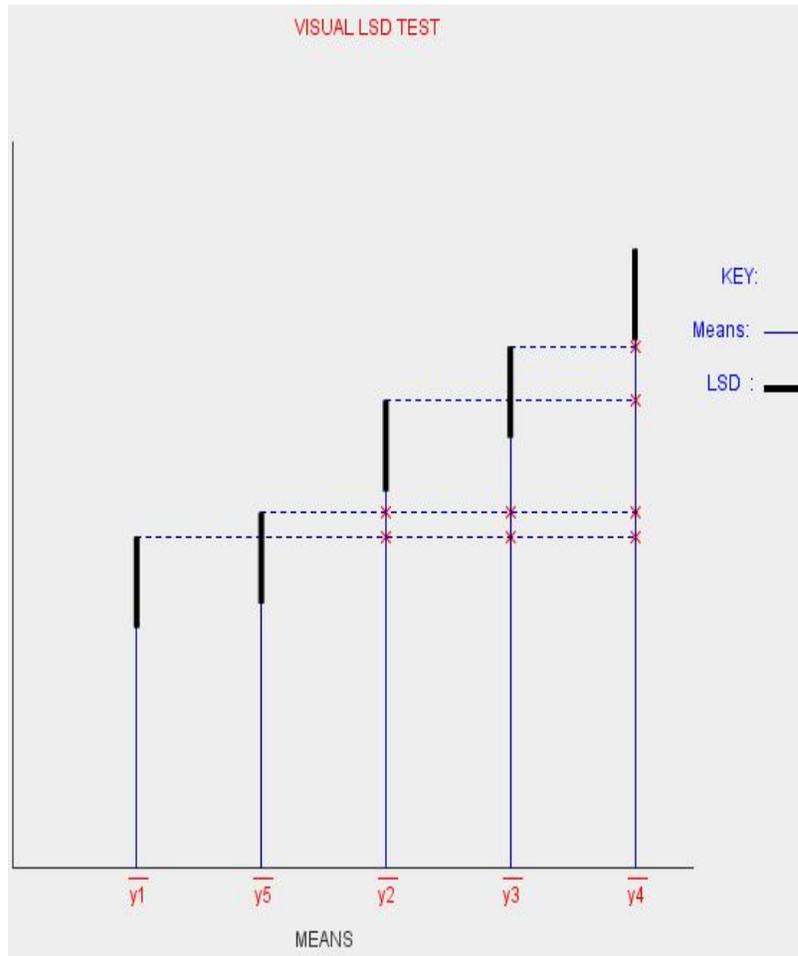


Figure: 2

Significant pairs are

\bar{y}_1 & \bar{y}_2	\bar{y}_2 & \bar{y}_5
\bar{y}_1 & \bar{y}_3	\bar{y}_3 & \bar{y}_4
\bar{y}_1 & \bar{y}_4	\bar{y}_3 & \bar{y}_5
\bar{y}_2 & \bar{y}_4	\bar{y}_4 & \bar{y}_5

It is also clear that y_1 and y_5 form a group which is homogenous, whereas y_2 and y_3 form another group, whereas y_4 is significantly different from all others.

