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Research Article

A New Hybrid Exponentially Weighted Moving Average control chart using Mixture Ratio Estimator of Mean

Hafiz Zain Pervaiz[1], Syed Muhammad Muslim Raza[1], Muhammad Moeen Butt[1], Saira Sharif[1], and Muhammad Haider[2]

[1]Department of Economics, School of Business and Economics, University of Management and Technology, Lahore, Pakistan
[2]Office of Controller of Examination, Minhaj University, Lahore, Pakistan

Abstract: In this paper, we propose a Hybrid Exponentially Weighted Moving Average (HEWMA) control chart based on a mixture ratio estimator of mean using a single auxiliary variable and a single auxiliary attribute (Moeen et al., [1]). We call it as Z- HEWMA control chart. The proposed control chart performance is evaluated using out-of-control-Average Run Length (ARL1). The control limits of the proposed chart is based on estimator, its mean square errors. A simulated example is used to compare the proposed Z-HEWMA, traditional/simple EWMA chart and CUSUM control chart. From this study the fact is revealed that Z-HEWMA control chart shows more efficient results as compared to traditional/simple EWMA and CUSUM control charts. The Z-HEWMA chart can be used for efficient monitoring of the production process in manufacturing industries where auxiliary information about a numerical variable and an attribute is available.

Keywords: Hybrid, EWMA, CUSUM, Mixture Ratio Estimator, Average Run Length.

1. INTRODUCTION

The Statistical Process Control (SPC) is a powerful collection of problem solving tools useful in achieving process stability and improving capability through the reduction of variability. SPC builds an environment which leads to continuous improvement in quality and productivity. (Montgomery, [2]). The Control charts commonly used to differentiate between the assignable and un-assignable causes. The purpose of the effective process monitoring system is to detect the presence of an assignable cause as rapidly as possible without stopping checking too often or too late. The control charts are of different types. Some are memory control charts and other is memory-less control charts. Shewhart charts are called memory-less control charts and are being used to detect a large size shift whereas the memory type charts are used for dealing with small size shifts. Grant Leavenworth [3] describes the Statistical Process Control (SPC) as a useful and important tool used commonly in the engineering field to monitor the overall process. SPC can be applied to all kinds of engineering operations. The significant application of the SPC analysis of the process will make the process more consistent and reliable. Kotz and Johnson [4] defined the causes of variations. It can either be due to random (chance) causes and/or assignable causes. Some stable systems of chances are inherent in any particular scheme of production and inspection. But for products to maintain their standards, the assignable causes due to personnel, machines or material must be eliminated or at least reduced. Aczel [5] also stated that the capability of any process is the natural behavior of the particular process after disturbances are eliminated. Montgomery [2] states that the processes that are operating in the presence of assignable/special causes can be abbreviated as out of control. Gupta and Gupta [6] defined statistical quality control as one of the most useful and economically important applications of the theory of sampling in the industrial field. Keller [7] and Mandel [8] defined

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*Corresponding Author: Syed Muhammad Muslim Raza <razamuslim4@gmail.com>

quality control as Statistical Process Control which refers to one of a variety of statistical techniques used to develop and maintain a firm ability to produce high-quality goods and services. Mandel [8]gave a regression control chart and Zhang [9] used the additional/auxiliary information in preparation of the cause-selecting-type control chart. Wade and Woodall [10] proposed the prediction limits by inserting some modifications in Zhang [9] control limits. Many efficient estimators can be obtained/ developed by the use of auxiliary variables, for example, see the researches by Kadilar and Cingi [11], Singh and Vishwakarma [12], Singh et al. [13], Hanif et al.[14], Awan and Shabbir [15]. Riaz [16] and Riaz and Does [17] proposed the control chart for the variability using one auxiliary variable for Phase-I. When the additional/auxiliary information is incorporated in the construction of quality control charting attributes, the efficiency of such charts is improved. Riaz [17, 18] suggested a control chart for location parameters using one auxiliary variable for Phase I. Woodall et al. [19], Haq [20] used Hybrid charts for monitoring of Mean using single auxiliary variable. Ahmed et al [21] also used a single auxiliary variable in designing their proposed control chart.

In this paper, we propose a New Hybrid Exponentially Weighted Moving Average (HEWMA) control chart based on a mixture ratio estimator with a single auxiliary variable and a single auxiliary attribute (cf. Moeen et al., [1]). In the later stage of this paper we have develop the control charts parameters based on this estimator. The performance is measured using average run length-out of control (ARL_1). The chart showing less ARL_1 has been awarded as a more efficient chart as it will be more sensitive to the change and it will be detecting the shift in the parameter(s) more rapidly than other charts in comparison. The control charting parameters will consist of an upper control limit (UCL), a lower control limit (LCL), a centerline (CL). We also compare the propose control charts with the existing control charts.

2. METHODOLOGY

2.1 The Proposed Z-Hybrid EWMA Control Chart

In this paper, we propose a new Hybrid Exponentially Weighted Moving Average (HEWMA) control chart based on mixture ratio estimator of mean based on single auxiliary variable and single attribute variable (cf. Moeen et al., [1]). We call it as "Z-HEWMA chart" which is named on primary author Hafiz Zain Pervaiz name. The estimator proposed by Moeen et al. [1] is given as:

$$\phi *_y (\tau_1, x_1) = \bar{y} \left[\frac{P_1}{p_1}\right]^{\alpha} \left[\frac{\bar{X}_1}{\bar{x}_1}\right]^{\beta} \tag{3.1}$$

Whereas y denote the study variable, denote the observed value, P denotes an auxiliary attribute, denote an auxiliary variable, and β use for generalized the estimator for using the minimum value of mean square error.

