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A MODIFIED POLYNOMIAL RESPONSE FUNCTION ALTERNATIVE TO LOG-LINEAR RESPONSE FUNCTION

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This article points out the difficulties being faced in fitting models for experimental data where either a saturation or toxic effect occurs and attempts to overcome these difficulties by proposing new methods. Polynomial response functions were objected due to their unboundedness and having symmetry about the optimum with quadratic form. Inverse polynomial response functions could not be recommended as they require some constraints and have biased estimates of unknown parameters with unknown sampling distributions. Log-linear response functions never give negative predictions and provide adequate representation of the data within its range. Although log-linear response functions are superior to other response functions but not the only alternative. In this work, the data are summarized successfully by employing ordinary polynomial response functions using suitable transformations and named as "Modified polynomial response functions". This method of model building is simple and has the capability of explaining the maximum variation of data.

Key words: inverse polynomial response functions, log-linear response functions, modified polynomial response functions

INTRODUCTION

For a unifactor experiment, the researcher has to know the following:

i) Is the factor applied effective?

ii) Are there differences in the effectiveness of different levels of factor?

iii) Can we form an appropriate relationship between the factor and the response?

The answers to questions *(i)* and *(ii)* are provided by F-test in analysis of variance (ANOVA) technique and Fisher's least significant difference test. To answer question *(iii)* i.e. to form the relationship between the response and the factor, the factor sum of squares is subdivided into linear, quadratic, cubic, etc. trend components through the use of orthogonal polynomial. The use of orthogonal polynomial requires the levels of the factor to be equally spaced. Also, the researcher in agriculture meets the response curve of the form shown in Fig. 1 and 2 i.e. if the factor applied is increased, the response will tend to be an asymptotic value (Fig.1) or the response after first rising, begins to fall again as the factor applied is further increased with no built-in symmetry (Fig.2).

There are various response functions available in literature being used by researchers in the biometric field. The most popular of these are the polynomial response functions, inverse polynomial response functions and log-linear response functions.

Polynomial response function can be generalized as:

$$E(Y) = \sum_{j_1=0}^{l_1-1}\sum_{j_2=0}^{l_2-1}\cdots\sum_{j_k=0}^{l_k-1} \beta_{j_1 j_2 \ldots j_k} X_1^{j_1} X_2^{j_2} \cdots X_k^{j_k} \quad \ldots (1)$$

Where $X_1, X_2, \ldots, X_k$ are k-quantitative factors with levels $l_1, l_2, \ldots, l_k$.

Box and Wilson (1951) used first order form of these functions in response surface methodology for multifactor experiments. Although polynomial response functions are simple and hypothesis testing is quite straight due to the assumption of normality, however, the following objections are raised against their use:

a). These functions are unbounded and polynomial can take a value as large (either -ve or +ve) as one pleases with increase in X.

b). These functions do not give asymptotic form of relationship (Fig. 1).

c). A specific disadvantage associated with a quadratic relationship is that it is symmetric about the optimum.

Nelder (1966) introduced inverse polynomial response functions which aimed to meet the objections raised to the polynomial response function. A full model inverse polynomial response function is defined as :-

$$E(Y)^{-1} = \sum_{j_1=0}^{l_1-1}\sum_{j_2=0}^{l_2-1}\cdots\sum_{j_k=0}^{l_k-1} \beta_{j_1 j_2 \ldots j_k} X_1^{j_1} X_2^{j_2} \cdots X_k^{j_k} \quad \ldots (2)$$

Where $l_1, l_2, \ldots, l_k$ are the corresponding levels of k factors respectively.

These have been successfully applied to data on the joint effect of light intensity and density on the growth of *Hibiscus moscheutos* by Hozumi et al. (1958) . For inverse polynomials, Nelder (1966) proposed two methods of estimating the unknown parameters i.e. the weighted least squares and iterative maximum likelihood methods. Ali et al. (1983) made the following objections on the use of inverse polynomial response function.

a). Inverse polynomial response functions are constrained such that the regression parameters should be positive so that the predicted response should be non-negative and bounded for X. The constrained method of estimation gives a residual sum of squares greater than that obtained by an unconstrained method and the assumption of a normal error distribution does not give either normally distributed constrained co-efficients or a residual sum of squares with standard chi-square distribution (Hudson, 1969) .

b). The estimates of unknown parameters are biased and so is the predicted response .

c). The sampling distribution of various quantities is unknown and therefore if hypothesis testing is important, the worker is in difficulty.