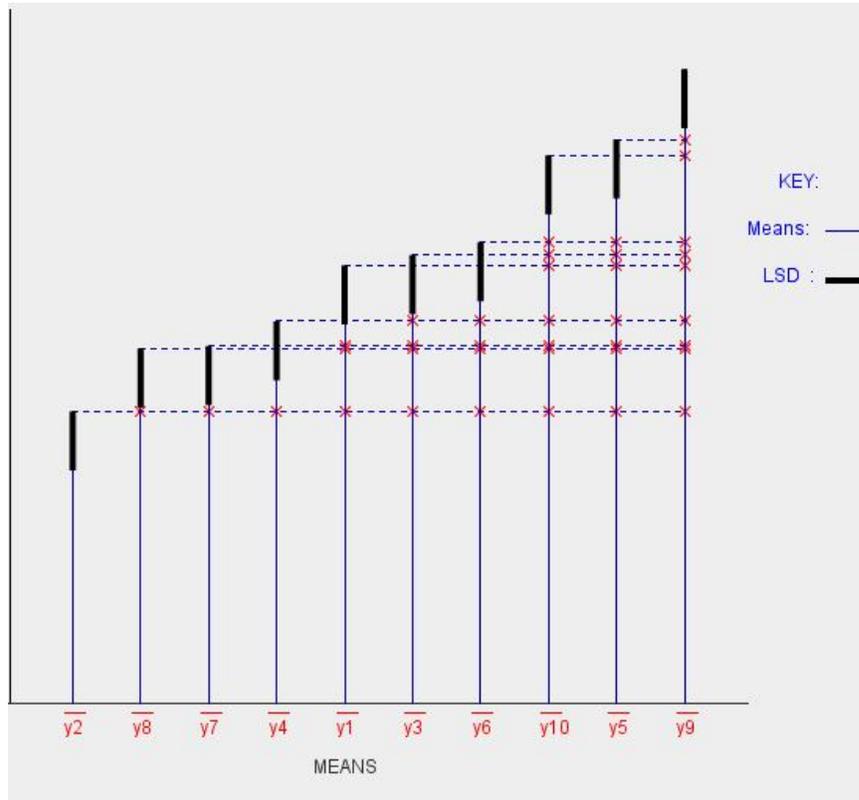


Figure: 3

It is evident that significant pairs are

- | | | |
|------------------------------|------------------------------|------------------------------|
| \bar{y}_2 & \bar{y}_8 | \bar{y}_8 & \bar{y}_5 | \bar{y}_1 & \bar{y}_{10} |
| \bar{y}_2 & \bar{y}_7 | \bar{y}_8 & \bar{y}_9 | \bar{y}_1 & \bar{y}_5 |
| \bar{y}_2 & \bar{y}_4 | \bar{y}_7 & \bar{y}_1 | \bar{y}_1 & \bar{y}_9 |
| \bar{y}_2 & \bar{y}_1 | \bar{y}_7 & \bar{y}_3 | \bar{y}_3 & \bar{y}_{10} |
| \bar{y}_2 & \bar{y}_3 | \bar{y}_7 & \bar{y}_6 | \bar{y}_3 & \bar{y}_5 |
| \bar{y}_2 & \bar{y}_6 | \bar{y}_7 & \bar{y}_{10} | \bar{y}_3 & \bar{y}_9 |
| \bar{y}_2 & \bar{y}_{10} | \bar{y}_7 & \bar{y}_5 | \bar{y}_6 & \bar{y}_{10} |
| \bar{y}_2 & \bar{y}_5 | \bar{y}_7 & \bar{y}_9 | \bar{y}_6 & \bar{y}_5 |
| \bar{y}_2 & \bar{y}_9 | \bar{y}_4 & \bar{y}_3 | \bar{y}_6 & \bar{y}_9 |
| \bar{y}_8 & \bar{y}_1 | \bar{y}_4 & \bar{y}_6 | \bar{y}_{10} & \bar{y}_9 |
| \bar{y}_8 & \bar{y}_3 | \bar{y}_4 & \bar{y}_{10} | \bar{y}_5 & \bar{y}_9 |
| \bar{y}_8 & \bar{y}_6 | \bar{y}_4 & \bar{y}_5 | |
| \bar{y}_8 & \bar{y}_{10} | \bar{y}_4 & \bar{y}_9 | |

4. Conclusions

From the above examples, it is obvious that the proposed new graphical display is far clearer than the VLSD proposed by Iqbal and Clarke (2003). It provides rapid comparisons and thus shows the results in a comprehensible way. In contrast to Iqbal and Clarke (2003) VLSD, which was subjective in nature, the proposed one is objective which particularly gives good results in marginal cases. It also carries an aesthetic element.

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