The Mean Square Error is given as:

$$\text{MSE}\left(\phi_y^*(\tau_{.,}x_{.}\right) = \frac{\theta \bar{Y}^2 C_y^2}{\left|\Delta_{(x_1,\tau_1)}\right|}\left[\left|\Delta_{(y,\tau_1,x_1)}\right|\right] \tag{3.2}$$

Where $\left|\Delta_{(x_1,\tau_1)}\right| = \begin{vmatrix} 1\rho_{P_{x1}b1} \\ \rho_{P_{x1}b1}1 \end{vmatrix}$ and

$$\left|\Delta_{(y,\tau_1,x_1)}\right| = \begin{vmatrix} 1\rho_{P_{x1}y}\rho_{P_yb1} \\ \rho_{P_{x1}y}1\rho_{x1b1} \\ \rho_{P_yb1}\rho_{x1b1}1 \end{vmatrix}$$

Whereas, Y denotes the main quality characteristic having mean μ_y and variance $\sigma_y{}^2$ and X denotes an auxiliary variable with mean and variance $\sigma_x{}^2$. Assume that the variables Y and X follow a bivariate normal distribution i.e. $(Y, X) \sim N(\mu_y, \mu_x, \sigma_y{}^2, \sigma_x{}^2, \rho)$ and P ~ binomial (n, p).

Now we define two new sequences and as follow: $E_1, E_2, \ldots$, and $HE_1, HE_2, \ldots$, as follow:

$$E_t = \lambda_1 \bar{G}_i + (1-\lambda_1) E_{t-1} \tag{3.3}$$

Where E_t shows EWMA charting statistics and $\bar{G}_i$ is the mean of the data and E_{t-1} shows previous statistics value information.

$$HE_t = \lambda_2 E_t + (1-\lambda_2) HE_{t-1} \tag{3.4}$$

The HE_t statistics is the Hybrid charting statistics, λ_1 and λ_2 are the smoothing parameters and the control limits of the proposed chart will be derived as:

$$E(HE_t) = \overline{\phi *_y (\tau_1, x_1)}$$

And the variance of HE_t (cf. Haq, [20]):

$$V(HE_t) = \frac{{\lambda_1}^2 {\lambda_2}^2 MSE(\phi *_y(\tau_1, x_1))}{(\lambda_1 - \lambda_2)^2} \left[\frac{(1-\lambda_1)^2(1-(1-\lambda_1)^{2i})}{1-(1-\lambda_1)^2} + \frac{(1-\lambda_2)^2(1-(1-\lambda_2)^{2i})}{1-(1-\lambda_2)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)\{1-(1-\lambda_1)^i(1-\lambda_2)^i\}}{1-(1-\lambda_1)(1-\lambda_2)} \right] \quad (3.5)$$

So control limits are:

$$UCL = \mu_0 + L_1\sqrt{Var(HE_t)}$$

$$LCL = \mu_0 - L_2\sqrt{Var(HE_t)} \quad (3.6)$$

Where L_1 and L_2 shows the 99.73 quantile points of the distribution under study.

A simulated example is used to compare the performance of Cumulative sum control chart (CUSUM) and Exponentially Weighted Moving Average (EWMA) control charts with the proposed Hybrid EWMA control chart. The performance will be measured on average run length-out of control (ARL_1). The chart showing less ARL_1 will be awarded as a more efficient chart as it will be more sensitive to the change and it will be detecting it more rapidly than other charts in comparison.

3. RESULTS AND DISCUSSION

3.1. Illustrative Example

To study the features of the proposed Z-HEWMA control chart, we have simulated the data and applied the Z-HEWMA chart. The simulated data consists of variables Y representing the study variable, P-auxiliary attribute, X-auxiliary variables. The procedure proposed in section 2 is evaluated using different choices of sample size (n), EWMA weight (λ_1) and HEWMA weight (λ2) and correlation coefficient among variables (ρ). Among these, some results are presented here for discussion purposes. Tables 1–2 give the ARLs of the proposed Z-HEWMA control and its comparison with the existing EWMA control chart. We have considered n = 25, 45 and λ_1 = 0.10 and λ_2 = 0.08 for ARL_0 = 220.00 and tables 3 report the results for ARL_0 = 370.00. Four cases of the correlation among variables are considered, which include $\rho P_y b_1 = \rho x_1 y = \rho P x_1 b_1 = \rho$ = 0.0,0.10,0.30, and 0.50.

Table 1. ARLs of the proposed Z-HEWMA chart and conventional EWMA chart for n = 25, ARL_0 = 220 and λ_1 = 0.10 and λ_2=0.08

ρ →	0	0.1	0.3	0.5		
Shift	L=2.82	L=3.9	L=5.4	L=5.8	L=2.35	H=5
δ ↓	Z-HEWMA				Conventional EWMA	Conventional CUSUM
0	218.22	216.16	222.46	221.18	218.72	220.11
0.1	107.60	79.73	78.09	89.65	144.81	130.66
0.2	30.34	17.15	19.56	22.88	74.53	62.13
0.3	7.69	2.55	5.53	8.40	44.15	31.60
0.5	1.99	1.11	1.00	1.41	29.82	23.24
0.8	1.00	1.00	1.00	1.00	16.86	11.94

Table 2. ARLs of the proposed Z-HEWMA chart and conventional EWMA chart for n = 45, ARL_0= 220 and λ_1 = 0.10 and λ_2=0.08

ρ →	0	0.1	0.3	0.5		
Shift	L=2.45	L=3.7	L=5.7	L=5.9	L=2.35	H=5
δ ↓	Z-HEWMA				Conventional EWMA	Conventional CUSUM
0	219.91	217.85	224.09	223.06	220.32	220.32
0.1	105.73	73.94	73.50	87.90	144.45	127.12
0.2	26.48	15.85	14.93	22.28	73.84	57.93
0.3	5.16	1.31	4.61	4.08	43.20	25.91
0.5	1.00	1.00	1.00	1.02	24.74	22.79
0.8	1.00	1.00	1.00	1.00	13.49	7.19

Table 3. ARLs of the proposed Z-HEWMA chart and conventional EWMA chart for n = 45, ARL_0 = 370 and λ_1 = 0.10 and λ_2 = 0.08

$\delta\rho$ →	**0**	**0.1**	**0.3**	**0.5**		
Shift ↓	**L=2.55**	**L=3.8 9**	**L=5.9**	**L=6.1**	**L=2.35**	**H=5**
	Z-HEWMA				Conventional EWMA	Conventional CUSUM
0	369.65	368.7	367.6	368.84	367.58	369.44
0.1	96.25	90.07	87.48	88.94	132.85	124.37
0.2	14.34	12.22	11.13	11.59	60.18	47.16
0.3	2.73	1.01	1.31	2.13	19.08	13.12
0.5	1.00	1.00	1.00	1.00	9.79	8.08
0.8	1.00	1.00	1.00	1.00	5.54	3.48

The Figure 2 shows the application of Z-HEWMA control chart on the first 50 subgroups.

