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Research Article

A Quasi Lindley Pareto Distribution

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Abstract: In applied sciences, the lifetime's data models have been constructed to facilitate better modeling and significant progress towards the reliability analysis or survival analysis. For this purpose, a new distribution "Quasi Lindley Pareto Distribution (QLPD)" has been introduced. It's Moments generating function, moments, mean, variance, coefficient of variation, skewness, kurtosis, survival function, hazard function, and distribution of extreme order statistics have been discussed. The maximum likelihood method has been discussed for estimating its parameters. The distribution has been fitted to real life data set to check its goodness of fit to which earlier the Lindley distribution and Quasi Lindley distribution have been fitted and it is found that QLPD provides closer fits than those by the Lindley distribution and Quasi Lindley distribution.

Keywords: Moments, Order statistics, Survival, Hazard, Goodness of fit

1. INTRODUCTION

The modeling of lifetime data is crucial in many fields of applied sciences like engineering, biological, finance and insurance. Exponential, Gamma, Weibull and Rayleigh distributions have been developed for this purpose. Lindley [1] had developed one parameter Lindley distribution for lifetime data analysis. Sankaran [2] developed discrete Poisson-Lindley distribution from Lindley and Poisson distributions. He also discussed its different properties, parameter estimation and goodness of fit. Ghitany et al. [3] worked on Lindley distribution with considering waiting times before service of the banking customer's. Shanker and Mishra [4] derived Quasi Lindley distribution (QLD). Shanker and Mishra [5] derived two-parameter Lindley distribution and discussed its various properties. Shanker et al. [6] introduced another two-parameter Lindley distribution. Shanker and Amanuel [7] developed new Quasi Lindley distribution. Shanker and Tekei [8] determined a new Quasi Poisson Lindley distribution. Shanker and Mishra [9] developed Poisson mixture of Quasi Lindley distribution and also described various statistical and mathematical properties, parameter estimation and applications. Lazri and Zeghdoudi [10] introduced a distribution for modeling lifetime data called Lindley Pareto distribution (LP) and also discussed its various properties and application. In this paper, we have derived Quasi Lindley Pareto distribution and its different properties are also discussed here.

Let Y be a random variable following the two parameter Quasi Lindley distribution (α, θ) with the following probability density function:

$$f(y;\, \alpha, \theta) = \frac{\theta(\alpha + y\theta)e^{-\theta y}}{(\alpha+1)} \quad y > 0,\ \alpha > -1,\ \theta > 0 \qquad (1)$$

Pareto distribution was established by Pareto [11] to elaborate the unequal distribution of wealth. There are many studies on Pareto distribution such as the generalized Pareto distribution was derived by Pickands [12], the beta-Pareto distribution was discussed by Akinsete et al. [13] and the beta generalized Pareto distribution was proposed by Mahmoudi [14].

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The mixed distribution is one of the most important concepts for developing a new distribution. For example, Zakerzadeh and Dolati [15] used gamma (α, θ) and gamma (α + 1, θ) to create a Generalized Lindley distribution (GLD). Nedjar and Zeghdoudi [16] determined a new distribution, based on gamma (2, θ) and one parameter Lindley distribution known as gamma Lindley distribution.

The T-X family of distributions defined by Alzaatreh et al. [17] is a new method to generate families of the continuous distributions. The cumulative distribution function (CDF) or distribution function (DF) of this family is defined as:

$$G(y) = \int_0^{\frac{F(y)}{1-F(y)}} f(t)\, dt \tag{2}$$

Using Quasi Lindley distribution, (2) becomes Quasi-Lindley X-family of distribution with CDF and Probability density function (pdf) such that:

$$G(y) = 1 - \exp(-\theta\delta)\{1 + \left(\frac{\theta}{\alpha+1}\right)\delta\} \tag{3}$$

Where $\delta = \frac{F(y)}{1-F(y)}$

$$g(y) = \left[\frac{\theta}{(\alpha+1)}\right]\left[\frac{f(y)}{\{1-F(y)\}^2}\right]\left[\gamma + \frac{F(y)}{1-F(y)}\right] \times e^{-\theta\left\{\frac{F(y)}{1-F(y)}\right\}} \tag{4}$$

We considered CDF and pdf a Pareto distribution as:

$$F(y) = 1 - \left[\frac{\beta}{y}\right]^k \qquad f(y) = \frac{k\beta^k}{y^{k+1}} \qquad y > \beta$$

