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MATHEMATICAL MODEL FOR EVALUATION AND MANAGEMENT OF TUBERCULOSIS IN MALE AND FEMALE PATIENTS

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ABSTRACT

The objective of this study was to evaluate and manage the T.B infection using mathematical model. The model based upon the biomedical knowledge of the pathogenesis of this disease and the current treatment regimes, which is useful for predicting the emergence of drug resistance T.B and assessing the probability that T.B will be eliminated. The model can be employed to evaluate difference of infection in male, female and both in order to answer a variety of health questions.

Key words: Tuberculosis, Mathematical model, male and female patients, *Mycobacterium tuberculosis.*

INTRODUCTION

Tuberculosis (caused by *Mycobacterium tuberculosis*) is the leading cause of death worldwide, being responsible for 3 million deaths annually (Zumla and Grange, 1998). One third of the world's population gets infection each year (WHO, 2000). T.B kills more adults than other infectious diseases such as Acquired Immune Deficiency Syndrome, malaria, diarrhea and leprosy combined (Grange, 2000) (Chee *et al*., 2000). It is responsible for 7% adult deaths worldwide, despite the availability of effective tools for its detection, treatment and prevention (Zulma *et al*., 1999) (Chee *et al.*, 2000). The pool of infection is maintained in magnitude by 5-6million new cases and 3 million deaths every year, (Park, 2002).

Eighty percent of all incident T.B cases are found in 22 developing countries, with more than half of the cases occurring in South East Asia (Anonymous, 1999b). The number of T.B cases in this region is increasing due to population growth, inadequate treatment and non-compliance of the instructions of physician/Doctor by the patient (Zulma, 1999). An estimated 95% T.B cases and 98% of tuberculosis deaths are in the developing world, among these 25% deaths are avoidable (Zumla, 1998) (Grange, 1999). India, China and Indonesia have the largest number of tuberculosis patients in South Asia (Shaw, 1997). Pakistan is ranked 5th among 22 high burden countries with 2,68,000 new cases in just 1998: an incidence of 181 per 1,00,000 (WHO,. 2000). This will increase to 5.2 million by the year 20011 (Javiad, 1997).

Tuberculosis drug-resistance and co-infection with Human Immuno Deficiency Virus (HIV) further adds to the burden associated with T.B, these development are considered to be the major obstacles in control and elimination of T.B (Gracia *et al*., 2000; Shaw, 1997).

The control of tuberculosis depends upon early diagnosis, correct treatment and maintained compliance with a considerably extended therapeutic regime. The estimated male patients in Pakistan are 250, 000 per year. Pakistan has high defaulter rate for the completion of T.B treatment ranging from 79% 1997 to 45% in 1999 (Anonymous, 1999a). Tuberculosis has strong implications on economic status of a family and of a country as a whole. Eighty percent (80%) of all T.B patients are in the most economically productive era of their lives. It has been estimated that a T.B patient alone can lead to 20-30% loss of house-hold income in developing countries because a T.B patient is economically unproductive and at the same time bears high cost of treatment (PHRI/SOROS/ICPH, 1997). In resource poor countries, nearly all patients die because of treatment difficulties and in-affordability (PHRI/SOROS/ICPH, 1997).

RESULTS

Table 1 summarized the data of T.B. patients registered during 2000-2007 at Ojha Institute of Chest Diseases, Nazimabad, Karachi. The number of patient according to this data are varying during the years 2000 to 2007.

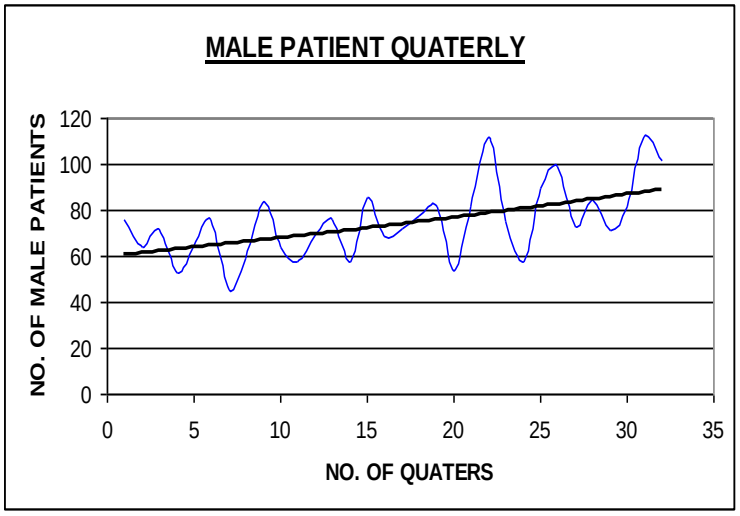
MATHEMATICAL MODEL FOR EVALUATION OF T.B. PATIENTS (MALE AND FEMALE)