Ali et al. (1983) proposed log-linear response function. The full model is shown below:

$$E(LogY)= \sum_{j_1=0}^{n_1-1} \sum_{j_2=0}^{n_2-1} \cdots \sum_{j_k=0}^{n_k-1} \beta_{j_1 j_2 \ldots j_k} X_1^{j_1} X_2^{j_2} \ldots X_k^{j_k} \quad \ldots(3)$$

For log-linear response functions the asymptotic form of the relationship exists and are unsymmetrical about the optimum. Assuming log Y distributed as a normal random variable, the estimation of βs is achieved by carrying out linear regression of log Y on the predictor variables. The resulting estimates are therefore minimum variance linear unbiased estimators. The objective of this work is to suggest better form of models by pointing out the deficiencies of the existing forms.

MATERIAL AND METHODS

Bould (1969) has conducted a series of experiments on black current crop at the Longashton research centre, Bristol, England to study the utility of leaf analysis as a guide to nutritional status of a plant. It is a well recognised approach by researchers in fruit nutrition. The study was made using factorial sand-culture pot experiments. The standard Longashton nutrition solution, as described by Bould (1969), was applied to the surface of the sand. Sand was thoroughly washed before use but was not purified by any acidic treatment. Four hardwood cuttings cv. Baldwin, were inserted in each container in August-September and watered. They were allowed over-winter in a cold glasshouse to encourage callus formation and rooting. In the following spring they were transferred to covered concrete pots in outdoor cages where they remained for two seasons. In the spring, when shoots were 6 to 9 inches long, they were thinned to two per cutting, thus leaving eight shoots per pot. New basal shoots formed in the second growing season were removed. Treatments applied and layout for the black current experiments 1 and 2 are given in Table. 1.

Table 1.Treatments and layout of black current experiments 1 and 2

Treatments	BPE-1	BPE-2
N	10,12,14,16,20	20
P	1/2,1,2,4	4
Mg	4	1/2,1,2,4
K	4	1/2,1,2,4
Layout	3 RCB	3 RCB
No. of plants per pot	4	4

The concentrations of nutrients are in terms of milligram equivalent/l. BPE= Black current pot experiments 1 and 2..

Total shoot length was recorded for each plot after leaf fall in the first non-cropping season. Ascorbic acid was determined potentiometrically after extraction of fruit with meta-phosphoric acid.

Log-linear response functions are claimed to be superior to the inverse polynomial response functions and ordinary polynomial response functions but it does not mean that the log-linear response functions are the only alternatives. According to McCullagh and Nelder (1983), all the models are wrong, some though are better than others. Further they advised not to fall in love with one model. The object of the present work is similar to that as guided by these two statisticians. It is suggested that if we are facing the data which show an asymptotic trend, it may also be appropriate to use the model.

$$E(Y) = \beta_0 + \beta_1 LogX \quad \ldots(\sim)$$

i.e. Y is linearly related to log X and for negative β_1, Y tends to be an asymptotic value β_0 as X increases i.e. if the trend is as shown in Fig. 1.