1.1 A Quasi Lindley Pareto Distribution

By utilizing (3) and (4), we get CDF and pdf of Quasi Lindley Pareto distribution (QLPD) with scale parameter(s) α and γ , shape parameter(s) θ, β and k is given by respectively:

$$Q(y) = 1 - \exp\left[\gamma - \theta\left(\frac{y}{\beta}\right)^k - 1\right]\left[1 + \left(\frac{\theta}{\alpha+1}\right)\left\{\left(\frac{y}{\beta}\right)^k - 1\right\}\right] \qquad y > 0, k, \alpha, \gamma, \theta, \beta > 0 \tag{5}$$

and

$$q(y) = \left[\frac{\theta k e^{\theta}}{\beta(1+\alpha)}\right]\left[(\gamma - \theta)\left(\frac{y}{\beta}\right)^{k-1} + \theta\left(\frac{y}{\beta}\right)^{2k-1}\right]\exp\{-\theta\left(\frac{y}{\beta}\right)^k\} \tag{6}$$

The nature of QLPD for different values of its parameters $\alpha, \gamma, \theta, \beta$ *and* k has shown graphically in the Fig. 1 (a), (b), (c) and (d). In Fig.1 (d) seven choices of QLPD parameters have been combined for comparisons.

2. SOME PROPERTIES

Survival Function: The Survival function for random variable Y from QLPD(Y; $k, \gamma,\ \alpha,\ \theta,\ \beta$) is given as:

$$\mathrm{S(y)} = \exp\left[-\theta\left(\frac{y}{\beta}\right)^k - 1\right]\left[1 + \left(\frac{\theta}{\alpha+1}\right)\left\{\left(\frac{y}{\beta}\right)^k - 1\right\}\right] \tag{7}$$

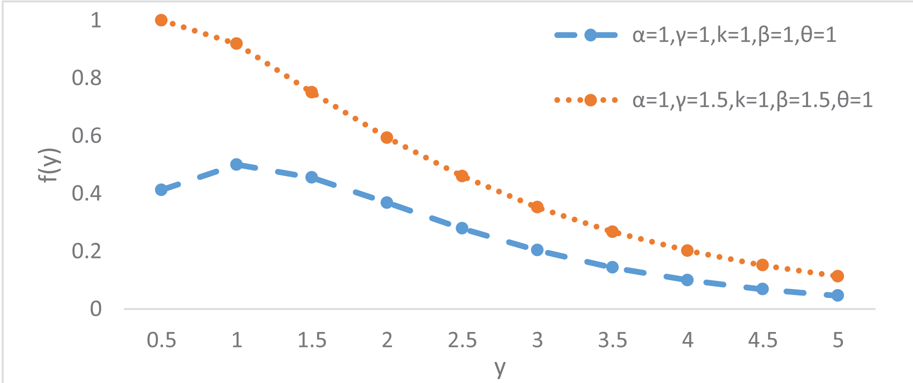

Fig.1 (a)

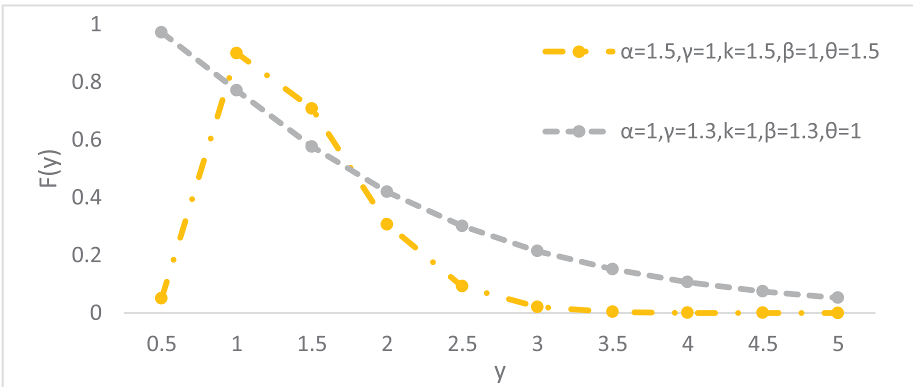

Fig.1 (b)

