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Research Article

Stringency Criterion for Modality Tests

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Abstract: Different techniques in the field of multimodality testing have aimed at different goals. In this study, we compare four nonparametric modality tests kernel density estimation test or Silverman Bandwidth test proposed by Silverman [13], Hartigan DIP test proposed by Hartigan and Hartigan [8], proportional mass test by Cavallo and Ringobon [4] and excess mass test by Muller and Sawitzki [12]. The most stringent test is found for different sample sizes 50, 100, 200 when only only μ_2 , μ_2 and σ^2_2 , μ_2 and α are considered period.

Keywords: Multimodality, Modality tests, Stringent test, Nonparametric test, Hypothesis tests, Period

1. INTRODUCTION

A stringency criterion is used to measure the performance of hypothesis tests. This criterion provides a unified view of optimality properties of tests. In most of the situations, size and power of any test are very useful for the comparison of different tests. However, in some situations this approach does not provide a satisfactory conclusion. Assume that there are two tests T_1 and T_2 for comparison of modality. These two tests can be compared on the basis of stringency criteria. For some alternatives,T_1 may be more powerful as compared to T_2 and for some other alternative situations, T_2 test may be more powerful as compared to T_1. To solve this type of problem, a technique has been introduced by Zaman [15] to compare the tests of modality. The approach of Zaman [15] is based on the density function of parametric assumption but this study discusses only non-parametric modality tests. There is no possible way to calculate most stringent test by the approach of likelihood ratio test when null and alternative hypotheses are tested. A modified method of Zaman [15] has been used in this study for estimating more stringent test by taking the minimum value of maximum difference between Maximum Power Approximation (MPA) and the power of different alternatives. Most of the nonparametric modality tests are developed for testing modality. The present study explores the following most popular tests only in univariate case:

(1) Kernel Density Estimation Test or Silverman Bandwidth Test proposed by Silverman [13].
(2) Hartigan Dip Test proposed by Hartigan and Hartigan [8].
(3) Proportional Mass Test by Cavallo and Ringobon [4].
(4) Excess Mass Test by Muller and Sawitzki [12].

Now-a-days these tests are used to measure modality of the distribution. These four tests are very useful measures to test the homogeneity and heterogeneity in the data. Most of the researcher applied these tests in different fields of life especially in economics, Bianchi [1] used modality tests to test the convergence by two nonparametric techniques (a) bootstrap multimodality (b) nonparametric density estimation tests, in a cross-

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section annual per capita GDP at constant US dollar of 119 countries. Cavallo and Ringobon [4] applied different theories of price stickiness having different implications of the distributions of price changes. Chen et al. [2] anticipated a modified likelihood ratio test for homogeneity in the finite mixture models. This criterion has never been used to test modality tests on the basis of stringency criterion. It also estimates the most stringent test. This research measures the modality tests on the basis of stringency for estimating best and worst test.

2. METHODOLOGY

Step-wise procedure for testing modality to compare the four tests under Stringency Criteria;

1. Find out critical values for each test of modality by Monte Carlo simulation technique.
2. The Power curve is drawn for each test of modality by plotting different alternatives along x-axis and Power of that test along y-axis. Let

   $P\left(L_{\pi i}{}^{h}\right)$ is denoted by different alternatives and

   $P\left(L_{\pi_j}^{*\ h}\right)$ is denoted by the Power of test.
3. For the deduction of the Approximated Power Envelope (APE), this study plots the different alternatives along X-axis and the Maximum Power Approximation (MPA) along Y-axis.
4. Short comings have been detected by each test through measuring maximum difference between the Power Curve and Maximum Power Approximation (MPA) and approximated Power Envelope has also been developed with the maximum difference. The short coming of the modality tests is given as

   $$S_C\left(L_{\pi_h}\right)=\left[\text{Max}\left\{P^*\left(L_{\pi j}{}^{h}\right)\text{-}P\left(L_{\pi i}{}^{h}\right)\right\}\right],$$

   where $P^*\left(L_{\pi j}{}^{h}\right)$ is denoted by MPA and $P\left(L_{\pi i}{}^{h}\right)$ is denoted by Power of different alternatives.
5. The most stringent test is identified by taking the minimum value of maximum differences between Maximum Power Approximation (MPA) and the Power of different alternatives. The function for most stringency test of this research is given as

   $$M_S\left(L_{\pi_h}\right)=\text{Min}\left[\text{Max}\left\{P^*\left(L_{\pi j}{}^{h}\right)\text{-}P\left(L_{\pi i}{}^{h}\right)\right\}\right],$$

   where $M_S\left(L_{\pi_h}\right)$ is denoted by most stringent test.
6. Steps 1 to 5 have been repeated for different sample sizes 50, 100, 200 when only μ_2 , μ_2 and σ_2^2 ,μ_2 and α are varying and finally found the most σ stringent test.

