Pak. J. Commer. Soc. Sci.
2010 Vol. 4 (2), 185-193

Group Acceptance Sampling Plan for Lifetime Data Using Generalized Pareto Distribution

Muhammad Aslam (Corresponding author)
Department of Statistics, Forman Christian College University, Lahore, Pakistan.
E-mail: aslam_ravian@hotmail.com

Munir Ahmad
Professor of Statistics
National College of Business Administration and Economics, Lahore, Pakistan
E-mail: drmunir@brain.net.pk

Abdul Razzaqe Mughal
National College of Business Administration and Economics, Lahore, Pakistan
E-mail: abdur_razzaque@live.com

Abstract

In this paper, a group acceptance sampling plan (GASP) is introduced for the situations when lifetime of the items follows the generalized Pareto distribution. The design parameters such as minimum group size and acceptance number are determined when the consumer's risk and the test termination time are specified. The proposed sampling plan is compared with the existing sampling plan. It is concluded that the proposed sampling plan performs better than the existing plan in terms of minimum sample size required to reach the same decision.

Keywords: Group sampling plan, generalized Pareto distribution, consumer's risk, producer's risk, operating characteristics

1. Introduction

The life of electronic accessories cannot be pre-accessed. This can only be observed through statistical distributions. In case, there is a failure time distribution of such items, a sampling plan is established which lead to an ample justification to accept or reject the submitted lot. The acceptance sampling plans are essential tools to determine the trustworthiness of such product with regard to their life time. In ordinary acceptance sampling plan, a single item is inspected at a time where as in group acceptance sampling plan (GASP), a multiple number of items are inspected on the basis of the number of testers available to the experimenter for testing. This reflects that GASP has advantage over the ordinary sampling plan because substantial testing time and cost can be reduced. In this paper, a group acceptance sampling plan is proposed when lifetime of the items follow the generalized Pareto distribution with known shape parameters. Sudden death testing is an example for such situations. Some applications of sudden death testing can be found in Pascual and Meeker, (1998), Vlcek et al. (2003) and Jun et al. (2006). Recently, Aslam and Jun (2009), Rao (2009), Aslam et al. (2010) introduced some group acceptance sampling plans for failure time distributions. Some ordinary acceptance sampling plans have been appeared in Epstein (1954), Goode and Kao (1961), Kantam and Rosaiah

(1998), Baklizi (2003), Baklizi and EI Masri (2004), Rosaiah and Kantam (2005), Kantam et al. (2006), Tsai and Wu (2006), Balakrishnan et al. (2007), Rosaiah et al. (2006, 2007), Aslam and Shahbaz (2007), Rosaiah et al. (2008) and Aslam and Kantam (2008).
The Generalized Pareto distribution was first introduced by Abd Elfattah et al. (2007).
The probability density function and the cumulative distribution function of generalized Pareto distribution are given below.

$$f(t;\alpha,\beta,\lambda,\delta)=\frac{\delta\alpha}{\beta}\left(\frac{t-\lambda}{\beta}\right)^{\delta-1}\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-(\alpha+1)} \tag{1}$$

and

$$F(t;\alpha,\beta,\lambda,\delta)=1-\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-\alpha}, \tag{2}$$

where $\lambda<t<\infty$, $\beta>0,\alpha>0,\delta>0$, λ is the location parameter, β is the scale parameter and (α,δ) are shape parameters.

The mean and the variance of generalized Pareto distribution are

$$\mu=\beta\left[\frac{\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\Gamma(\alpha)}\right]+\lambda \tag{3}$$

and

$$\sigma^2=\beta^2\left[\frac{\Gamma\left(1+\frac{2}{\delta}\right)\Gamma\left(\alpha-\frac{2}{\delta}\right)}{\Gamma(\alpha)}-\frac{\Gamma\left(1+\frac{1}{\delta}\right)\Gamma\left(\alpha-\frac{1}{\delta}\right)}{\Gamma(\alpha)}\right]^2. \tag{4}$$

2. The Proposed GASP

In this section, a new GASP is introduced. Let μ be the true mean and μ_0 denote the specified mean of an item follow by a lifetime distribution respectively. An item is rejected due to bad if the true value of the μ is smaller than the specified value μ_0, that is, $\mu<\mu_0$, otherwise accepted. The GASP also considers the mutual development of the both producer's risk and the consumer's risk, denoted by α and β respectively. The probability of acceptance of a bad item is the consumer's risk and the probability of rejection of a good item is called the producer's risk

The GASP based on truncated life tests was proposed by Aslam et al. (2010), which is described as under:

Step 1: Determine the group size g .