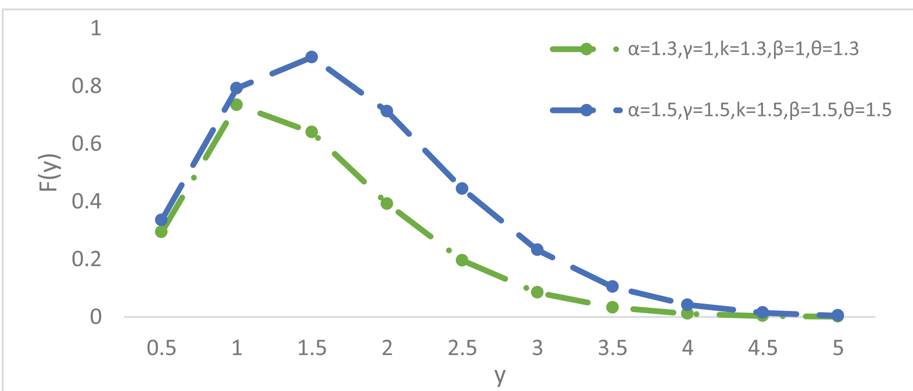

Fig.1 (c)

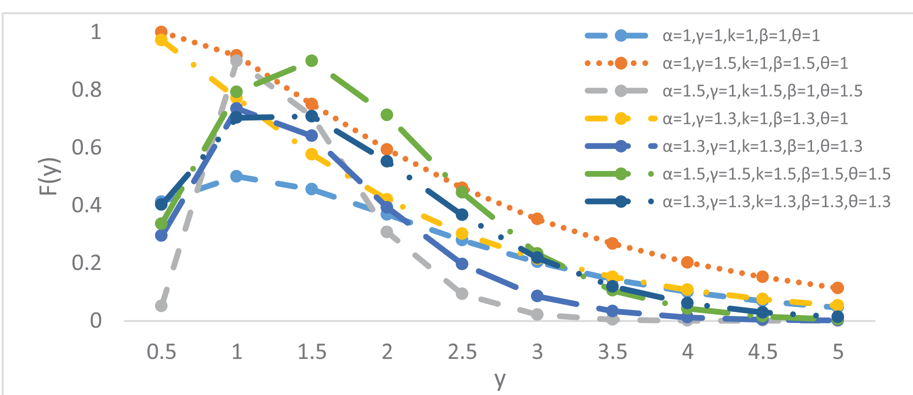

Fig.1 (d)

Hazard Function: The failure rate function has also been called hazard function, determined, by Eq. (6) over survival function Eq. (7). So, hazard function of $\mathrm{QLPD}(\mathrm{Y}; k,\ \gamma, \alpha,\ \theta,\ \beta)$ is:

$$H(y) = \frac{\theta k[(\alpha-\theta)+\theta\left(\frac{y}{\beta}\right)^{k}]\left(\frac{y}{\beta}\right)^{k-1}}{\beta\left[(1+\alpha)+\theta\left\{\left(\frac{y}{\beta}\right)^{k}-1\right\}\right]} \tag{8}$$

Moment Generating Function: The moment generating function of $\mathrm{QLPD}(\mathrm{Y}; k,\gamma,\ \alpha,\ \theta,\ \beta)$ is:

$$M_y(t) = \frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\sum_{i=0}^{\infty}\{\Gamma\left(\frac{i}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{i}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^{i}}{\theta^{i/k}}.\frac{t^{i}}{i!} \tag{9}$$

This function has been utilized to determine i^{th} number of moment about origin.

Moments: The i^{th} moment about origin of random variable Y from $\mathrm{QLPD}(\mathrm{Y}; k,\gamma,\ \alpha,\ \theta,\ \beta)$ are given below:

$$E(y^{i}) = \frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{i}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{i}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^{i}}{\theta^{\frac{i}{k}}} \tag{10}$$

By putting i = 1, 2, 3 and 4 in Eq. (10), first four moments about origin have been derived to determine the mean, variance, coefficient of variation, skewness and kurtosis.

First four moments about mean are defined as:

$$\mu_1 = m_1 - m_1 \tag{11}$$

$$\mu_2 = m_2 - (m_1)^2 \tag{12}$$

$$\mu_3 = m_3 - 3m_2m_1 + 2(m_1)^3 \tag{13}$$

$$\mu_4 = m_4 - 6m_3m_1 + 3m_2m_1 - 4(m_1)^{4} \tag{14}$$

From Eq. (10), we have,

$$m_1 = E(y^1) = \frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta}{\theta^{\frac{1}{k}}} \tag{15}$$

$$m_2 = E(y^2) = \frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{2}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{2}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^2}{\theta^{\frac{2}{k}}} \tag{16}$$

$$m_3 = E(y^3) = \frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{3}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{3}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^3}{\theta^{\frac{3}{k}}} \tag{17}$$

$$m_4 = E(y^4) = \frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{4}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{4}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^4}{\theta^{\frac{4}{k}}} \tag{18}$$