3. IDENTIFICATION OF MOST STRINGENT TEST

For identification of most stringent test from each test of modality, first we subtract the power of each test from the Maximum Power Approximation (MPA) then we take the value of maximum of these differences that is called short comings of the tests and finally get the minimum value of that maximum difference at different alternatives and different sample sizes. A test that has minimum short coming is called most stringent test. The "**Max**" is used for the maximum value of the short comings and **"MPA**" is used for the maximum power approximation. All of these results at different alternatives and different sizes are given in the Tables 1 to 9.

The results of sample size 50 has shown the shortcoming of Silverman that has minimum value from those maximum shortcomings. So Silverman test is the most stringent test because of minimum shortcoming. The shortcomings of the four tests at sample size 100 produced the maximum values of these shortcomings and concluded that the minimum value of that maximum is the most stringent test. So Silverman is the most stringent test because of minimum shortcoming. The shortcomings of the four tests at sample size 200 are an estimate then finding the maximum values of these shortcomings and finally concluded that the minimum value of that maximum is the most stringent test. So PM test is the most stringent test because of minimum shortcoming. The shortcomings of the four tests at sample size 50 calculated and found the maximum values of these shortcomings and finally concluded that the minimum value of that maximum is the most stringent test. So Silverman test is the most stringent test because of minimum shortcoming. The shortcomings of the four tests at sample size 100 calculated and found the maximum values of these shortcomings and finally concluded that the minimum value of that maximum is the most stringent test. So Silverman test is also the most

Table 1. Comparison of most stringent test for sample size is 50 and only μ_2 is varying.

Short comings of various test at sample size 50				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
11.3	6.25	0	7.3	5.5
44	34.86	0	33	34.8
97.4	67.31	0	87.4	67.6
100	23.8	0	88	26.2
100	3.08	0	81	2.1
100	0.27	0	58	0.1
100	0.02	0	47	0
100	0.01	0	47	0
100	0	0	51	0
Max	67.31	0	88	67.6

Table 2. Comparison of most stringent test for sample size is 100 and only μ_2 is varying.

Short comings of various test at sample size 100				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
20	14.61	13	0	15.9
15	9.6	0	7	10.6
24	7.38	0	7	7.3
100	16.69	0	84	16.9
100	0.1	0	90	0.1
100	0	0	84	0
100	0	0	59	0
100	0	0	54	0
100	0	0	37	0
Max	16.69	13	90	16.9

Table 2. Comparison of most stringent test for sample size is 200 and only μ_2 is varying.

Short comings of various test at sample size 200				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
98	92.88	86	0	91.9
94	88.98	61	0	90.1
95	74.22	47	0	73.8
100	2.5	0	3	2.5
100	0	0	2	0
100	0	0	9	0
100	0	0	5	0
100	0	0	0	0
100	0	0	0	0
Max	92.88	86	9	91.9

Table 4. Comparison of most stringent test for sample size is 50 when μ_2 and σ^2_2 arevarying.

Short comings of various test at sample size 50				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
7	0.96	4	0	2.2
35	29.56	0	24	29.3
78	71.17	0	64	69.6
88	78.94	0	64	77.6
90	75.97	0	59	77
91	73.39	0	56	75.7
96	75.71	0	52	73.7
100	76.43	0	44	73.4
100	73.5	0	42	73.3
Max	78.94	4	64	77.6

Table 5. Comparison of most stringent test for sample size is 100 when μ_2 and σ^2_2 arevarying.

Short comings of various test at sample size 100				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
15.6	9.91	0	6.3	10.5
57.4	46.07	0	55.7	48.5
98.9	31.25	0	97.9	34.4
100	0.14	0	100	0.1
100	0	0	100	0
100	0	0	99.9	0
100	0	0	99.7	0
100	0	0	99.4	0
100	0	0	99.3	0
Max	46.07	0	100	48.5

Table 6. Comparison of most stringent test for sample size is 200 when μ_2 and σ^2_2 arevarying.

