



# A Novel First Order Dithered Hybrid MASH-EFM with Cancellation Transfer Function Sigma-delta Modulator for Eight-bit Audio DAC

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**Abstract:** This work proposes a new architecture named “Dithered hybrid multi-stage noise shaping-error feedback modulator with cancellation transfer function” sigma-delta modulator for audio digital-to-analog (D/A) converter. Multi-stage noise shaping (MASH) architecture possesses noise shaping technique which produces high signal-to-noise ratio with greater stability. Error feedback modulator (EFM) has an advantage of providing greater stability with time varying inputs. The state-of-the-art technology under the name of dithered hybrid MASH-EFM with cancellation transfer function (CTF) sigma-delta modulator provides better noise shaping, high SNR and greater stability than the traditional sigma-delta architecture. MATLAB simulation shows that 1st order Dithered hybrid MASH-EFM SDM with CTF achieves high signal-to-noise ratio (SNR) of 184db over 22kHz bandwidth when oversampling ratio (OSR) is 128, which is approximately equal to the SNR of 2nd order traditional architecture. Other performance parameters like signal-to-quantization noise ratio (SQNR), effective number of bits (ENOB), signal-to-noise and distortion ratio (SNDR) and dynamic range (DR) are also recorded in this work. Noise transfer function (NTF) graphs are also plotted for traditional and proposed architectures.

**Keywords:** Sigma-delta modulator (SDM), multi-stage noise shaping (MASH), over sampling ratio (OSR), noise transfer function (NTF), signal transfer function (STF)

## 1. INTRODUCTION

Sigma-delta modulation is a technique widely used in high resolution analog-to-digital (A/D) and digital-to-analog (D/A) converters. In perspective of analog-to-digital converter, it is effectively used in super audio CD format because of providing greater SNR. It is extensively used in wireless communications field and technology such as GSM and CDMA, etc. where high speed ADC's with wide bandwidth are required [1-3]. Portable audio playback devices such as IPOD and MP3 which perform digital-to analog conversion also utilize it. Other applications of sigma-delta modulator are found in instrumentation, seismic activity measurements, speech, video, ISDN, digital cellular radio, frequency synthesizer, ultrasound,

chromatography [4], etc.

Analog-to-digital and digital-to-analog conversion produce quantization noise [5], so the signal is not reconstructed properly. Sigma-delta modulator (SDM) uses noise shaping and oversampling which transfers this noise from region of interest to higher frequency region [6]. A new proposed architecture removes the quantization noise more efficiently than the old architecture. Hence, it results in better noise shaping and higher SNR values.

A fundamental structure of this modulator consists of “Sigma” and “Delta” blocks [7]. Fig. 1 shows first order SDM structure for ADC system. Sigma block consists of a discrete-time filter “F (z)”

which comprises of accumulator and delay blocks. A Comparator performs quantization denoted by “Q(.)” and a DAC is located in the feedback path. Delta block subtracts output “V” from the input [8, 9]. DAC is replaced by new transfer function “G(Z)” in case of digital delta-sigma modulator as delineated in Fig. 2. The filter “F” is designed digitally.

A lot of researchers have shown a greater interest in the existing architecture. This work presents a new architecture named “Dithered hybrid MASH-EFM with CTF” which uses the characteristics of both MASH and EFM architectures.

### 1.1 Traditional SDM

The input is applied to a Feed forward filter  $F(z)$  followed by a quantizer. The quantized output is then fed back to the summer to subtract it from the input. The second order SDM involves Feed forward filter  $(Z^{-1}/1-Z^{-1})$  in all the two stages with a single quantizer as shown in Fig. 3. A high order SDM can be easily designed and implemented with higher accuracy achieving high performance parameters [10, 11]. The second order and nth order traditional SDM are shown in Fig. 3[12]. The output  $Y(z)$  is generally analyzed by an expression given as:

$$Y(z) = STF(z)X(z) + NTF(z)E(z) \quad (1)$$

### 1.2 Error Feedback modulator (EFM)

In error feedback modulator, the error signal is then given back to the input signal by passing through the loop filter “H(Z)” while the workings of other components of EFM are the same [13]. The architecture is shown in Fig. 4.

## 2. A NEW PROPOSED ARCHITECTURE (FIRST ORDER DITHERED HYBRID MASH-EFM WITH CANCELLATION TRANSFER FUNCTION)

The state-of-the-art technology is introduced in this work under the name of Dithered hybrid MASH-EFM with CTF SDM architecture. MASH architecture provides high valued performance parameters due to its multi-stage noise shaping and EFM provides more stability because of loop filter

in the feedback path. In the first stage of this work, a new architecture “1st order hybrid MASH-EFM” is developed and then it is modified into 1st order Dithered hybrid MASH-EFM architecture.