From above equations, we have,

$$\mu_1 = 0 \tag{19}$$

$$\mu_2 = Var(y) \tag{20}$$

$$\mu_3 = \frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{3}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{3}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^3}{\theta^{\frac{3}{k}}} - 3\frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{2}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{2}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^2}{\theta^{\frac{2}{k}}} \times$$

$$\frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta}{\theta^{\frac{1}{k}}} + 2\left(\frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta}{\theta^{\frac{1}{k}}}\right)^3 \quad (21)$$

$$\mu_4 = \frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{4}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{4}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^4}{\theta^{\frac{4}{k}}} - 6\frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{3}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{3}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^3}{\theta^{\frac{3}{k}}} \times$$

$$\frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta}{\theta^{\frac{1}{k}}} + 3\frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{2}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{2}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta^2}{\theta^{\frac{2}{k}}} \times$$

$$\frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta}{\theta^{\frac{1}{k}}} - 4\left(\frac{e^{\theta}}{(1+\alpha)}\left[(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right]\frac{\beta}{\theta^{\frac{1}{k}}}\right)^4 \quad (22)$$

Mean: The mean of random variable Y from QLPD(Y; $k, \gamma,\ \alpha,\ \theta,\ \beta$) is defined as:

$$E(y) = \frac{e^{\theta}}{(1+\alpha)}\left(\frac{\beta}{\theta^{\frac{1}{k}}}\right)\left[(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right] \quad (23)$$

Variance (Var): The variance of random variable Y from QLPD(Y; $k, \gamma,\ \alpha,\ \theta,\ \beta$) is obtained as follow:

$$\mathrm{Var(y)} = \frac{e^{\theta}}{(1+\alpha)}\left(\frac{\beta}{\theta^{\frac{2}{k}}}\right)\left[\left\{(\alpha-\theta)\Gamma\left(\frac{2}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{2}{\mathrm{k}}+2,\ \theta)\right\} - \left\{(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right\}^2\frac{e^{\theta}}{(1+\alpha)}\right] \quad (24)$$

Coefficient of Variance (C.V): The coefficient of variance for random variable Y from QLPD(Y; $k, \gamma,\ \alpha,$ $\theta,\ \beta$) is obtained as follow:

$$C.V = \sqrt{\frac{Var(y)}{E(y)}} \quad (25)$$

$$C.V = \sqrt{\frac{(1+\alpha)}{e^{\theta}}\frac{\left[(\alpha-\theta)\Gamma\left(\frac{2}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{2}{\mathrm{k}}+2,\ \theta)\right]}{\left[(\alpha-\theta)\Gamma\left(\frac{1}{\mathrm{k}}+1,\ \theta\right)+\Gamma(\frac{1}{\mathrm{k}}+2,\ \theta)\right]} - 1} \quad (26)$$

Similarly, Skewness and Kurtosis will be determined by using Eq. (21, 22 and 24):

$$Skewness = \frac{\mu_3^2}{\mu_2^3} \quad (27)$$

$$kurtosis = \frac{\mu_4}{\mu_2^2} \quad (28)$$

Order Statistics (OS): Ordering of positive continuous random variables is an important tool for judging the comparative behavior. Let, $Y_1, Y_2, \dots, Y_n$ are random samples from QLPD(Y; $k, \gamma, \alpha,\ \theta,\ \beta$) and $Y_{r.n}$ is the r^{th} order statistics with pdf given as follow;

$$q_{r.n}(y) = \frac{n!}{(r-1)!(n-1)!}q(y)[Q(y)]^{r-1}[1-Q(y)]^{n-r} \quad (29)$$

$$q_{r.n}(y) = \frac{n!}{(r-1)!(n-1)!}\left[\left\{\frac{\theta k e^{\theta}}{\beta(1+\alpha)}\right\}\left\{(\gamma-\theta)\left(\frac{y}{\beta}\right)^{k-1} + \theta\left(\frac{y}{\beta}\right)^{2k-1}\right\}\exp\{-\theta\left(\frac{y}{\beta}\right)^{k}\}\right] \times \left[1-\exp\left[-\theta\left(\frac{y}{\beta}\right)^{k} - 1\right]\left[1+\left(\frac{\theta}{\alpha+1}\right)\left\{\left(\frac{y}{\beta}\right)^{k}-1\right\}\right]\right]^{r-1} \times \left[\exp\left\{-\theta\left(\frac{y}{\beta}\right)^{k}-1\right\}\left[1+\left(\frac{\theta}{\alpha+1}\right)\left\{\left(\frac{y}{\beta}\right)^{k}-1\right\}\right]\right]^{n-r} \quad (30)$$