The Model Equation of the Male infected T.B patients is the growth rate function. The figure shows the trend of male infected patient on quarterly basis (three months). The model equation of this trend function is exponentially

growth rate function, but the value of coefficient of determination R^2 has value 0.2984 i.e 30% of growth per quarter of year. Its also indicate as 70% of male patients were not being approached to the hospital to proper checkup of health or may be there were those who infected by disease and visit only once in their life and not approaching to the doctors further. In other words it can be interpreted as 30% infected person is varied over the total variations of male infected patient in quarter of years and the remaining 70% is accounted as those who are not approaching to the doctors by any reason and can be referred as the error term of the function.

Table 1. Statistical data of male & female patients of T.B. (reported at Chest Hospital Nazimabad Karachi)

		NO. OF MALE PATIENTS	NO. OF FEMALE PATIENTS	NO OF MALE & FEMALE PATIENTS
		M	F	M+F
2000	Jan-mar	76	126	202
2000	Apr-Jun	64	96	160
2000	Jul-Sep	72	94	166
2000	Sep-Dec	53	120	173
2001	Jan-mar	65	131	196
2001	Apr-Jun	76	112	188
2001	Jul-Sep	46	89	135
2001	Sep-Dec	59	98	157
2002	Jan-mar	84	128	212
2002	Apr-Jun	64	98	162
2002	Jul-Sep	58	106	164
2002	Sep-Dec	70	104	174
2003	Jan-mar	76	120	196
2003	Apr-Jun	58	98	156
2003	Jul-Sep	86	102	188
2003	Sep-Dec	69	96	165
2004	Jan-mar	72	120	192
2004	Apr-Jun	78	98	176
2004	Jul-Sep	82	113	195
2004	Sep-Dec	54	77	131
2005	Jan-mar	82	129	211
2005	Apr-Jun	112	116	228
2005	Jul-Sep	75	81	156
2005	Sep-Dec	58	63	121
2006	Jan-mar	90	91	181
2006	Apr-Jun	99	111	210
2006	Jul-Sep	73	85	158
2006	Sep-Dec	85	79	164
2007	Jan-mar	71	107	178
2007	Apr-Jun	82	94	176
2007	Jul-Sep	112	75	187
2007	Sep-Dec	102	67	169



Model Equation (Male) $y = 59.903e^{0.0124x}$

$R^2 = 0.2984$

Application of Model on Data (male) received from Nazimabad Chest Hospital

Model Equation (Male) $y = 59.903e^{0.0124x}$

$R^2 = 0.2984$

$y = 59.903\ e^{0.0124x}$ ---------- (1)

Now From equation (A) $N = N_o e^{Kt}$ ----------(2)

Now we calculate the value of K

$$K = \frac{\text{Ln } N - \text{Ln } No}{t}$$

$$K = \frac{\text{Ln } 60.650 - \text{Ln } 59.903}{1}$$

$= 4.1059 - 4.0928$

$= 0.0124$

Now comparing the equation (1) and (2) N_0=59.903 This is the initial patient of T.B

N= y Number of patient K=0.0124 , t = x Number of Quarter(year)

To check the validity of the Model the number of Male Patient at Nazimabad Chest Hospital has been calculated, which is appended below

