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Economic Reliability Acceptance Sampling Plans from Truncated Life Tests based on the Burr Type XII Percentiles

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Abstract

In this article, economic reliability acceptance sampling plan (ERASP) is developed for the Burr type XII distribution when the life test is truncated at pre-specified designed parameters. The minimum termination time is necessary to ensure that the specified life percentile is found under a given producer's risk. The operating characteristic values of the proposed plan are presented for various parameters. A comparative study of proposed plan and existing plan developed by Lio et al. (2010) is also discussed. The result is illustrated by a real dataset example**.**

Keywords: Reliability test plan, minimum sample size, truncated life test, producer's risk, operating characteristics.

1. Introduction

In the global world, statistical quality control is of great importance. This is considered necessary to ascertain the visible operating characteristic values of the proposed plan. As such the inspection of the various manufacture products is considered essential for ensuring the trustworthiness of an item with regards to its lifetime. The main purpose of the acceptance sampling plan is to ensure that the lifetime of the product is according to the desired standard of the consumer or not. In case the life test shows that the mean (average) life of the product is above the desired standard, the submitted lot is accepted otherwise the same is rejected.

The other advantage of acceptance sampling plan is that it provides a strict inspection of an assembled product before it can be sent for the consumer's use according to the desired standard. When the confident limit of the mean life is established it becomes easier to arrive at a definite decision as to whether the submitted lot may be accepted or rejected. It is therefore important to note that whatever the type of sampling plans may be, the producer's as well as the consumer's risks are always attached with the sampling plans. The purpose of this study is to minimize the experimental time in view of the given designed parameters and also satisfying the producer's risks. So, we can say that the proposed plan is more economical in the means of saving cost, time, energy and labor. No attention from the scholars have been performed to develop an Economic reliability acceptance sampling plan (ERASP) for Burr type XII distribution assuming that the lifetime of an item basis this distribution with pre-assume shape parameters. Hence, economic reliability acceptance sampling plan (ERASP) is adopted for a truncated life test when the lifetime of an item follows the Burr type XII distribution where as, in the single/ordinary acceptance sampling plan is a primary to all acceptance sampling schemes. Life test can be conducted by inspecting the single item or inspection result can be defined into two branches of outcomes is called the single accepting sampling schemes. Some ordinary acceptance sampling plan can be found by Epstein (1954) Baklizi (2003), Jun et al. (2006), Kantam et al. (2006), Aslam and Shahbaz (2007) Balakrishnan et al. (2007), Rosaiah et al. (2008), Aslam et al. (2010) and Mughal et al. (2010).We must therefore understand the necessity of economic reliability acceptance sampling plans (ERASP) and the techniques available for the determination of the quality and ensuring future improvements in this regard. The Burr type XII distribution (Burr XII) was first presented in the literature by Burr (1942). The Burr XII has many applications in variety of context see, for example, Lio et al. (2010).

An Advisory Group on Reliability of Electronic Equipment (1957) defined reliability as follows: ''Reliability is the probability of performing without failure specified function under given conditions for a specified period of time.'' Reliability sampling plans considered obtaining the acceptability of an item at future point of view, it usually includes some type of life testing where as acceptance sampling plans are used on a lot-by-lot basis and very beneficial approach to maximizing the quality at lowest cost. Acceptance sampling must be proposed in a usual way when the inspection performed and it is also the primary tool of acceptance control.

2. Economic Reliability Sampling Plans

The probability density function (pdf) of three parameter Burr XII random variable T can be written as,

$$f(t)=\frac{\beta}{\alpha k}\left(\frac{t}{\alpha}\right)^{\beta-1}\left[1+\left(\frac{t}{\alpha}\right)^{\beta}\right]^{k-1},\ t>0,\beta>0,\ k>0,\ \alpha>0. \qquad (2.1)$$

Where (β, k) >0, are shape parameters and α is the scale parameter. The cumulative distribution function (cdf) of Burr XII is given by,

$$F(t)=1-\left[1+\left(\frac{t}{\alpha}\right)^{\beta}\right]^{-k}, \quad t>0,\beta>0,k>0,\alpha>0. \qquad (2.2)$$