So, the pdf of the smallest OS at r=1, $q_{1.n}(y)$ and the largest OS at r=n, $q_{n.n}(y)$ are obtained by:

$$q_{1.n}(y) = n\left[\left\{\frac{\theta k e^{\theta}}{\beta(1+\alpha)}\right\}\left\{(\gamma-\theta)\left(\frac{y}{\beta}\right)^{k-1} + \theta\left(\frac{y}{\beta}\right)^{2k-1}\right\}\right] \times \left[1+\left(\frac{\theta}{\alpha+1}\right)\left\{\left(\frac{y}{\beta}\right)^{k}-1\right\}\right]^{n-1} \times \left[\exp\{-\theta\left(\frac{y}{\beta}\right)^{k}\}\right]^{n} \quad (31)$$

and

$$q_{n.n}(y) = n\left[\left\{\frac{\theta k e^{\theta}}{\beta(1+\alpha)}\right\}\left\{(\gamma-\theta)\left(\frac{y}{\beta}\right)^{k-1} + \theta\left(\frac{y}{\beta}\right)^{2k-1}\right\}\exp\{-\theta\left(\frac{y}{\beta}\right)^{k}\}\right] \times \left[1-\exp\left[-\theta\left(\frac{y}{\beta}\right)^{k} - 1\right]\left[1+\left(\frac{\theta}{\alpha+1}\right)\left\{\left(\frac{y}{\beta}\right)^{k}-1\right\}\right]\right]^{n-1} \quad (32)$$

3. ESTIMATION OF PARAMETERS

Let, $y_1, y_2, \ldots, y_n$ be a random sample of size n from the Quasi Lindley Pareto distribution and the likelihood function of QLPD is defined as:

$$L = \left[\frac{\theta k e^{\theta}}{\beta(1+\alpha)}\right]^{n} \prod_{i=1}^{n}\left[(\gamma-\theta)\left(\frac{y_i}{\beta}\right)^{k-1} + \theta\left(\frac{y_i}{\beta}\right)^{2k-1}\right] \exp^{\sum_{i=1}^{n}\left\{-\theta\left(\frac{y_i}{\beta}\right)^{k}\right\}} \quad (33)$$

Further, the log likelihood function is obtained as follow:

$$logL = nlog\left[\frac{\theta k e^{\theta}}{\beta(1+\alpha)}\right] + log\left[\frac{(\gamma-\theta)}{\beta^{k-1}}\prod_{i=1}^{n} {y_i}^{k-1} + \frac{\theta}{\beta^{k-1}}\prod_{i=1}^{n} {y_i}^{2k-1}\right] - \sum_{i=1}^{n}\left\{-\theta\left(\frac{y_i}{\beta}\right)^{k}\right\} \quad (34)$$

By taking partial derivatives with respect to parameters, following estimates are obtained:

$$\widehat{\alpha_{\mathrm{mle}}} = -1 \quad (35)$$

$$\widehat{\beta_{\mathrm{mle}}} = \left[\frac{-k\theta\sum_{i=1}^{n} {y_i}^{k}}{-n-3k+2}\right]^{1/k} \quad (36)$$

$$\widehat{\theta_{\mathrm{mle}}} = \frac{n}{\sum_{i=1}^{n}\left(\frac{y_i}{\beta}\right)^{k} - n} \quad (37)$$

$$\widehat{\gamma_{\mathrm{mle}}} = \theta \quad (38)$$

$$\widehat{k_{mle}} = \frac{3\ln[\ln\beta - \sum_{i=1}^{n}\ln y_i] - \ln n + \ln\theta - \ln\left[\sum_{i=1}^{n}\ln\frac{y_i}{\beta}\right] + \ln\left(\frac{y_i}{\beta}\right)}{-2ln} \quad (39)$$

4. GOODNESS OF FIT

The Quasi Lindley Pareto Distribution (QLPD) has been fitted to a data set to which earlier the Lindley distribution (LD) and Quasi Lindley Distribution (QLD) have been fitted and it was found that QLPD provides better fit than those by LD and QLD. Here the fitting of the QLPD to the data set have been presented in the Table 1. The data is regarding the survival times (in days) of 72 guinea pigs infected

Table 1. Data of survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal [18]

Survival Time (in days)	Observed frequency	Expected frequency		
		LD	QLD	QLPD
0-80	8	16.14	10.71	10.24
80-160	30	21.91	26.95	27.73
160-240	18	15.39	17.71	17.66
240-320	8	9.00	9.14	8.59
320-400	4	5.47	4.26	4.53
400-480	3	1.80	1.86	2.31
480-560	1	2.29	1.34	0.94
Total	72	72.00	72.00	72.00

with virulent tubercle bacilli, observed and reported by Bjerkedal [18].