Step 2: Sample gr items from a lot randomly and allocate r items to each group for the life test. The required sample size in the life test is $n = gr$.

Step 3: Determine the acceptance number c for every group and specify the termination time of the life test t_0 .

Step 4: Implement the life test based on the g groups of items, simultaneously. Accept the lot if at most c failed items are found in every group by the termination time. Truncate the life test and reject the lot if more than c failures are found in any group.

It is important to note that if $r = 1$, then GASP reduces to the ordinary acceptance sampling plan. The lot acceptance probability for the proposed plan is given by

$$L(p) = \left[\sum_{i=0}^{c} \binom{r}{i} p^{i} (1-p)^{r-i} \right]^{g} \tag{5}$$

where p is the function of F (t) given in (2). It would be convenient to take the termination time as a multiple of the specified number 'a', that is, $\mu_0 = t_0 / a$. Therefore, the distribution function can be written as

$$p = F(t) = 1 - \left[1 + \left[\frac{a\,\Gamma\left(\alpha - \frac{1}{\delta}\right)\Gamma\left(1 + \frac{1}{\delta}\right)}{\left(\frac{\mu}{\mu_0}\right)\Gamma(\alpha)} \right]^{\delta} \right]^{-\alpha} \tag{6}$$

where p can be evaluated when the shape parameter, the multiplier a and the ratio μ/μ_0 are specified. The minimal group size can be determined such that following inequality (7) is satisfied

$$L(p_0) \le \beta . \tag{7}$$

Table 1 gives the minimal group size of the GASP for the shape parameters $(\alpha, \delta) = 2$, the termination ratio $a =$ 0.7, 0.8, 1.0, 1.2, 1.5, 2.0, and the number of tester $r = 2(1)9$, the acceptance number $c = 0(1)7$, for consumer's risk $\beta =$ 0.25, 0.10, 0.05, 0.01. The choices of parameters are used for the comparison purpose. This table indicates that the required minimal group size decreases as the termination ratio increases. Moreover, the required minimal group size in a life test increases as the number of testers increases.

Some operating characteristic (OC) values and the minimal ratios μ/μ_0 are also tabulated to help experimenters to select the GASPs. The minimal ratios μ/μ_0 in Table 2 are computed based on the group size given in Table 1. Once if you find the minimum group size, then one may be interested to obtain the probability when the quality of the item is reliable. For given values of g and a, the probability of acceptance is increased as the mean ratio increased. The OC values are given in Table 2 for $c = 2$ and $r = 4$.

Table 1: Number of groups required for the proposed plan using generalized Pareto distribution with $\alpha = \delta = 2$.

			a					
β	r	c	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	2	1	1	1	1
	3	1	4	3	2	1	1	1
	4	2	7	5	2	2	1	1
	5	3	14	8	4	2	1	1
	6	4	30	14	5	3	2	1
	7	5	64	26	7	3	2	1
	8	6	138	47	11	4	2	1
	9	7	301	87	16	6	2	1
0.10	2	0	3	2	2	1	1	1
	3	1	6	3	3	2	1	1
	4	2	11	7	4	2	2	1
	5	3	24	13	6	3	2	1
	6	4	49	23	8	4	2	1
	7	5	106	42	12	5	3	2
	8	6	228	78	18	7	3	2
	9	7	500	144	26	9	4	2
0.05	2	0	3	3	2	2	1	1
	3	1	7	5	3	2	2	1
	4	2	15	9	5	3	2	1
	5	3	30	17	7	4	2	2
	6	4	64	30	10	5	3	2
	7	5	137	55	15	7	3	2
	8	6	297	101	23	9	4	2
	9	7	651	187	34	12	5	2
0.01	2	0	5	4	3	2	2	1
	3	1	11	8	5	3	2	2
	4	2	22	14	7	4	3	2
	5	3	46	25	11	6	3	2
	6	4	98	46	16	8	4	2
	7	5	211	84	23	10	5	3
	8	6	456	154	35	14	6	3
	9	7	999	287	52	18	7	3

Table 2: OC values for the proposed plan using generalized Pareto distribution with $\alpha=\delta=2$, $r=4$ and $c=2$.