Short comings of various test at sample size 200				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
17	11.43	0	6.9	10.8
67	62.98	0	63.21	63
91	86.37	0	81.24	85.2
100	89.61	0	95.28	88.6
100	81.04	0	97.27	81.7
100	70.76	0	90.33	70.6
100	59.56	0	73.01	60.6
100	51.84	0	54.54	50
100	44.02	0	47.46	49.1
Max	89.61	0	97.27	88.6

Table 7. Comparison of Most Stringent Test for sample size is 50, when μ_2 and α are varying.

Short comings of various test at sample size 50				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
6.5	1.75	3.5	3.5	0
11	5.26	0	9	5.5
21	6.28	0	16	5.9
67	5.44	0	54	2
95.6	0.2	6.6	62.6	0
99.8	0	4.8	61.8	0.1
99.9	0	2.9	47.9	0.2
100	0.85	1	54	0
100	0.06	0	73	0.3
Max	6.28	6.6	73	5.9

Table 8. Comparison of Most Stringent Test for sample size is 100, when μ_2 and α are varying.

Short comings of various test at sample size 100				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
21	15.88	14	0	16.9
15	9.98	0	11	10.6
24	3.22	0	19	7.3
100	2.5	0	94	16.9
100	0	0	87	0.1
100	0	0	78	0
100	0	0	66	0
100	0	0	58	0
100	0	0	87	0
Max	15.88	14	94	16.9

Table 9. Comparison of Most Stringent Test for sample size is 200, when μ_2 and α are varying.

Short comings of various test at sample size 200				
MPA	**Hartigan DIP test**	**Silverman test**	**PM test**	**EM test**
99.6667	94.2767	87.6667	0	93.5667
94	88.6	61	0	90.1
87	70.38	39	0	65.8
100	16.69	0	6	2.5
100	0.1	0	2.6667	0
100	0	0	3	0
100	0	0	2	0
100	0	0	0.6667	0
100	0	0	0.6667	0
Max	94.2767	87.6667	6	93.5667

stringent test because of minimum shortcoming. The shortcomings of the four tests at sample size 200 calculates and finds the maximum values of these shortcomings and finally concludes that the minimum value of that maximum is the most stringent test. So Silverman test is the most stringent test because of minimum shortcoming. These short- comings of the four tests at sample size 50 calculated and estimated the maximum values of these shortcomings and finally concluded that the minimum value of that maximum is the most stringent test. In this table the shortcomings of Hartigan Dip test, Excess mass Test and Silverman test are looking very close, but Excess Mass test is the most stringent test because of minimum shortcoming. It may occur due to random fluctuation of the density. The short- comings of the four tests at sample size 100 are estimated and found the maximum values of these shortcomings and finally concluded that the minimum value of that maximum is the most stringent test. The shortcomings of Hartigan Dip test, Excess Mass Test and Silverman test are looking very close but Silverman test has the minimum shortcoming, so it is the most stringent test. These shortcomings of the four tests at sample size 200 also calculated and then found the maximum values of these shortcomings and finally conclude that the minimum value of that maximum is the most stringent test. The shortcomings of PM test have the minimum shortcoming, so it is the most stringent test.

4. CONCLUSION

It is concluded from Tables 1 to 3, when only μ_2 is varying the shortcomings of the four tests at sample size 50 and 100, the Silverman is the most stringent test because of minimum short coming. But the shortcomings of PM test for all four tests at sample size 200 are least, so the PM test is most stringent due to large bumps and large sample size. From Tables 4 to 6 the shortcomings of the four tests at sample sizes 50,100 and 200, when are varying, it is concluded that the Silverman test is the most stringent test because of minimum short coming and small bumps. From Tables 7 to 9 when μ_2 and σ^2_2 are varying at sample size 50 , the short comings of Hartigan DIP test, Excess Mass Test and Silverman test are looking very close but Excess Mass test is the most stringent test because of minimum shortcoming. It may occur due to random fluctuation of the density. The shortcomings of the four tests at sample size 100 Silverman test is looking the most stringent test due to large bumps. At sample size 200 it is found that the shortcomings of PM test has the minimum value, so it is the most stringent test due to large sample size and large bumps. Finally it is concluded that the Silverman test is the most stringent test as compared to the other tests except in the large samples and large bumps.

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