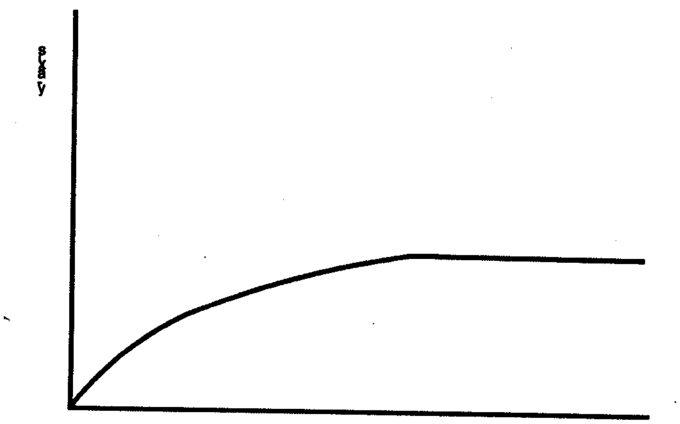

Fig. 1

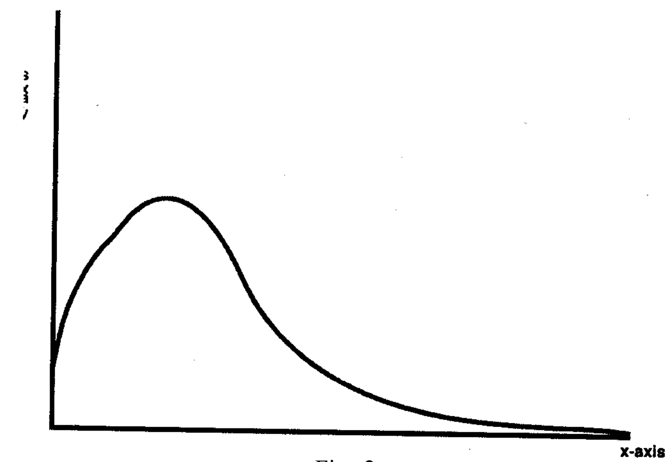

Fig. 2

The shape of the response curve found in agricultural and biometric fields.

The model given below:

$$E(Y) = \beta_0 + \beta_1 \, Log X + \beta_2 \, (Log x)^2 \quad \ldots (5)$$

may also be useful to summarize the data which are unsymmetrical about the optimum i.e. as shown in Fig. 2. For this model, the optimum value is at $X = Exp(-\beta_1/2\beta_2)$

The layout of experiments for which the data were analysed are given in Table 1. The levels P for BPE-1 and the levels of Mg and K for BPE-2 are applied on logarithmic scale. It is suggested to take the logarithmic transformation for these factors and take the other factors as they are. Use the ordinary polynomial response functions to summarize the data from such experiments. The resulting response functions, are called "Modified polynomial response functions".

Consider an experiment with two factors X_1 and X_2 at levels l_1 and l_2 respectively. Assuming that levels of X_1 are equally spaced in terms of log-scale and the levels of X_2 are not equally spaced in terms of any scale (e.g. log, geometric, etc.). The full model for such a situation is presented below:

$$E(Y) = \beta_0 + \beta_{11}(Log X_1) + \beta_{12}(Log X_1)^2 + \ldots + \beta_{1(l_1-1)}(Log X_1)^{l_1-1} + \beta_{01}X_2 + \beta_{02}X_2^2 + \ldots + \beta_{0(l_2-1)}X_2^{l_2-1} + \beta_{11}(Log X_1)X_2 + \ldots + \beta_{(l_1-1)(l_2-1)}(Log X_1)^{l_1-1}X_2^{l_2-1} \quad \ldots (6)$$

Since the values of $log X_1$ are equally spaced therefore so called co-efficients of orthogonal polynomials can be used for the values of $log X_1$ and ~, thus co-efficients obtained from Gram-Schmidt method are used to form the analysis of variance table. After selecting the appropriate response function, the researcher tends to keep those terms in the model which help in explaining the variation of data clearly. Ali's method of model specification from ANOVA table has been used (Ali, 1983).

This method consists of the following steps:

1) Set up ANOVA table and test the significance of the main effects and interactions.
2) Partition the sum of squares for the main effects of each factor for single degree of freedom such that the variation of the data explained due to inverse, linear, quadratic, cubic, etc. can be examined separately.
3) Test the simplification of these effects for each of the factors in the same conforming order i.e. inverse, linear, quadratic, cubic, etc. Further test the significance of two factors, three factors,---, and. k-factors interaction effects which are partitioned for single degree of freedom e.g. inverse * inverse, inverse * linear, linear * cubic, etc.
4) Apply multiple regression analysis including all those terms in the model which are shown to be significant, keeping in view the conforming order of systematic model building method.