In Table 1, the expected frequencies according to the Lindley distribution and Quasi Lindley distribution have also been given for ready comparison with those obtained by the Quasi Lindley Pareto distribution. It can be seen that the QLPD gives much closer fits than the LD and QLD of Shanker and Mishra [4] and thus provides a better alternative to the LD and QLD.

5. SUMMARY

In applied sciences, the lifetime's data models are constructed to facilitate better modeling and significant progress towards the reliability analysis or survival analysis. In this paper, we introduced five-parameter Quasi Lindley Pareto Distribution (QLPD). The probability distribution function with different values of its parameters has been shown graphically. Several properties such as probability density function, distribution function, survival function, hazard function, moment generating function, moments, mean, variance, coefficient of variance, skewness, kurtosis and distribution of extreme order statistics have been derived. The estimation of parameters by the method of maximum likelihood has been discussed. Finally, the proposed distribution has been fitted to a real life data set to check its goodness of fit to which earlier the Lindley distribution (LD) and Quasi Lindley distribution (QLD) have been fitted and it is found that the QLPD provides closer fits than those by the LD and QLD. Therefore, it is suggested to use QLPD as a lifetime's data model for better estimation.

6. REFERENCES

1. Lindley, D. V. Fiducial distributions and Bayes' theorem. Journal of the Royal Society, series B, 20: 102-107 (1958).
2. Sankaran, M. The Discrete Poisson-Lindley Distribution. Biometrics, 26(1): 145-149 (1970).
3. Ghitany, M. E., Atieh, B. & Nadarajah, S. Lindley Distribution and Its Application. Mathematics and Computers in Simulation, 78: 493-506 (2008).
4. Shanker, R. & Mishra, A. A quasi Lindley distribution, African Journal of Mathematics and Computer Science Research, 6(4): 64 – 71 (2013 a).
5. Shanker, R. & Mishra, A. A two-parameter Lindley distribution, Statistics in Transition-new series, 14(1): 45- 56 (2013 b).
6. Shanker, R., Sharma, S. & Shanker, R. A two-parameter Lindley distribution for modeling waiting and survival times data, Applied Mathematics, 4: 363 – 368 (2013).
7. Shanker, R. & Amanuel, A. G. A new quasi Lindley distribution, International Journal of Statistics and Systems, 8(2): 143 – 156 (2013).
8. Shanker, R. & Tekei, A. L. A new quasi Poisson-Lindley distribution, International Journal of Statistics and Systems, 9(1): 79 – 85 (2014).
9. Shanker, R. & Mishra, A. A quasi Poisson- Lindley distribution, to appear in Journal of Indian Statistical Association. (2016).
10. Lazri, N. & Zeghdoudi, H. On Lindley-Pareto Distribution Properties and Application. GSTF Journal of Mathematics, Statistics and Operations Research (JMSOR),3(2) (2016).

11. Pareto, V. Cours d'Économie Politique. Geneva: Droz. (1896).
12. Pickands, J. Statistical inference using extreme order statistics. Annals of Statistics, 3: 119-131 (1975).
13. Akinsete, A., Famoye, F. & Lee C. The Beta-Pareto distribution. Statistics, 42(6): 547–563 (2008).
14. Mahmoudi, E. The beta generalized Pareto distribution with application to lifetime data. Mathematics and Computers in Simulation, 81(11): 2414–2430 (2011).
15. Zakerzadeh, H. & Dolati, A. Generalized Lindley distribution, Journal of Mathematical extension, 3(2): 13 –25 (2009).
16. Nedjar, S., & Zeghdoudi, H. Gamma Lindley distribution and its application. Journal of Applied Probability and Statistics, 11(1) (2016).
17. Alzaatreh, A., Lee, C. & Famoye, F. A new method for generating families of distributions. Metron, 71: 63-79 (2013).
18. Bjerkedal, T. Acquisition of resistance in guinea pigs infected with different doses of virulent tubercle bacilli. American Journal of Epidemiol, 72(1): 130-148 (1960).