From table 1-3 and Figure 1 we observed that Z-HEWMA performs better than the traditional-EWMA and CUSUM control charts. The efficiency is evaluated using ARL1 criterion. The efficiency of the Z-HEWMA chart increases (i.e. ARL1 values reduces) with the increase in Shift from 0 to 0.8 respectively. The Z-HEWMA chart has better performance at correlation ρ = 0.1 and 0.3 than at ρ = 0.5.

The Appendix A1 shows Illustrated Example and Charting Statistics where values given CL, LCL, UCL, and Z-HEWMA.

Tables 1–3 and Figure 1 shows that:

(i) The control chart constants are directly proportional in relation to the correlation coefficient i.e. control chart constants increase as the correlation coefficient increases. However, with the increase in sample size the correlation coefficient and charting constants become stable/constant.

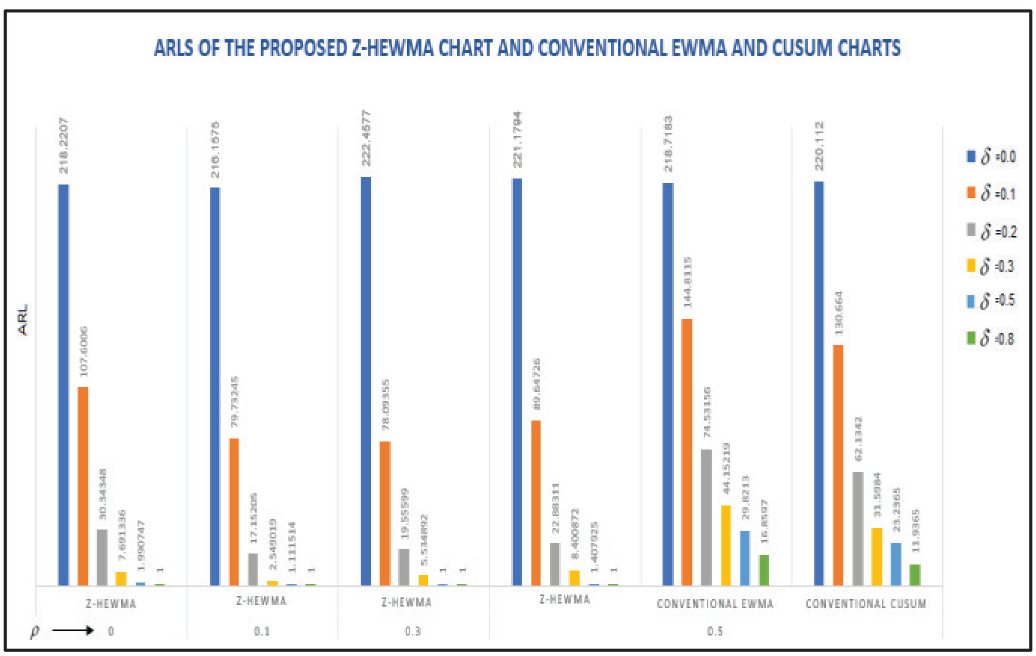

Fig. 1. ARLs of the proposed Z-HEWMA chart and conventional EWMA chart for n = 25, ARL_0 = 220 and λ_1 = 0.10 and λ_2=0.08

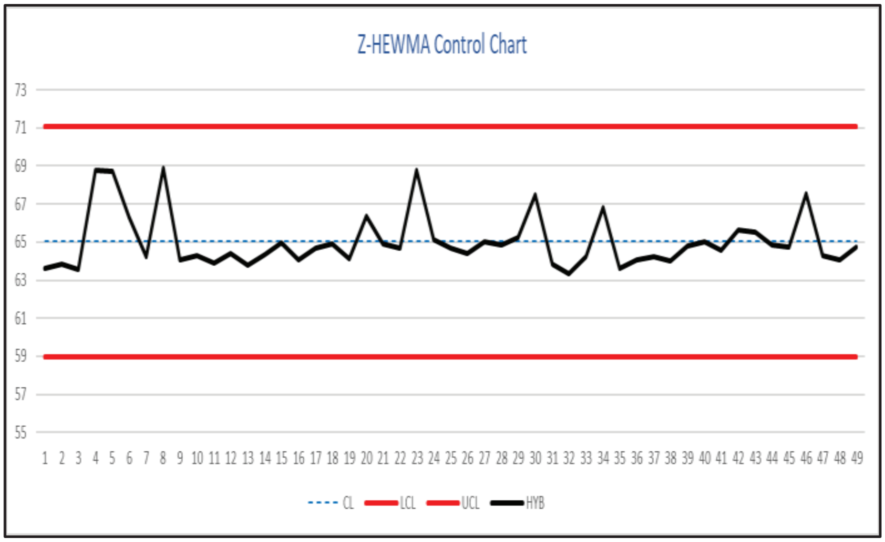

Fig. 2. Z-HEWMA chart for n = 25, ARL_0 = 220, λ_1 = 0.10, λ_2 = 0.08 and L=2.5.

(ii) When the variables are correlated and the process is shifted, the ARL values decrease rapidly as compared to the case when ρ = 0.

(iii) The proposed control chart has better performance when the correlation ρ = 0.1 and 0.3 than at ρ = 0.5.

(iv) The ARL1values decrease as sample size increases for a fixed value of ARL0 and λ's.

4. CONCLUSION AND RECOMMENDATIONS

This article proposed Z-HEWMA control chart using the auxiliary information for efficient monitoring of process mean. The control chart constants have been determined using Monte Carlo Simulation for various values of correlation coefficients. It is observed that the choice of correlation coefficient has a significant and directly proportional effect on the value of control chart constants. These charting constants are used in the construction of the proposed Z-HEWMA control chart. The performance of the proposed control chart is evaluated using the average run length (out-of-control); ARL1. The ARL1 values are calculated using different values of correlation coefficient and sample sizes.