It is important to note that when k =1, the Burr XII reduces to the log-logistic distribution. Wu et al (2005) show that the shape parameter β plays a vital role for the Burr XII. For the objective of saving the cost, test time, labor and energy, an economic reliability acceptance sampling plan (ERASP) can be conducted to find the smallest termination time to determine a certain mean life of items when the life test is terminated at a pre-assigned sample size n and the number of failures recorded does not exceed a given terminating number r. So, the life test is terminated if the termination time t ends or the rth failure occurs if we inspect the rn sample, whichever is observed first. The Burr XII is not a symmetrical distribution, then the mean life can not be moderate to describe the measures of central location for this distribution. Therefore, we propose an ERASP for the percentile of Burr XII under the truncated life test.

The 100q-th percentile of equation (2.2) is,

$$t_q = \alpha \left[\left(\frac{1}{(1-q)} \right)^{1/k} - 1 \right]^{\beta} \tag{2.3}$$

For simplification, let $\delta_q = t/t_q$ then cdf of equation (2.2) can be written as,

$$F(t,\delta) = 1 - \left[1 + \delta^{\beta} \left(\left(\frac{1}{1-q} \right)^{1/k} - 1 \right) \right]^{-k} \quad ,t > 0 \tag{2.4}$$

Considering if the life test schedule is t, the probability of accepting a good lot can be at least $1 - P^*$, and the maximum number of acceptable bad items in the lot (batch) is r. The ERASP for the percentile under the truncated life test is to build up the minimum termination time t, for this given terminating number r and sample size n, such that the producer's risk P^*, the probability of rejecting a good lot, does exceed $1 - P^*$. A good lot define as a true 100q-th percentile, t_q is greater then the specified percentile, t_q^0 which can be written as $t_q > t_q^0$. Therefore, for a given P^*, the design parameters of proposed ERASP are $(n, r, t_q/t_q^0)$. The given proposed ERASP is independent of the scale parameter α. Consider that the lot size is large enough; the probability of accepting a lot can be evaluated by the cumulative binomial probability because decision about the submitted lot is either accepted or rejected. For more justification one may refer to Stephens (2001). The lot acceptance probability for the proposed plan can be written as,

$$L(p) = \sum_{r=1}^{r-1} \binom{n}{r} p^r (1-p)^{n-r} \tag{2.5}$$

Where $p = F(t)$ is the probability of a failure recorded during the time t given a pre-assumed 100q-th percentile of lifetime t_q^0, so $p = F\left((t/t_q^0)(1/d_q)\right)$ and $d_q = t_q/t_q^0$. The minimum values of sample size (n) is determined by solving the following inequality, (2.6),

$$L(p) = \sum_{r=1}^{r-1} \binom{n}{r} p^r (1-p)^{n-r} \leq 1 - P* \qquad (2.6)$$

Now, we can determine the minimum termination time for given information at the various values of $\left(n, r, k, \beta, t_q / t_q^0\right)$ and $P*$ respectively, when the following (2.7) inequality is satisfied,

$$L(p) \geq 1 - P* \qquad (2.7)$$

Table 2.1, 2.2, 2.3, 2.4 describe the minimum termination time for given information (defined above) at the different values of designed parameters. The selection of designed parameters are used for comparison purpose which is already suggested by Lio et al.(2010) in your research article, for more detail see Lio et al. (2010). From these tables, we can see the different behaviors of termination time, when the value of shape parameter increases, the trend of the termination time decreases.

Table 2.1: Test termination ratios for the proposed plan incase of Burr type XII distribution with $k = 0.08$ and $\beta = 5.47$ for specified values of terminating number *r* and producer risk $P* = 0.05$.

n / *r*	*2r*	*3r*	*4r*	*5r*	*6r*	*7r*	*8r*	*9r*	*10r*
1	0.6950	0.6400	0.6043	0.5780	0.5580	0.5420	0.5280	0.5166	0.5060
2	0.9900	0.8660	0.8000	0.7570	0.7250	0.7000	0.6800	0.6630	0.6480
3	1.1870	0.9920	0.9000	0.8430	0.8030	0.7720	0.7480	0.7280	0.7108
4	1.3390	1.0790	0.9660	0.8986	0.8520	0.8170	0.7890	0.7670	0.7479
5	1.4630	1.1450	1.0140	0.9380	0.8864	0.8480	0.8180	0.7940	0.7735
6	1.5740	1.1990	1.0510	0.9680	0.9120	0.8710	0.8390	0.8140	0.7924
7	1.6670	1.2420	1.0820	0.9924	0.9330	0.8900	0.8560	0.8290	0.8071
8	1.7490	1.2800	1.1070	1.0120	0.9500	0.9050	0.8700	0.8423	0.8191
9	1.8220	1.3130	1.1290	1.0290	0.9640	0.9175	0.8817	0.8529	0.8290