			2	4	6	8	10	12
β	g	a						
0.25	7	0.7	0.9391	0.9987	0.9999	1.0000	1.0000	1.0000
	5	0.8	0.9150	0.9979	0.9998	1.0000	1.0000	1.0000
	2	1.0	0.9018	0.9971	0.9997	0.9999	1.0000	1.0000
	2	1.2	0.7954	0.9921	0.9992	0.9998	1.0000	1.0000
	1	1.5	0.7601	0.9873	0.9986	0.9997	0.9999	1.0000
	1	2.0	0.4944	0.9496	0.9931	0.9985	0.9996	0.9999
0.10	11	0.7	0.9059	0.9989	0.9998	1.0000	1.0000	1.0000
	7	0.8	0.8830	0.9971	0.9997	0.9999	1.0000	1.0000
	4	1.0	0.8132	0.9942	0.9994	0.9999	1.0000	1.0000
	2	1.2	0.7954	0.9921	0.9992	0.9998	1.0000	1.0000
	2	1.5	0.5778	0.9747	0.9972	0.9994	0.9998	0.9999
	1	2.0	0.4944	0.9496	0.9931	0.9985	0.9996	0.9999
0.05	15	0.7	0.8740	0.9971	0.9997	1.0000	1.0000	1.0000
	9	0.8	0.8522	0.9963	0.9996	0.9999	1.0000	1.0000
	5	1.0	0.7722	0.9927	0.9993	0.9999	1.0000	1.0000
	3	1.2	0.7093	0.9882	0.9988	0.9998	0.9999	1.0000
	2	1.5	0.5778	0.9747	0.9972	0.9994	0.9998	0.9999
	1	2.0	0.4944	0.9496	0.9931	0.9985	0.9996	0.9999
0.01	22	0.7	0.8207	0.9958	0.9996	0.9999	1.0000	1.0000
	14	0.8	0.7798	0.9942	0.9995	0.9999	1.0000	1.0000
	7	1.0	0.6963	0.9898	0.9990	0.9998	0.9999	1.0000
	4	1.2	0.6326	0.9842	0.9983	0.9997	0.9999	1.0000
	3	1.5	0.4392	0.9623	0.9958	0.9991	0.9998	0.9999
	2	2.0	0.2444	0.9017	0.9861	0.9971	0.9992	0.9997

Table 2 indicates that OC function increase more quickly than quality. For example, for $\beta=0.25$, $r=4$, $c=2$ and $a=0.7$, the number of group required is $g=7$. If the true mean is twice the specified value $\mu/\mu_0=2$, the producer's risk is approximately $\alpha=0.0609$, while $\alpha=0.0001$ when the true value of the mean is 6 times than the specified one.

When the producer's risk is given, the minimal ratios of the true mean life to the specified mean life μ/μ_0 can be determined which satisfies the following inequality

$$L(p_1)\geq 1-\alpha . \tag{8}$$

The minimal ratio, μ/μ_0 , found in (8) is another indicator which helps the producer to choose the desired GASP. The minimal ratio of μ/μ_0 in Table 3 is computed based on the group size given in Table 1.

Table 3: Minimum ratio of true average life to specified life for the producer's risk of 0.05 using generalized Pareto distribution with $\alpha = \delta = 2$.