It is observed that ARL1values are significantly small for low correlation (i.e. between 0.1–0.3) while it is large for correlation coefficient (0.5). The performance of the proposed Z-HEWMA chart is also compared with a simple EWMA chart and CUSUM control charts for a fixed sample size and correlation coefficient. It is observed that the proposed chart is more efficient to detect a small shift in the process mean than other control charts under study. Therefore, the use of the proposed Z-HEWMA chart is recommended when the study variable and auxiliary variables are statistically correlated with each other.

5. ACKNOWLEDGEMENT

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6. CONFLICT OF INTEREST

The authors declare no conflict of interest.

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Appendix A1. Illustrated Example and Charting Statistics

married aux.	Weight gain (y)	age (X)	$\bar{Y}$	X1	X2	X3	X4	X5	Mean	P(j)	p1	p2	p3	p4	p5	p(i)	$\bar{X}$	Phi	Zi	CL	LCL	UCL	Z-HEWA
1	22	31	32.2792	35	18	46	14	45	31.6	0.7824	0	1	1	1	1	0.8	31	30.866	58.173	65.216	62.135	69.354	63.635
1	50	23		31	41	44	24	43	36.6		1	0	1	1	0	0.6		35.532	59.106	65.216	62.135	69.354	62.729
0	38	24		45	33	17	21	17	26.6		1	1	1	1	1	1		29.334	57.867	65.216	62.135	69.354	61.757
1	15	26		18	16	31	17	40	24.4		0	0	0	1	0	0.2		159.895	83.979	65.216	62.135	69.354	66.201
0	17	34		17	19	32	26	30	24.8		0	0	0	0	1	0.2		157.316	83.463	65.216	62.135	69.354	69.653
0	12	15		24	24	15	22	15	20		1	0	1	0	0	0.4		97.536	71.507	65.216	62.135	69.354	70.024
1	46	24		44	23	40	16	18	28.2		1	0	0	1	1	0.6		46.116	61.223	65.216	62.135	69.354	68.264
1	28	30		35	15	18	27	25	24		0	1	0	0	0	0.2		162.560	84.512	65.216	62.135	69.354	71.514
0	38	23		46	43	19	34	15	31.4		0	0	1	1	1	0.6		41.417	60.283	65.216	62.135	69.354	69.268
1	47	37		26	20	27	43	23	27.8		1	1	1	0	0	0.6		46.780	61.356	65.216	62.135	69.354	67.685
0	35	20		45	45	21	25	38	34.8		1	1	1	0	0	0.6		37.370	59.474	65.216	62.135	69.354	66.043
0	30	35		29	26	35	20	21	26.2		1	1	0	1	0	0.6		49.637	61.927	65.216	62.135	69.354	65.220
1	66	28		38	43	23	37	45	37.2		1	1	1	0	0	0.6		34.959	58.992	65.216	62.135	69.354	63.974
1	50	22		24	28	41	18	24	27		1	0	1	1	0	0.6		48.166	61.633	65.216	62.135	69.354	63.506
1	31	26		45	26	23	14	44	30.4		0	0	0	1	1	0.4		64.168	64.834	65.216	62.135	69.354	63.772
0	26	23		15	39	45	43	16	31.6		1	0	0	1	1	0.6		41.154	60.231	65.216	62.135	69.354	63.063
0	25	36		37	44	41	15	34	34.2		1	0	0	1	0	0.4		57.039	63.408	65.216	62.135	69.354	63.132
0	27	32		28	30	32	23	43	31.2		0	1	0	1	0	0.4		62.523	64.505	65.216	62.135	69.354	63.407
1	23	24		38	24	37	27	25	30.2		0	0	1	1	1	0.6		43.062	60.612	65.216	62.135	69.354	62.848
1	36	18		41	46	37	39	35	39.6		0	1	0	0	0	0.2		98.521	71.704	65.216	62.135	69.354	64.619
0	60	26		23	42	44	27	18	30.8		0	1	1	0	0	0.4		63.335	64.667	65.216	62.135	69.354	64.629
1	33	27		16	44	34	31	44	33.8		1	0	1	0	0	0.4		57.714	63.543	65.216	62.135	69.354	64.412
0	30	17		19	35	16	14	39	24.6		0	0	0	0	1	0.2		158.595	83.719	65.216	62.135	69.354	68.273
1	31	26		28	39	23	26	27	28.6		1	0	0	1	0	0.4		68.207	65.641	65.216	62.135	69.354	67.747
1	10	32		18	14	16	37	30	23		1	1	1	0	0	0.6		56.543	63.309	65.216	62.135	69.354	66.859
1	73	35		44	17	32	23	15	26.2		0	0	1	1	1	0.6		49.637	61.927	65.216	62.135	69.354	65.873
1	20	39		16	18	29	20	16	19.8		1	1	0	1	0	0.6		65.681	65.136	65.216	62.135	69.354	65.725
0	16	27		31	25	39	22	42	31.8		0	0	0	1	1	0.4		61.343	64.269	65.216	62.135	69.354	65.434
1	28	31		16	14	26	18	17	18.2		1	1	0	1	0	0.6		71.455	66.291	65.216	62.135	69.354	65.605
1	34	38		14	29	22	46	42	30.6		0	0	0	0	1	0.2		127.498	77.500	65.216	62.135	69.354	67.984
1	15	26		18	16	31	17	40	24.4		0	0	0	1	0	0.2		159.895	83.979	65.216	62.135	69.354	66.201
0	17	34		17	19	32	26	30	24.8		0	0	0	0	1	0.2		157.316	83.463	65.216	62.135	69.354	69.653