Table 2.2: Test termination ratios for the proposed plan incase of Burr type XII distribution with $k = 0.08$ and $\beta = 5.47$ for specified values of terminating number *r* and producer risk $P^* = 0.01$.

n / *r*	*2r*	*3r*	*4r*	*5r*	*6r*	*7r*	*8r*	*9r*	*10r*
1	0.5040	0.6390	0.6040	0.4250	0.4110	0.3990	0.3890	0.3800	0.3740
2	0.7800	0.7040	0.6600	0.6280	0.6050	0.5860	0.5700	0.5570	0.5460
3	0.9450	0.8290	0.7670	0.7260	0.6960	0.6730	0.6530	0.6370	0.6230
4	1.0700	0.9150	0.8370	0.7880	0.7530	0.7260	0.7040	0.6860	0.6697
5	1.1750	0.9800	0.8890	0.8320	0.7930	0.7630	0.7390	0.7190	0.7017
6	1.2630	1.0300	0.9290	0.8670	0.8240	0.7900	0.7650	0.7430	0.7260
7	1.3440	1.0770	0.9620	0.8950	0.8480	0.8130	0.7860	0.7630	0.7442
8	1.4100	1.1140	0.9900	0.9170	0.8680	0.8310	0.8030	0.7790	0.7596
9	1.4800	1.1480	1.0140	0.9370	0.8850	0.8470	0.8170	0.7920	0.7722

Table 2.3: Test termination ratios for the proposed plan incase of Burr type XII distribution with $k = 5.49$ and $\beta = 0.85$ for specified values of terminating number *r* and producer risk $P^* = 0.05$.

n / *r*	*2r*	*3r*	*4r*	*5r*	*6r*	*7r*	*8r*	*9r*	*10r*
1	0.1880	0.1160	0.0824	0.0636	0.0514	0.0429	0.0364	0.0319	0.0280
2	0.9695	0.5630	0.3891	0.2941	0.2345	0.1939	0.1646	0.1426	0.1250
3	1.7204	0.9719	0.6638	0.4980	0.3956	0.3264	0.2762	0.2390	0.2100
4	2.3256	1.2950	0.8790	0.6579	0.5212	0.4293	0.3630	0.3143	0.2760
5	2.8210	1.5560	1.0510	0.7845	0.6200	0.5100	0.4320	0.3730	0.3276
6	3.2280	1.7670	1.1910	0.8870	0.7000	0.5760	0.4877	0.4210	0.3690
7	3.5700	1.9400	1.3070	0.9729	0.7680	0.6315	0.5339	0.4600	0.4040
8	3.8700	2.0940	1.4050	1.0430	0.8250	0.6780	0.5730	0.4940	0.4337
9	4.1300	2.2250	1.4910	1.1070	0.8740	0.7176	0.6064	0.5235	0.4590

Table 2.4: Test termination ratios for the proposed plan incase of Burr type XII distribution with $k = 5.49$ and $\beta = 0.85$ for specified values of terminating number *r* and producer risk $P^* = 0.01$.

n / r	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.0270	0.0170	0.0120	0.0093	0.0075	0.0063	0.0053	0.0046	0.0041
2	0.3417	0.2000	0.1390	0.1040	0.0839	0.0692	0.0590	0.0510	0.0444
3	0.8100	0.4610	0.3150	0.2360	0.1880	0.1550	0.1310	0.1140	0.0987
4	1.2530	0.7080	0.4810	0.3610	0.2860	0.2350	0.1990	0.1720	0.1508
5	1.6800	0.9290	0.6290	0.4700	0.3720	0.3060	0.2590	0.2240	0.1968
6	1.9999	1.1200	0.7560	0.5640	0.4460	0.3670	0.3100	0.2680	0.2350
7	2.3500	1.2860	0.8680	0.6470	0.5110	0.4200	0.3550	0.3070	0.2690
8	2.6400	1.4300	0.9650	0.7190	0.5680	0.4670	0.3940	0.3405	0.2985
9	2.8900	1.5600	1.0530	0.7830	0.6180	0.5080	0.4290	0.3701	0.3250