1st quarter 2000 $y = 59.903e^{0.012*1}$

y = 60.65042 Male T.B Patient

2nd quarter 2000 $y = 59.903e^{0.0124*2}$

= 61.40717 Male T.B Patient

3rd quarter 2000 $y = 59.903e^{0.0124*3}$

= 62.17336 Male T.B Patient

4th quarter 2000 $y = 59.903e^{0.0124*4}$

= 62.94911 Male T.B Patient

The above Equation shows that the male patient of T.B is Exponential Growth Rate Function .As per the actual

data the male T.B patients reported Nazimabad Chest Hospital during the year 2000 were 247, However, model indicates male patients reported is 267, both results are almost same. It confirms that the model is appropriate.
Now, we have calculated the male infected T.B patient for the years 2008, 2012 on quarterly basis with the help of the same model the results are as follows.
We write the equation of exponentially growth rate function of Male T.B. Patient

$$y = 59.903\, e^{0.0124x}$$

1st Quarter 2008 $y = 59.903e^{0.0124*33}$
= 90.19072 Male T.B Patient

2nd Quarter 2008 $y = 59.903e^{0.0124*34}$
= 91.31605 Male T.B Patient

3rd Quarter 2008 $y = 59.903e^{0.0124*35}$
= 92.45542 Male T.B Patient

4th Quarter 2008 $y = 59.903e^{0.0124*36}$
= 93.60901 Male T.B Patient

Total Number of Patients of male T.B.infected Patients, predicted for the year 2008 is 368.

1st Quarter At 2012 $y = 59.903e^{0.0124*49}$
= **109.9831** Male T.B Patient

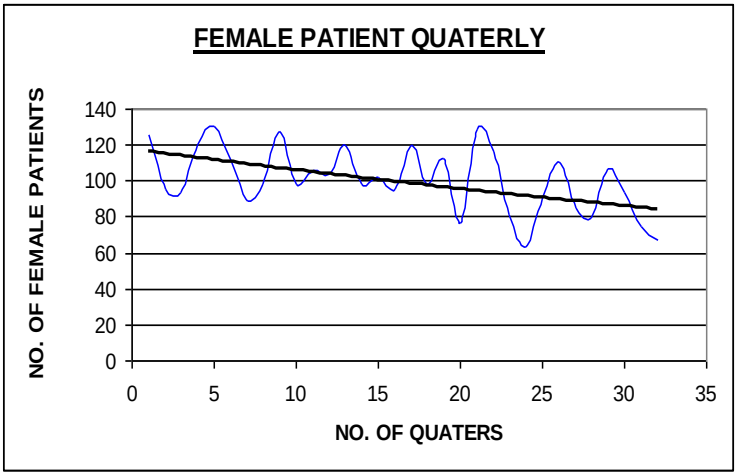
2nd Quarter At 2012 $y = 59.903e^{0.0124*50}$
=111.3554 Male T.B Patient

3rdQuarter At 2012 $y = 59.903e^{0.0124*51}$
= **112.7448** Male T.B Patient

4thQuarter At 2012 $y = 59.903e^{0.0124*52}$
= 114.1515 Male T.B Patient

The above results indicate that the expected T.B infected patients to visit Nazimbabd Chest Hospital during the year 2012 will be around 448

The figure shows the trend of female infected patient on quarterly basis (three months).the trend of the function is exponentially decay rate function. The Model Equation of the Female infected T.B patient i.e the decay rate function. here coefficient of determination R^2 is 0.2651 i.e 26% of decay per quarter of a year (three months). So we can say 74% of female patients were not being approached to the hospital to proper checkup of health or in other words 26% infected person is varied over the total variations of female infected patient in quarter of years (three months)



Model Equation (Female) $y = 117.57e^{-0.0104x}$
$R^2 = 0.2651$

Application of Model on Data (female) received from Nazimabad Chest Hospital

Model Equation (Female) $y = 117.57e^{-0.0104x}$

$y = 117.57e^{-0.0104x}$ ----------(1)

Now From equation (B) $K < 0$ $N = N_o e^{-Kt}$ ----------(2)

Now comparing the equation (1) and (2) N0=117.57 this is the initial patient of T.B
N= y Number of patient K= - 0.0104, t = x Number of Quarter (year)

To check the validity of the Model the number of Female Patient at Nazimabad Chest Hospital has been calculated, which is appended below