married aux.	Weight gain (y)	age (X)	$\overline{Y}$	X1	X2	X3	X4	X5	Mean	P(j)	p1	p2	p3	p4	p5	p(i)	$\overline{X}$	Phi	Zi	CL	LCL	UCL	Z-HEWA
0	12	15		24	24	15	22	15	20		1	0	1	0	0	0.4		97.536	71.507	65.216	62.135	69.354	70.024
1	46	24		44	23	40	16	18	28.2		1	0	0	1	1	0.6		46.116	61.223	65.216	62.135	69.354	68.264
1	28	30		35	15	18	27	25	24		0	1	0	0	0	0.2		162.560	84.512	65.216	62.135	69.354	71.514
0	38	23		46	43	19	34	15	31.4		0	0	1	1	1	0.6		41.417	60.283	65.216	62.135	69.354	69.268
1	47	37		26	20	27	43	23	27.8		1	1	1	0	0	0.6		46.780	61.356	65.216	62.135	69.354	67.685
0	35	20		45	45	21	25	38	34.8		1	1	1	0	0	0.6		37.370	59.474	65.216	62.135	69.354	66.043
0	30	35		29	26	35	20	21	26.2		1	1	0	1	0	0.6		49.637	61.927	65.216	62.135	69.354	65.220
1	66	28		38	43	23	37	45	37.2		1	1	1	0	0	0.6		34.959	58.992	65.216	62.135	69.354	63.974
1	50	22		24	28	41	18	24	27		1	0	1	1	0	0.6		48.166	61.633	65.216	62.135	69.354	63.506
1	31	26		45	26	23	14	44	30.4		0	0	0	1	1	0.4		64.168	64.834	65.216	62.135	69.354	63.772
0	26	23		15	39	45	43	16	31.6		1	0	0	1	1	0.6		41.154	60.231	65.216	62.135	69.354	63.063
0	25	36		37	44	41	15	34	34.2		1	0	0	1	0	0.4		57.039	63.408	65.216	62.135	69.354	63.132
0	27	32		28	30	32	23	43	31.2		0	1	0	1	0	0.4		62.523	64.505	65.216	62.135	69.354	63.407
1	23	24		38	24	37	27	25	30.2		0	0	1	1	1	0.6		43.062	60.612	65.216	62.135	69.354	62.848
1	36	18		41	46	37	39	35	39.6		0	1	0	0	0	0.2		98.521	71.704	65.216	62.135	69.354	64.619
0	60	26		23	42	44	27	18	30.8		0	1	1	0	0	0.4		63.335	64.667	65.216	62.135	69.354	64.629
1	15	26		18	16	31	17	40	24.4		0	0	0	1	0	0.2		159.895	83.979	65.216	62.135	69.354	66.201
0	17	34		17	19	32	26	30	24.8		0	0	0	0	1	0.2		157.316	83.463	65.216	62.135	69.354	69.653
0	12	15		24	24	15	22	15	20		1	0	1	0	0	0.4		97.536	71.507	65.216	62.135	69.354	70.024
1	46	24		44	23	40	16	18	28.2		1	0	0	1	1	0.6		46.116	61.223	65.216	62.135	69.354	68.264
1	28	30		35	15	18	27	25	24		0	1	0	0	0	0.2		162.560	84.512	65.216	62.135	69.354	71.514
0	38	23		46	43	19	34	15	31.4		0	0	1	1	1	0.6		41.417	60.283	65.216	62.135	69.354	69.268
1	47	37		26	20	27	43	23	27.8		1	1	1	0	0	0.6		46.780	61.356	65.216	62.135	69.354	67.685
0	35	20		45	45	21	25	38	34.8		1	1	1	0	0	0.6		37.370	59.474	65.216	62.135	69.354	66.043
0	30	35		29	26	35	20	21	26.2		1	1	0	1	0	0.6		49.637	61.927	65.216	62.135	69.354	65.220
1	66	28		38	43	23	37	45	37.2		1	1	1	0	0	0.6		34.959	58.992	65.216	62.135	69.354	63.974
1	50	22		24	28	41	18	24	27		1	0	1	1	0	0.6		48.166	61.633	65.216	62.135	69.354	63.506
1	31	26		45	26	23	14	44	30.4		0	0	0	1	1	0.4		64.168	64.834	65.216	62.135	69.354	63.772
0	26	23		15	39	45	43	16	31.6		1	0	0	1	1	0.6		41.154	60.231	65.216	62.135	69.354	63.063
0	25	36		37	44	41	15	34	34.2		1	0	0	1	0	0.4		57.039	63.408	65.216	62.135	69.354	63.132
0	27	32		28	30	32	23	43	31.2		0	1	0	1	0	0.4		62.523	64.505	65.216	62.135	69.354	63.407
1	23	24		38	24	37	27	25	30.2		0	0	1	1	1	0.6		43.062	60.612	65.216	62.135	69.354	62.848
1	36	18		41	46	37	39	35	39.6		0	1	0	0	0	0.2		98.521	71.704	65.216	62.135	69.354	64.619
0	60	26		23	42	44	27	18	30.8		0	1	1	0	0	0.4		63.335	64.667	65.216	62.135	69.354	64.629
1	33	27		16	44	34	31	44	33.8		1	0	1	0	0	0.4		57.714	63.543	65.216	62.135	69.354	64.412
0	30	17		19	35	16	14	39	24.6		0	0	0	0	1	0.2		158.595	83.719	65.216	62.135	69.354	68.273