RESULTS AND DISCUSSION

Table 2 presents information about the significance of the main effects and interaction of BPE-1 and BPE-2. With the help of these results and using Ali's method of model specification, the data are presented in appropriate mathematical forms. It is

Table 2. Analysis of variance of data obtained from black current experiments 1 and 2

Response variates	BPE-1				BPE-2			
	Blocks	N	P	NP	Blocks	Mg	K	MgK
Yield	n.s.	***	***	***	n.s.	***	***	**
Shoot length	F<1	n.s.	***	n.s.	F<1	*	***	F<1
Asc. acid	F<1	n.s.	***	n.s.	F<1	**	***	F<1

*,**, *** denote significant at 5%, 1% or 0.1% level respectively.

Table 3. Modified polynomial response function analysis for BPE-l
(N*P factorial experiments)

Respouse varrables

	Yield			Shoot length			Asc. acid		
Terms	Reg.co-efficient	t	Sig.	Reg.co-efficients	t	Sig.	Reg.co-efficient	t	Sig.
Const.	262.06	0,467	0.643	1057.77	22.05	0.0	507.09	30.7	0.0
N	215.09	2.8	0.006	-6.57	-2.63	0.04	-2.15	-1,95	0.05
N'	-6.69	-2.70	0.009	_	_	_	_	_	_
N'	_	_	_	_	_	_	_	_	_
N'	_	_	_	_	_	_	_	_	_
LogP	2497.10	4.59	0.0	943.24	6.54	0.0	-53.71	-4.7	0.0
(LogP)'	-2204.4	-1.5	0.1	-1955.8	-5.5	0.0	_	_	_
(LogP)'	-6172.01	-4.2	.0001	_	_	_	_	_	_
N(LogP)	45.26	1,3	0.1	-21,05	-2.16	0.03	_	_	_
N(LogP)'	206.21	2,4	0.02	94.23	3.91	.0003	_	_	_
	F=93.17 Sig.F=O			F=98.91 Sig.F=O			F=13.00 Sig.F=O		

Table 4. Modified polynomial response function analysis for BPE-2
(Mg*K factorial experiment)

Response variables

	Yield			Shoot length			Asc. acid		
Terms	Reg.co-effient	t	Sig.	Reg, eo-efficient	t	Sig.	Reg ..co-efficient	t	Sig.
Const.	1923.14	21,84	0	935.33	37.83	0	398.76	71,1	0
LogMg	155.92	0.869	0.38	-24.79	-0.55	0.585	-61,98	-4.15	.0001
(Log Mg)2	-469.78	-1,078	0.28	_	_	_	_	_	_
LogK	1740.75	4.92	0	534.19	8.16	0	71,66	4.99	0
(Log K)2	1246.78	1,18	0.24	-183.05	-1,10	0.279	_	_	_
(Log K)3	-3392.8	-1,49	0.14	_	_	_	_	_	_
LogMg LogK	1480.98	4.53	.0001	-	-	-	-	-	-
	F=34.32 Sig.F=O			F=43.10 Sig.F=O			F=18.5S ig.F=O		

considered that modified polynomial response function is an appropriate form of the response and Ali's method of model specification seems suitable to select terms used in the model. Yield, shoot length and ascorbic acid are taken as the response variables for both BPE-l and BPE-2. The results obtained by applying these methods are given in Tables 3 and 4.

To check whether the proposed model is practicable or not, F-values are calculated. Tables 3 and 4 indicate that F-values are highly significant for all regressands under BPE-l and BPE-2. There is further indication of the goodness of fit of the "Modified polynomial response function" to the data. The inclusion of some insignificant values in the model is due to the order mentioned in Ali's method of model simplification i.e. the inclusion of higher order terms conform to the existence of lower order terms in equation. All the main effects are assumed to be significant. The results given in Tables 3 and 4 provide ample indication about the goodness of the fit of data. It is evident that modified polynomial response function proved to be a successful alternative to the log-linear response function.

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