1st quarter 2000 $y = 117.57e^{-0.0104*1}$
y = 116.3536 Female T.B Patient

2nd quarter 2000 $y = 117.57e^{-0.0104*2}$
y = 115.1498 Female T.B Patient

3rd quarter 2000 $y = 117.57e^{-0.0104*3}$
y = 113.9584 Female T.B Patient

4th quarter 2000 $y = 117.57e^{-0.0104*4}$
y = 112.7794 Female T.B Patient

The above Equation shows that the female patient of T.B is Exponential Decay Rate Function .As per the actual data the female T.B patients reported Nazimabad Chest Hospital during the year 2000 were 458, However, model indicates female patients reported is 436, both results are almost same. It confirms that the model is appropriate.
Now, we have calculated the female infected T.B patient for the years 2008, 2012 on quarterly basis with the help of the same model the results are as follows.
We write the equation of exponentially decay rate function of Female T.B Patient
$y = 117.57e^{-0.0104x}$

1st Quarter 2008 $y = 117.57e^{-0.0104*33}$
= 83.41548 Female T.B Patient

2nd Quarter 2008 $y = 117.57e^{-0.0104*34}$
= 82.55245 Female T.B Patient

3rd Quarter 2008 $y = 117.57e^{-0.0104*35}$
= 81.69836 Female T.B Patient

4th Quarter 2008 $y = 117.57e^{-0.0104*36}$
= 80.8531 Female T.B Patient

Total Number of Patients o f female infected, predicted for the year 2008 is 329

1st Quarter At 2012 $y = 117.57e^{-0.0104*49}$
= 70.62851 Female T.B Patient

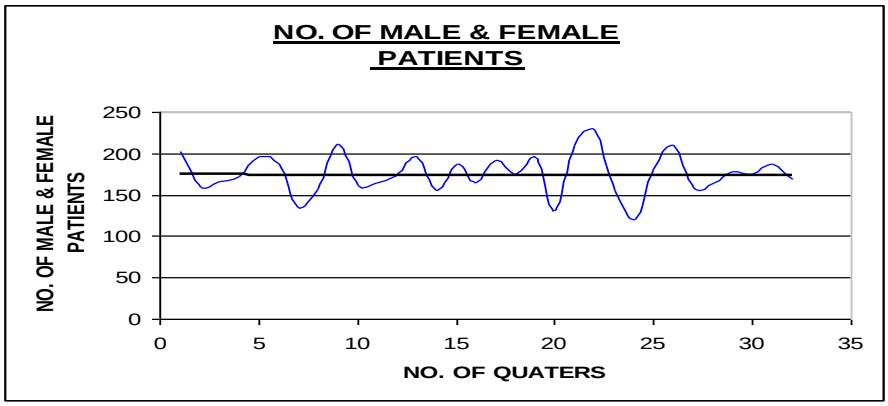
2nd Quarter At 2012 $y = 117.57e^{-0.0104*50}$
y = 69.89778 Female T.B Patient

3rdQuarter At 2012 $y = 117.57e^{-0.0104*51}$
y = 69.17461 Female T.B Patient

4thQuarter At 2012 $\mathbf{y = 117.57e^{-0.0104*52}}$

$\mathbf{y}$ = 68.45892 Female T.B Patient

The above results indicate that the expected T.B infected patients to visit Nazimbabd Chest Hospital during the year 2012 will be around 278



Model Equation (Male + Female) $\mathbf{y = 174.68e^{-0.0002x}}$

$\mathbf{R^2 = 0.0001}$

The Model Equation of the Male and Female infected T.B patient is the smooth rate function. The figure shows the trend of male and female infected patient on quarterly basis (three months). The model equation of this trend function is exponentially smooth rate function, but value of coefficient of determination R^2 has value 0.0001 i.e. 0.01% of smooth per quarter of year. Its also interpret as 99.99 % male and female patients have constant number of varied patients out of the constant increase in the population so there is no variation in infected male and female patients. In other words it indicates that 0.01% of infected male and female person variation is negligible over the total variations of male and female infected patient in quarter of years.