married aux.	Weight gain (y)	age (X)	$\overline{Y}$	X1	X2	X3	X4	X5	Mean	P(j)	p1	p2	p3	p4	p5	p(i)	$\overline{X}$	Phi	Zi	CL	LCL	UCL	Z-HEWA
1	31	26		28	39	23	26	27	28.6		1	0	0	1	0	0.4		68.207	65.641	65.216	62.135	69.354	67.747
1	10	32		18	14	16	37	30	23		1	1	1	0	0	0.6		56.543	63.309	65.216	62.135	69.354	66.859
1	73	35		44	17	32	23	15	26.2		0	0	1	1	1	0.6		49.637	61.927	65.216	62.135	69.354	65.873
1	20	39		16	18	29	20	16	19.8		1	1	0	1	0	0.6		65.681	65.136	65.216	62.135	69.354	65.725
0	16	27		31	25	39	22	42	31.8		0	0	0	1	1	0.4		61.343	64.269	65.216	62.135	69.354	65.434
1	33	27		16	44	34	31	44	33.8		1	0	1	0	0	0.4		57.714	63.543	65.216	62.135	69.354	64.412
0	30	17		19	35	16	14	39	24.6		0	0	0	0	1	0.2		158.595	83.719	65.216	62.135	69.354	68.273
1	31	26		28	39	23	26	27	28.6		1	0	0	1	0	0.4		68.207	65.641	65.216	62.135	69.354	67.747
1	10	32		18	14	16	37	30	23		1	1	1	0	0	0.6		56.543	63.309	65.216	62.135	69.354	66.859
1	73	35		44	17	32	23	15	26.2		0	0	1	1	1	0.6		49.637	61.927	65.216	62.135	69.354	65.873
1	20	39		16	18	29	20	16	19.8		1	1	0	1	0	0.6		65.681	65.136	65.216	62.135	69.354	65.725
0	16	27		31	25	39	22	42	31.8		0	0	0	1	1	0.4		61.343	64.269	65.216	62.135	69.354	65.434
0	39	18		29	46	38	40	28	36.2		1	0	1	0	1	0.6		35.925	59.185	65.216	62.135	69.354	66.224
0	85	16		44	44	22	21	39	34		1	1	1	1	1	1		22.950	56.590	65.216	62.135	69.354	64.298
0	30	40		26	27	21	33	33	28		1	1	0	0	1	0.6		46.446	61.289	65.216	62.135	69.354	63.696
1	44	26		37	40	41	46	14	35.6		0	1	0	0	0	0.2		109.591	73.918	65.216	62.135	69.354	65.740
0	37	17		30	25	16	42	46	31.8		1	0	1	1	1	0.8		30.672	58.134	65.216	62.135	69.354	64.219
1	34	29		24	36	27	34	36	31.4		1	1	0	1	0	0.6		41.417	60.283	65.216	62.135	69.354	63.432
1	43	30		40	15	14	32	39	28		1	1	1	0	0	0.6		46.446	61.289	65.216	62.135	69.354	63.003
1	26	30		27	35	29	15	15	24.2		1	1	0	1	1	0.8		40.304	60.061	65.216	62.135	69.354	62.415
1	19	36		38	46	36	26	17	32.6		0	0	0	1	1	0.4		59.838	63.968	65.216	62.135	69.354	62.725
0	12	23		33	18	37	40	22	30		0	0	1	0	1	0.4		65.024	65.005	65.216	62.135	69.354	63.181
0	25	18		38	46	34	39	23	36		0	0	1	0	1	0.4		54.187	62.837	65.216	62.135	69.354	63.113
0	45	23		16	32	16	35	22	24.2		1	1	0	0	0	0.4		80.608	68.122	65.216	62.135	69.354	64.114
0	35	19		19	40	35	15	17	25.2		0	0	1	1	0	0.4		77.410	67.482	65.216	62.135	69.354	64.788
1	0	23		18	45	39	29	28	31.8		1	1	0	0	0	0.4		61.343	64.269	65.216	62.135	69.354	64.684
0	30	27		26	45	30	33	33	33.4		0	1	1	0	0	0.4		58.405	63.681	65.216	62.135	69.354	64.483
1	35	25		39	18	40	27	28	30.4		1	0	0	0	0	0.2		128.337	77.667	65.216	62.135	69.354	67.120
1	37	20		37	39	23	14	25	27.6		1	0	0	1	1	0.6		47.119	61.424	65.216	62.135	69.354	65.981
1	46	31		33	34	38	26	24	31		1	0	1	1	0	0.6		41.951	60.390	65.216	62.135	69.354	64.863
1	54	21		25	38	44	27	31	33		0	0	1	1	0	0.4		59.113	63.823	65.216	62.135	69.354	64.655
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		29.737	57.947	65.216	62.135	69.354	63.313
1	21	30		41	17	44	29	33	32.8		1	0	1	1	1	0.8		130.000	78.000	65.216	62.135	69.354	66.251
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		187.000	89.400	65.216	62.135	69.354	70.880
0	46	31		41	17	44	29	33	32.8		1	0	1	1	1	0.8		155.000	83.000	65.216	62.135	69.354	73.304
1	45	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		118.000	75.600	65.216	62.135	69.354	73.764