3. Example

A manufacturer would like to develop an ERASP and know whether the life of his products are above than the specified percentile life, t_q = 2000 hours. Consider that the lifetime of products follows the Burr type XII distribution, the designed parameters of the proposed ERASP are $\left(n, r, k, \beta, t_q / t_q^0\right)$= (5, 2, 5.49, 0.85, 0.1040) for $P^* = 0.01$. From table 2.4, the manufacturer needs to select a sample of 10 products and put on test, the lot is rejected if more than 2 failures occur during 208 hours, otherwise accept it. For comparison purpose, the same real life example as quoted by Lio et al. (2010) has been illustrated. “The dataset is regarding the first failure times of small electric carts used for internal transportation and delivery in a large manufacturing facility, an experimenter would like to establish the true unknown 10th percentile lifetime for the small electric cart mentioned above to be at least 2 months and the life test would be ended at 6 months, which should have led to the ratio $t/t_{0.1}^0 = 3$months”. Thus, with $r = 1$, $P^* = 0.05$, $k = 0.08$, and $\beta = 5.47$,the experimenter should obtain from Table 4.1 that the sample size n must be 6 and the sampling plan $\left(n, r, k, \beta, t_q / t_q^0\right)$=(6,1,0.08,5.47,3.0). The experimenter select six items and would accept the null hypothesis if no more than one failure occur during the 3 months.

4. Comparison

In order to compare the proposed ERASP, our measurements are less than the existing plan developed by Lio et al. (2010) for common designed parameters and various risks.
In Tables 4.1, 4.2, 4.3and 4.4 the upper values of cells presenting the proposed plan and lowest values denoting the existing plan. Suppose that we want to develop an ERASP with pre-specified percentile is t_q / t_q^0 =5000 hours when $P^* = 0.01, k = 0.08$

and $\beta = 5.47$ .Choose 18 items from the lot and put on test, if 2nd failure recorded during the termination time 2785 hours, we reject the lot otherwise the same is accepted. For the same designed parameters if the exiting plan using 10000 hours produces the same result, we can say that the proposed ERASP is more beneficial in the sense of saving cost, time, energy and labor.

Table 4.1: Comparison of the test termination ratios of the proposed plan with that of Lio et al. (2010) when $P^* = 0.05, k = 0.08$ and $\beta = 5.47$

n / *r*	*2r*	*3r*	*4r*	*5r*	*6r*	*7r*	*8r*	*9r*	*10r*
1					0.558 3.000	0.542 2.500	0.528 2.000		
2									
3				0.843 2.500	0.803 2.000				
4			0.966 3.000				0.789 1.500		
5									
6				0.968 2.000					
7			1.082 2.500			0.890 1.500			
8			1.107 2.500						
9									

Table 4.2: Comparison of the test termination ratios of the proposed plan with that of Lio et al. (2010) when $P^* = 0.01, k = 0.08$ and $\beta = 5.47$

n / *r*	*2r*	*3r*	*4r*	*5r*	*6r*	*7r*	*8r*	*9r*	*10r*
1								0.380 3.000	0.374 2.500
2								0.557 2.000	
3									
4				0.788 3.000		0.726 2.000			
5									
6				0.867 2.500					
7					0.848 2.000				
8									
9			1.014 3.000						

Table 4.3: Comparison of the test termination ratios of the proposed plan with that of Lio et al. (2010) when $P^* = 0.05, k = 0.08$ and $\beta = 5.47$

n / r	2r	3r	4r	5r	6r	7r	8r	9r	10r
1									
2									
3									
4								0.3143 2.5000	
5									
6									
7							0.5339 2.5000		
8									
9								0.5235 2.0000	

Table 4.4: Comparison of the test termination ratios of the proposed plan with that of Lio et al. (2010) when $P^* = 0.01, k = 0.08$ and $\beta = 5.47$