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	6.86	7.84	6.92	8.30	10.40	13.90
	3	1	2.94	3.11	3.48	3.44	4.29	5.72
	4	2	2.08	2.23	2.33	2.79	3.01	4.01
	5	3	1.76	1.85	2.07	2.21	2.44	3.26
	6	4	1.59	1.65	1.80	2.01	2.36	2.82
	7	5	1.48	1.54	1.65	1.78	2.10	2.53
	8	6	1.41	1.45	1.56	1.67	1.92	2.33
	9	7	1.35	1.39	1.49	1.61	1.78	2.17
0.10	2	0	8.40	7.90	9.80	8.30	10.40	13.90
	3	1	3.30	3.20	3.88	4.20	4.30	5.72
	4	2	2.27	2.38	2.67	2.79	3.49	4.01
	5	3	1.91	1.99	2.21	2.37	2.76	3.26
	6	4	1.69	1.76	1.92	2.09	2.36	2.82
	7	5	1.56	1.62	1.76	1.90	2.22	2.80
	8	6	1.47	1.52	1.65	1.78	2.02	2.55
	9	7	1.41	1.45	1.56	1.68	1.93	2.37
0.05	2	0	8.40	9.60	9.79	11.75	10.40	13.90
	3	1	3.41	3.57	3.88	4.20	5.21	5.72
	4	2	2.40	2.50	2.79	3.03	3.49	4.01
	5	3	1.97	2.07	2.26	2.48	2.76	3.68
	6	4	1.75	1.82	1.98	2.16	2.51	3.15
	7	5	1.60	1.67	1.80	1.98	2.22	2.80
	8	6	1.51	1.56	1.69	1.83	2.09	2.55
	9	7	1.44	1.48	1.60	1.73	1.98	2.37
0.01	2	0	10.9	11.10	12.0	11.75	14.7	13.90
	3	1	3.84	4.03	4.45	4.66	5.21	6.95
	4	2	2.58	2.71	2.98	3.20	3.78	4.65
	5	3	2.09	2.19	2.42	2.65	2.96	3.68
	6	4	1.84	1.92	2.10	2.30	2.61	3.15
	7	5	1.68	1.74	1.89	2.06	2.37	2.96
	8	6	1.57	1.62	1.76	1.92	2.19	2.69
	9	7	1.48	1.54	1.66	1.80	2.05	2.49

2.1. Example

Suppose a bulb manufacturer would like to know whether the mean life of their bulbs is longer than the specified life $\mu_0 = 1000$ hours. Suppose further that the manufacturer wants to run a life experiment in 1000 hours by using testers equipped with 4 items each. Assuming that the lifetime of bulbs follow a generalized Pareto distribution with $(\alpha, \delta) = 2$. If the manufacturer would like to select the acceptance number $c = 2$ and the multiplier in termination time $a = 0.70$ under the consumer's risk =0.25. Table 1 gives the required minimal group size $g = 7$. The GASP (g, r, c, a) = (7, 4, 2, 0.7). In practice, the manufacture needs to draw a random sample of size 28 bulbs from the lot and allocate 4 items to 7 groups on the life test. The lot is accepted if no more than 2 failed bulbs are found in 700 hours in every group. Otherwise, the lot is rejected.

The OC values for the GASP (g, r, c, a) = (7, 4, 2, 0.70) are as follows:

μ/μ_0	2	4	6	8	10	12
p_a	0.9391	0.9987	0.9999	1.0000	1.0000	1.0000

That is, a lot of items will be accepted with probability 0.9391 if the true mean life of items in lot is 2 times than the specified mean life. The probability of accepting the lot increases up to 0.9999 if the true mean life is 6 times than the specified mean life.

3. A Comparative Study

In order to compare proposed GASP with that of Rao (2009), we consider the above example. The GASP for the generalized exponential distribution are (g, r, c, a) = (12, 4, 2, 0.70) and (g, r, c, a) = (7, 4, 2, 0.70) respectively. So, our proposed GASP requires 28 $(n = r \times g)$ items and the existing plan given by Rao (2009) requires 48 items, respectively, to reach on a same decision about the submitted items.

4. Concluding Remarks

In this paper, the design of GASP is developed for the generalized Pareto distribution. The minimal group size, OC values and the minimal ratio of the true mean life to the specified mean life are determined and tabulated for illustration. Thus our proposed plan performs better than the existing plan in terms of minimum sample size required to reach the same decision.

Acknowledgments. We thank the editor and anonymous referees for their insightful and helpful comments on the earlier draft of this paper.