married aux.	Weight gain (y)	age (X)	$\overline{Y}$	X1	X2	X3	X4	X5	Mean	P(j)	p1	p2	p3	p4	p5	p(i)	$\overline{X}$	Phi	Zi	CL	LCL	UCL	Z-HEWA
0	32	22		41	17	44	29	33	32.8		1	0	1	1	1	0.8		158.000	83.600	65.216	62.135	69.354	75.731
1	27	23		41	17	44	29	33	32.8		1	0	1	1	1	0.8		115.000	75.000	65.216	62.135	69.354	75.585
1	20	29		41	17	44	29	33	32.8		1	0	1	1	1	0.8		112.000	74.400	65.216	62.135	69.354	75.348
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		144.000	80.800	65.216	62.135	69.354	76.438
1	27	24		41	17	44	29	33	32.8		1	0	1	1	1	0.8		107.000	73.400	65.216	62.135	69.354	75.831
1	10	36		41	17	44	29	33	32.8		1	0	1	1	1	0.8		177.000	87.400	65.216	62.135	69.354	78.144
1	0	29		41	17	44	29	33	32.8		1	0	1	1	1	0.8		117.000	75.400	65.216	62.135	69.354	77.596
1	29	25		41	17	44	29	33	32.8		1	0	1	1	1	0.8		172.000	86.400	65.216	62.135	69.354	79.356
0	65	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		197.000	91.400	65.216	62.135	69.354	81.765
0	29	18		41	17	44	29	33	32.8		1	0	1	1	1	0.8		214.000	94.800	65.216	62.135	69.354	84.372
0	45	26		41	17	44	29	33	32.8		1	0	1	1	1	0.8		202.000	92.400	65.216	62.135	69.354	85.978
0	20	38		41	17	44	29	33	32.8		1	0	1	1	1	0.8		203.000	92.600	65.216	62.135	69.354	87.302
0	30	21		41	17	44	29	33	32.8		1	0	1	1	1	0.8		152.000	82.400	65.216	62.135	69.354	86.322
1	16	28		41	17	44	29	33	32.8		1	0	1	1	1	0.8		197.000	91.400	65.216	62.135	69.354	87.337
0	68	21		41	17	44	29	33	32.8		1	0	1	1	1	0.8		108.000	73.600	65.216	62.135	69.354	84.590
1	25	26		41	17	44	29	33	32.8		1	0	1	1	1	0.8		185.000	89.000	65.216	62.135	69.354	85.472
1	26	18		41	17	44	29	33	32.8		1	0	1	1	1	0.8		209.000	93.800	65.216	62.135	69.354	87.138
0	0	16		41	17	44	29	33	32.8		1	0	1	1	1	0.8		126.000	77.200	65.216	62.135	69.354	85.150
1	8	38		41	17	44	29	33	32.8		1	0	1	1	1	0.8		186.000	89.200	65.216	62.135	69.354	85.960
1	18	18		41	17	44	29	33	32.8		1	0	1	1	1	0.8		176.000	87.200	65.216	62.135	69.354	86.208
0	36	26		41	17	44	29	33	32.8		1	0	1	1	1	0.8		175.000	87.000	65.216	62.135	69.354	86.366
1	28	31		16	14	26	18	17	18.2		1	1	0	1	0	0.6		71.455	66.291	65.216	62.135	69.354	65.605
1	34	38		14	29	22	46	42	30.6		0	0	0	0	1	0.2		127.498	77.500	65.216	62.135	69.354	67.984
0	39	18		29	46	38	40	28	36.2		1	0	1	0	1	0.6		35.925	59.185	65.216	62.135	69.354	66.224
0	85	16		44	44	22	21	39	34		1	1	1	1	1	1		22.950	56.590	65.216	62.135	69.354	64.298
0	30	40		26	27	21	33	33	28		1	1	0	0	1	0.6		46.446	61.289	65.216	62.135	69.354	63.696
1	44	26		37	40	41	46	14	35.6		0	1	0	0	0	0.2		109.591	73.918	65.216	62.135	69.354	65.740

married aux.	Weight gain (y)	age (X)	$\overline{Y}$	X1	X2	X3	X4	X5	Mean	P(j)	p1	p2	p3	p4	p5	p(i)	$\overline{X}$	Phi	Zi	CL	LCL	UCL	Z-HEWA
0	37	17		30	25	16	42	46	31.8		1	0	1	1	1	0.8		30.672	58.134	65.216	62.135	69.354	64.219
1	34	29		24	36	27	34	36	31.4		1	1	0	1	0	0.6		41.417	60.283	65.216	62.135	69.354	63.432
1	43	30		40	15	14	32	39	28		1	1	1	0	0	0.6		46.446	61.289	65.216	62.135	69.354	63.003
1	26	30		27	35	29	15	15	24.2		1	1	0	1	1	0.8		40.304	60.061	65.216	62.135	69.354	62.415
1	19	36		38	46	36	26	17	32.6		0	0	0	1	1	0.4		59.838	63.968	65.216	62.135	69.354	62.725
0	12	23		33	18	37	40	22	30		0	0	1	0	1	0.4		65.024	65.005	65.216	62.135	69.354	63.181
0	25	18		38	46	34	39	23	36		0	0	1	0	1	0.4		54.187	62.837	65.216	62.135	69.354	63.113
0	45	23		16	32	16	35	22	24.2		1	1	0	0	0	0.4		80.608	68.122	65.216	62.135	69.354	64.114
0	35	19		19	40	35	15	17	25.2		0	0	1	1	0	0.4		77.410	67.482	65.216	62.135	69.354	64.788
1	0	23		18	45	39	29	28	31.8		1	1	0	0	0	0.4		61.343	64.269	65.216	62.135	69.354	64.684
0	30	27		26	45	30	33	33	33.4		0	1	1	0	0	0.4		58.405	63.681	65.216	62.135	69.354	64.483
1	35	25		39	18	40	27	28	30.4		1	0	0	0	0	0.2		128.337	77.667	65.216	62.135	69.354	67.120
1	37	20		37	39	23	14	25	27.6		1	0	0	1	1	0.6		47.119	61.424	65.216	62.135	69.354	65.981
1	46	31		33	34	38	26	24	31		1	0	1	1	0	0.6		41.951	60.390	65.216	62.135	69.354	64.863
1	54	21		25	38	44	27	31	33		0	0	1	1	0	0.4		59.113	63.823	65.216	62.135	69.354	64.655
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		29.737	57.947	65.216	62.135	69.354	63.313
1	21	30		41	17	44	29	33	32.8		1	0	1	1	1	0.8		130.000	78.000	65.216	62.135	69.354	66.251
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		187.000	89.400	65.216	62.135	69.354	70.880
0	46	31		41	17	44	29	33	32.8		1	0	1	1	1	0.8		155.000	83.000	65.216	62.135	69.354	73.304
1	45	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		118.000	75.600	65.216	62.135	69.354	73.764
0	32	22		41	17	44	29	33	32.8		1	0	1	1	1	0.8		158.000	83.600	65.216	62.135	69.354	75.731
1	27	23		41	17	44	29	33	32.8		1	0	1	1	1	0.8		115.000	75.000	65.216	62.135	69.354	75.585
1	20	29		41	17	44	29	33	32.8		1	0	1	1	1	0.8		112.000	74.400	65.216	62.135	69.354	75.348
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		144.000	80.800	65.216	62.135	69.354	76.438
1	27	24		41	17	44	29	33	32.8		1	0	1	1	1	0.8		107.000	73.400	65.216	62.135	69.354	75.831
1	10	36		41	17	44	29	33	32.8		1	0	1	1	1	0.8		177.000	87.400	65.216	62.135	69.354	78.144
1	0	29		41	17	44	29	33	32.8		1	0	1	1	1	0.8		117.000	75.400	65.216	62.135	69.354	77.596
1	29	25		41	17	44	29	33	32.8		1	0	1	1	1	0.8		172.000	86.400	65.216	62.135	69.354	79.356
0	65	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		197.000	91.400	65.216	62.135	69.354	81.765
0	29	18		41	17	44	29	33	32.8		1	0	1	1	1	0.8		214.000	94.800	65.216	62.135	69.354	84.372
0	45	26		41	17	44	29	33	32.8		1	0	1	1	1	0.8		202.000	92.400	65.216	62.135	69.354	85.978