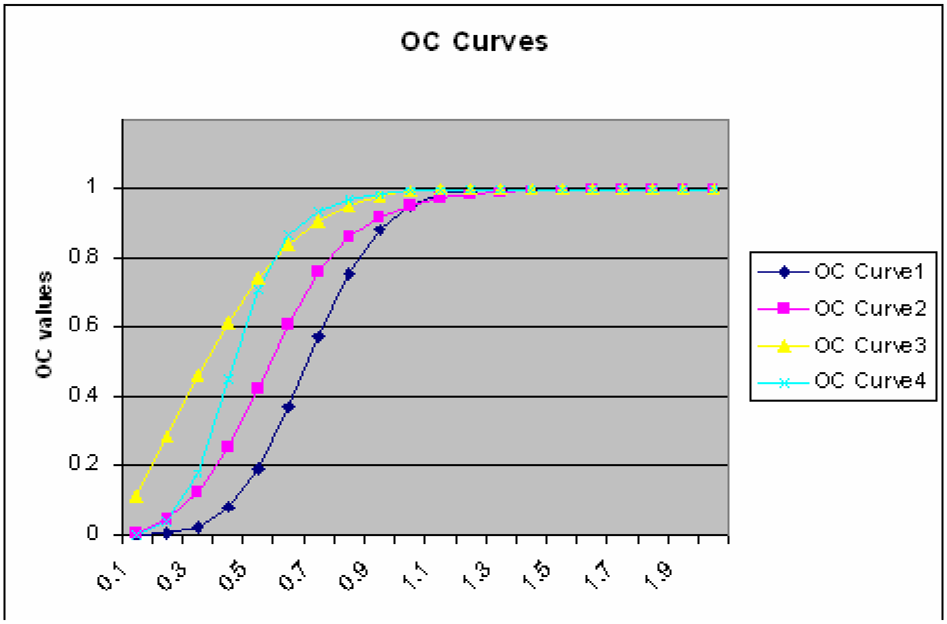
n / r	2r	3r	4r	5r	6r	7r	8r	9r	10r
1									
2									
3									
4									0.1508 3.0000
5									
6									
7									
8									
9								0.3701 2.5000	

In quality control, Operating Characteristic (OC) Curves plays a significant role to determine the probability of accepting manufactured lots or batches when using different sampling plans. It shows the relationship between the percentage-defectives and the probability of acceptance when we conduct a lifetime experiment. OC curves help us in the selection of acceptance sampling plans and also help in reducing risks. The different behavior of OC curves presented in Table 4.5 and Fig 4.6.

Table 4.5: Operating Characteristic values for the proposed plan incase of Burr type XII distribution with $k = 0.08$ and $\beta = 5.47$ for specified values of terminating number *r* and producer's risk $P^* = 0.01, 0.05$

$d_q = \frac{t_q}{t_q^0}$	(OC Curve1) $n=15, \alpha=0.05$ $r=2$ and $t/t_q=0.843$	(OC Curve 2) $n=6, \alpha=0.05$ $r=0$ and $t/t_q=0.558$	(OC Curve 3) $n=6, \alpha=0.01$ $r=2$ and $t/t_q=0.945$	(OC Curve 4) $n=9, \alpha=0.01$ $r=0$ and $t/t_q=0.38$
0.1	8.78699E-05	0.006762422	0.112409251	0.002524702
0.2	0.003052847	0.041707530	0.285285722	0.038411743
0.3	0.021527943	0.120307735	0.458693319	0.178423905
0.4	0.078064739	0.250540972	0.613001430	0.446573754
0.5	0.193156331	0.423817309	0.740162183	0.710056601
0.6	0.367342388	0.606211560	0.837743360	0.864249662
0.7	0.570424401	0.756080961	0.906878828	0.935724665
0.8	0.753450141	0.856517366	0.951408967	0.967763442
0.9	0.880833601	0.916227562	0.977063328	0.982760121
1.0	0.950387306	0.950172482	0.990154072	0.990223825
1.1	0.981432993	0.969477643	0.996090098	0.994168045
1.2	0.993447246	0.980695761	0.998526484	0.996367068
1.3	0.997728600	0.987408300	0.999458299	0.997651574
1.4	0.999204361	0.991550282	0.999801078	0.998432770
1.5	0.999713465	0.994182708	0.999925756	0.998924799
1.6	0.999892943	0.995902097	0.999971523	0.999244319
1.7	0.999958344	0.997053432	0.999988706	0.999457463
1.8	0.999983108	0.997841952	0.999995357	0.999603064
1.9	0.999992868	0.998393098	0.999998020	0.999704654
2.0	0.999996871	0.998785492	0.999999125	0.999776891