References

Abd Elfattah, A.M., Elsherpieny, E.A. and Hussein, E.A. (2007). A new generalized Pareto distribution. *Interstat*, 12, 1-6.

Aslam, M., Mughal, A. R., Ahmed, A. and Zafar Yab, M. (2010). Group acceptance sampling plan Pareto distribution of the second kind. *Journal of Testing and Evaluation*, 38(2), 1-8.

Aslam, M. and Jun, C.-H. (2009). A group acceptance sampling plan for truncated life tests based the inverse Rayleigh distribution and log-logistic distribution. *Pakistan Journal of Statistics*, 25(2), 107-119.

Aslam, M. and Kantam, R.R.L. (2008). Economic acceptance sampling based on truncated life tests in the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, 24(4), 269-276.

Aslam, M. and Shahbaz, M. Q. (2007). Economic reliability tests plans using the generalized exponential distribution. *Journal of Statistics*, 14, 52-59.

Baklizi, A. (2003). Acceptance sampling based on truncated life tests in the Pareto distribution of the second kind. *Advances and Applications in Statistics*, 3(1), 33-48.

Baklizi, A. and EI Masri, A.E.K. (2004). Acceptance sampling based on truncated life tests in the Birnbaum-Saunders model. *Risk Analysis*, 24(6), 1453-1457.

Balakrishnan, N., Leiva, V. and Lopez, J. (2007). Acceptance sampling plans from truncated life tests based on the generalized Brinbaum-Saunders distribution. *Communication in Statistics-Simulation and Computation*, 36, 643-656.

Epstein, B. (1954). Truncated life tests in the exponential case. *Annals of Mathematical Statistics*, 25, 555-564.

Goode, H.P. and Kao, J.H.K. (1961). Sampling plans based on the Weibull distribution. *Proceeding of the Seventh National Symposium on Reliability and Quality Control:* Philadelphia, pp. 24-40.

Jun, C.-H., Balamurali, S. and Lee, S.-H. (2006). Variables sampling plans for Weibull distributed lifetimes under sudden death testing. *IEEE Transactions on Reliability*, 55(1), 53-58.

Kantam, R.R.L. and Rosaiah, K. (1998). Half logistic distribution in acceptance sampling based on life tests. *IAPQR Transactions*, 23(2), 117-125.

Kantam, R.R.L., Rao, G.S. and Sriram, G. (2006). An economic reliability test plan: log-logistic distribution. *Journal of Applied Statistics*, 33(3), 291-296.

Pascual, F.G. and Meeker, W.Q. (1998). The modified sudden death test: planning life tests with a limited number of test positions. *Journal of Testing and Evaluation*, 26(5), 434 - 443.

Rosaiah, K. and Kantam, R.R.L. (2005). Acceptance sampling based on the inverse Rayleigh distribution. *Economic Quality Control*, 20(2), 277-286.

Rosaiah, K., Kantam, R. R. L. and Reddy, J. R. (2008). Economic reliability test plan with inverse Rayleigh variate. *Pakistan Journal of Statistics*, 24, 57-65.

Rosaiah, K., Kantam, R.R.L. and Chaudhary, S.K. (2006). Reliability of test plans for exponentiated log-logistic distribution. *Economic Quality Control*, 21(2), 165-175.

Rosaiah, K., Kantam, R.R.L. and Chaudhary, S.K. (2007). Exponentiated log-logistic distribution: An economic reliability test plan. *Pakistan Journal of Statistics*. 23(2), 147-146.

Rao, G. S. (2009). A group acceptance sampling plan for lifetimes following a generalized exponential distribution. *Economic Quality Control*, 24(1), 75-85.

Stephens, K. S. (2001). *The Handbook of Applied Acceptance Sampling: Plans, Procedures and Principles*. Milwaukee, WI: ASQ Quality Press.

Tsai, T.-R and Wu, S.-J. (2006). Acceptance sampling based on truncated life tests for generalized Rayleigh distribution. *Journal of Applied Statistics*, 33(6), 595-600.

Vlcek, B.L., Hendricks, R.C. and Zaretsky, E.V. (2003). *Monte Carlo Simulation of Sudden Death Bearing Testing*. NASA, Hanover, MD, USA.