married aux.	Weight gain (y)	age (X)	$\overline{Y}$	X1	X2	X3	X4	X5	Mean	P(j)	p1	p2	p3	p4	p5	p(i)	$\overline{X}$	Phi	Zi	CL	LCL	UCL	Z-HEWA
1	28	31		16	14	26	18	17	18.2		1	1	0	1	0	0.6		71.455	66.291	65.216	62.135	69.354	65.605
1	34	38		14	29	22	46	42	30.6		0	0	0	0	1	0.2		127.498	77.500	65.216	62.135	69.354	67.984
0	39	18		29	46	38	40	28	36.2		1	0	1	0	1	0.6		35.925	59.185	65.216	62.135	69.354	66.224
0	85	16		44	44	22	21	39	34		1	1	1	1	1	1		22.950	56.590	65.216	62.135	69.354	64.298
0	30	40		26	27	21	33	33	28		1	1	0	0	1	0.6		46.446	61.289	65.216	62.135	69.354	63.696
1	44	26		37	40	41	46	14	35.6		0	1	0	0	0	0.2		109.591	73.918	65.216	62.135	69.354	65.740
0	37	17		30	25	16	42	46	31.8		1	0	1	1	1	0.8		30.672	58.134	65.216	62.135	69.354	64.219
1	34	29		24	36	27	34	36	31.4		1	1	0	1	0	0.6		41.417	60.283	65.216	62.135	69.354	63.432
1	43	30		40	15	14	32	39	28		1	1	1	0	0	0.6		46.446	61.289	65.216	62.135	69.354	63.003
1	26	30		27	35	29	15	15	24.2		1	1	0	1	1	0.8		40.304	60.061	65.216	62.135	69.354	62.415
1	19	36		38	46	36	26	17	32.6		0	0	0	1	1	0.4		59.838	63.968	65.216	62.135	69.354	62.725
0	12	23		33	18	37	40	22	30		0	0	1	0	1	0.4		65.024	65.005	65.216	62.135	69.354	63.181
0	25	18		38	46	34	39	23	36		0	0	1	0	1	0.4		54.187	62.837	65.216	62.135	69.354	63.113
0	45	23		16	32	16	35	22	24.2		1	1	0	0	0	0.4		80.608	68.122	65.216	62.135	69.354	64.114
0	35	19		19	40	35	15	17	25.2		0	0	1	1	0	0.4		77.410	67.482	65.216	62.135	69.354	64.788
1	0	23		18	45	39	29	28	31.8		1	1	0	0	0	0.4		61.343	64.269	65.216	62.135	69.354	64.684
0	30	27		26	45	30	33	33	33.4		0	1	1	0	0	0.4		58.405	63.681	65.216	62.135	69.354	64.483
1	35	25		39	18	40	27	28	30.4		1	0	0	0	0	0.2		128.337	77.667	65.216	62.135	69.354	67.120
1	37	20		37	39	23	14	25	27.6		1	0	0	1	1	0.6		47.119	61.424	65.216	62.135	69.354	65.981
1	46	31		33	34	38	26	24	31		1	0	1	1	0	0.6		41.951	60.390	65.216	62.135	69.354	64.863
1	54	21		25	38	44	27	31	33		0	0	1	1	0	0.4		59.113	63.823	65.216	62.135	69.354	64.655
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		29.737	57.947	65.216	62.135	69.354	63.313
1	21	30		41	17	44	29	33	32.8		1	0	1	1	1	0.8		130.000	78.000	65.216	62.135	69.354	66.251
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		187.000	89.400	65.216	62.135	69.354	70.880
0	46	31		41	17	44	29	33	32.8		1	0	1	1	1	0.8		155.000	83.000	65.216	62.135	69.354	73.304
1	45	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		118.000	75.600	65.216	62.135	69.354	73.764
0	32	22		41	17	44	29	33	32.8		1	0	1	1	1	0.8		158.000	83.600	65.216	62.135	69.354	75.731
1	27	23		41	17	44	29	33	32.8		1	0	1	1	1	0.8		115.000	75.000	65.216	62.135	69.354	75.585
1	20	29		41	17	44	29	33	32.8		1	0	1	1	1	0.8		112.000	74.400	65.216	62.135	69.354	75.348
0	30	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		144.000	80.800	65.216	62.135	69.354	76.438
1	27	24		41	17	44	29	33	32.8		1	0	1	1	1	0.8		107.000	73.400	65.216	62.135	69.354	75.831
1	10	36		41	17	44	29	33	32.8		1	0	1	1	1	0.8		177.000	87.400	65.216	62.135	69.354	78.144

married aux.	Weight gain (y)	age (X)	$\overline{Y}$	X1	X2	X3	X4	X5	Mean	P(j)	p1	p2	p3	p4	p5	p(i)	$\overline{X}$	Phi	Zi	CL	LCL	UCL	Z-HEWA
1	0	29		41	17	44	29	33	32.8		1	0	1	1	1	0.8		117.000	75.400	65.216	62.135	69.354	77.596
1	29	25		41	17	44	29	33	32.8		1	0	1	1	1	0.8		172.000	86.400	65.216	62.135	69.354	79.356
0	65	20		41	17	44	29	33	32.8		1	0	1	1	1	0.8		197.000	91.400	65.216	62.135	69.354	81.765
0	29	18		41	17	44	29	33	32.8		1	0	1	1	1	0.8		214.000	94.800	65.216	62.135	69.354	84.372
0	45	26		41	17	44	29	33	32.8		1	0	1	1	1	0.8		202.000	92.400	65.216	62.135	69.354	85.978