Fig 4.6: OC Curves for the Table 4.5



t_q / t_q^0

5. Conclusion

In this article, the methodology to obtain the minimum termination ratio is presented. The tables are constructed for different designed parameters. The measurements are compared with the existing plan developed by Lio et al. (2010) and it is concluded the proposed ERASP is better the in reducing the termination time. Hence, the propose ERASP is more economical in reducing cost, time, energy and labor which led to the final conclusion about the lot submitted by the vendor. Investigating the design of ERASP to other lifetime distributions such as the generalized pareto and generalized exponential distribution in future research.

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REFERENCES

Ali, U., Mughal, A.R., (2010). *Statistics for graduate students,* 1st ed., Al-Fouzi Publishers, Lahore.

Aslam, M., Ahmed, M., and Mughal, A.R., (2010).Group Acceptance sampling plan for lifetimes having generalized pareto distribution. *Pakistan Journal of Commerce and Social Sciences.* 4(2), 185-193.

Aslam, M., Mughal, A.R., Hanif, M., and Ahmed, M. (2010). Economic Reliability Group Acceptance Sampling Based on Truncated Life Tests Using Pareto Distribution of the Second Kind. *Communications of the Korean Statistical Society*, 17(5), 725–732.

Aslam, M., and Shahbaz, M. Q. (2007). Economic reliability tests plans using the generalized exponential distribution, *Journal of Statistics*, 14, 52-59.

Baklizi, A. (2003). Acceptance sampling based on truncated life tests in the Pareto distribution of the second kind. *Advances and Applications in Statistics*, 3(1), 33-48.

Balakrishnan, N., Leiva, V., and Lopez, J. (2007). Acceptance sampling plans from truncated life tests based on the generalized Brinbaum-Saunders distribution. *Communication in Statistics-Simulation and Computation*, 36, 643-656.

Burr, I.W., (1942). Cumulative frequency functions, *Annals of Mathematical Statistics*, 13, 215–232.

Epstein, B. (1954). Truncated life tests in the exponential case. *Annals of Mathematical Statistics*, 25, 555-564.

Jun, C.-H., Balamurali, S. and Lee, S.-H. (2006). Variables sampling plans for Weibull distributed lifetimes under sudden death testing. *IEEE Transactions on Reliability*, 55(1), 53-58.

Kantam, R.R.L. and Srinivasa Rao, G. and Sriram, G. (2006). An economic reliability test plan: log-logistic distribution, *Journal of Applied Statistics*, 33(3), 291-296.

Lio, Y.L., Tsai, T.-R and Wu, S.-J. (2010). Acceptance sampling plans from truncated life tests based on the burr type XII percentiles. *Journal of the Chinese Institute of Industrial Engineers*, 27(4), 270-280.

Mughal, A.R., Aslam, M., Hussain, J., Rehman, A., (2010). Economic Reliability group Acceptance Sampling Plans for Lifetimes Following a Marshall-Olkin Extended Distribution. *Middle Eastern Finance and Economics*.7, 87-93.

Rosaiah, K., Kantam, R. R. L. and Reddy, J. R. (2008). Economic reliability test plan with inverse Rayleigh variate, *Pakistan Journal of Statistics*, 24, 57-65.

Schilling, E.G., Neubauer, D.V. (2008). *Acceptance Sampling in Quality Control*, CRC Press, Taylor & Francis Group, New York, USA.

Stephens, K.S. (2001). *The Handbook of Applied Acceptance Sampling: Plans, Procedures and Principles*. Milwaukee, WI: ASQ Quality Press.

Wu, J.W. and H.Y. Yu, (2005).Statistical inference about the shape parameter of the Burr type XII distribution under the failure-censored sampling plan, *Applied Mathematics and Computation*, 163,443–482.