Model Equation (Male + Female) $\mathbf{y = 174.68e^{-0.0002x}}$

Application of Model on Data (male and female) received from Nazimabad Chest Hospital

$\mathbf{y = 174.68e^{-0.0002x}}$ ----------(1)

Now From equation (B) $K < 0 \quad N = N_o e^{-Kt}$ ----------(2)

Now comparing the equation (1)and (2) N_0=174.68 This is the initial patient of T.B

N= y Number of patient K= - 0.0002, t = x Number of Quarter(year)

To check the validity of the Model the number of Male and Female Patients at Nazimabad Chest Hospital has been calculated, which is appended below

1st quarter 2000 $\mathbf{y = 174.68e^{-0.0002*1}}$

y = 174.6451 Male+Female T.B Patient

2nd quarter 2000 $\mathbf{y = 174.68e^{-0.0002*2}}$

y = 174.6101 Male+Female T.B Patient

3rd quarter 2000 $\mathbf{y = 174.68e^{-0.0002*3}}$

y = 174.5752 Male+Female T.B Patient

4th quarter 2000 $\mathbf{y = 174.68e^{-0.0002*4}}$

y = 174.5403 Male+Female T.B Patient

The above Equation shows that the male and female patient of T.B is Exponential Smooth Rate Function .As per the actual data the male and female T.B patients reported Nazimabad Chest Hospital during the year 2000 were 701, However, model indicates male and female patients reported is 698, both results are almost same. It confirms

that the model is appropriate.

Now, we have calculated the male and female infected T.B patients for the years 2008, 2012 on quarterly basis with the help of the same model the results are as follows.

We write the equation of exponentially smooth rate function of Male and Female T.B Patient

$$y = 174.68\, e^{-0.0002x}$$

1st Quarter 2008 $y = 174.68e^{-0.0002*33}$
= 173.5309 Male+Female T.B Patient

2nd Quarter 2008 $y = 174.68e^{-0.0002*34}$
= 173.4962 Male+Female T.B Patient

3rd Quarter 2008 $y = 174.68e^{-0.0002*35}$
= 173.4615 Male+Female T.B Patient

4th Quarter 2008 $y = 174.68e^{-0.0002*36}$
= 173.4268 Male+Female T.B Patient

Total Number of male **1st Quarter** At 2012 $y = 174.68e^{-0.0002*49}$
= 172.9765 Male+Female T.B Patient

TotalNumber of male andFemaleT.B infected patints , Predicted for the year2008is 694

2nd Quarter At 2012 $y = 174.68e^{-0.0002*50}$
= 172.9419 Male+Female T.B Patient

3rdQuarter At 2012 $y = 174.68e^{-0.0002*51}$
= 172.9073 Male+Female T.B Patient

4thQuarter At 2012 $y = 174.68e^{-0.0002*52}$
= 172.8727 Male+Female T.B Patient

The above results indicate that the expected T.B infected patients to visit Nazimbabd Chest Hospital during the year 2012 will be around 692

DISCUSSION

Karachi is the thickly populated city of the country, various hospitals are working with different kind of infected diseases and number of patients from far and wide is being approach in the city for proper cure.

T.B is one of the major infected disease among others like HIV, Influenza etc, Mainly five Hospitals in Karachi provide the care and cure facility for the T.B patients in which Nazimabad Chest Hospital is one, where most of the infected patients approach for their treatment.

The statistical data of male and female T.B patients, registered in Hospital during the year 2000 to 2007, which are recorded on quarterly basis. As refer to the actual data of Male , Female , and Combined male and female patients reported during eight years commencing from 2000 at the Nazimabad Chest Hospital Karachi we have plotted three graphs A, B and C for male , female , and both male & female infected patients respectively. According to the graph A, plotted for the male infected patients only over 2000 to 2007 on quarterly basis(three month) shows the growth trend where as the graph B for the female infected patients only shows the decay trend. If we ignore the individual sex trend and take combine i.e both male and female infected patients as the graph C shows a smooth trend, it is obvious that the smooth trend of graph does not clear the real picture of individual trend of either sex infected patient, and it is very difficult to conclude what the trend moving inside

The growth and decay trends of infected patients, individually gives the clear picture or difference between male and female infected patients. The actual and calculated data position of the year 2000 on quarterly (three month) basis and forecasting trend are given as under :

	Male Patients		Female Patients		Male and Female Patients	
	Actual	Calculated	Actual	Calculated	Actual	Calculated
2000	265	247	436	458	701	698
2008	--	368	--	329	--	694
2012 (Forecast)	--	448	--	278	--	692

From the above table, it is clear that the calculated value of the model function is almost equal to the actual value of the year 2000. This confirm the validity of the model, also the T.B. patient expected to visit the Nazimabad Chest Hospital in the year 2012 has been calculated, that the same model and it come out to be 448 and 278 Male and Female T.B patient respectively.

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