The block diagram of 1st order hybrid MASH-EFM is delineated in Fig. 5. Fig. 6 shows the internal structure of this proposed architecture. The two useful delays and are used in this architecture indicated as “delay 1” and “delay 2” respectively to make the STF unity. The feed forward filter along with the feedback path is the major characteristic of MASH architecture. While error signal and discrete transfer function in Fig. 6 are the characteristics of EFM. Limiter is responsible to prevent overflow when the input signal is closed to full scale so that the quantization error remains bounded. The other main role of limiter is to improve NTF. The limiter hierarchy is also represented in Fig. 6. In this model,

The output in Z-domain is expressed as:

$$Y(z) = \left[ (X(Z)Z^{-1} - X(Z)Z^{-2}) + \left( (E_1(Z) - 2Z^{-1}E_1(Z) + E_1(Z)Z^{-2} - E_1(Z))(1 - Z^{-1}) \right) \right] \left( \frac{1}{1 - Z^{-1}} \right) + E(Z) \quad (2)$$

Hence, it can be written as:

$$Y(Z) = X(Z)Z^{-1} + (-2Z^{-1}E_1(Z) + E_1(Z)Z^{-2}) + E(Z) \quad (3)$$

$$Y(Z) = X(Z)Z^{-1} - 2Z^{-1}E_1(Z) + E_1(Z)Z^{-2} + E(Z)^2 \quad (4)$$

And, for simplicity, consider that:

$$\begin{aligned} \text{Error signal} &= E_1(z) = E(z) \\ Y(Z) &= X(Z)Z^{-1} + (1 - 2Z^{-1} + Z^{-2})E_1(Z) \end{aligned} \quad (5)$$

Finally, the output expression for this model will be:

$$Y(Z) = X(Z)Z^{-1} + (1 - Z^{-1})^2 E_1(Z) \quad (6)$$

$$\text{Here, } STF = Z^{-1} \quad (7)$$

$$\text{And } NTF = (1 - Z^{-1})^2 \quad (8)$$

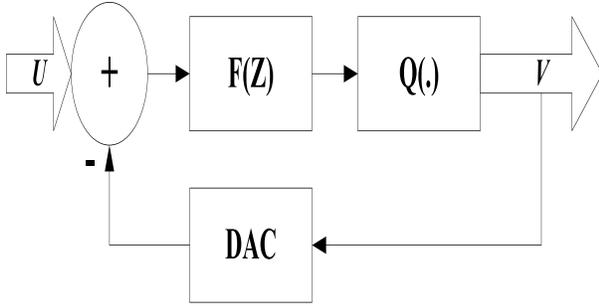


Fig. 1. First order Delta-sigma modulator.

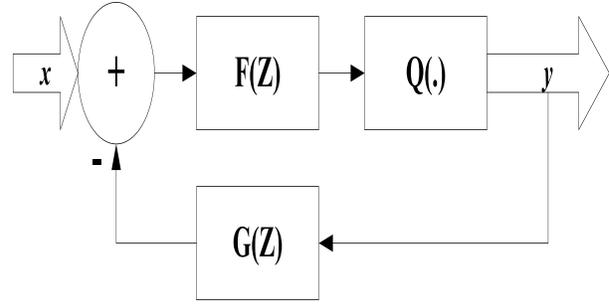


Fig. 2. Delta-sigma modulator for DAC.

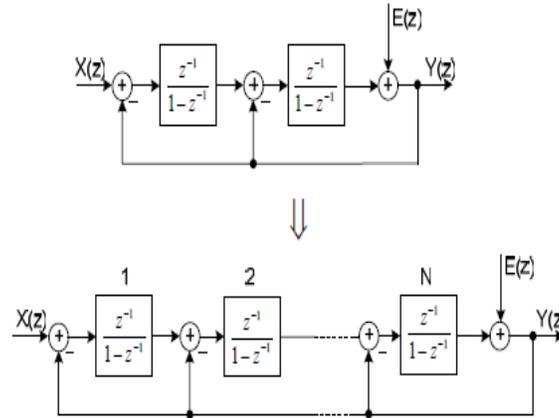


Fig. 3. Second order and nth order traditional SDM for DAC.

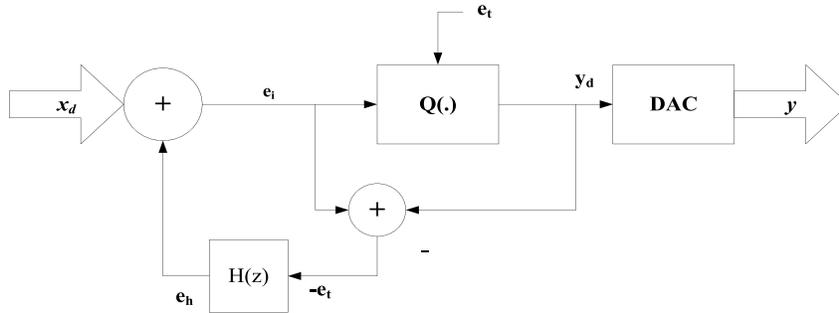


Fig. 4. Error feedback modulator (EFM) architecture.

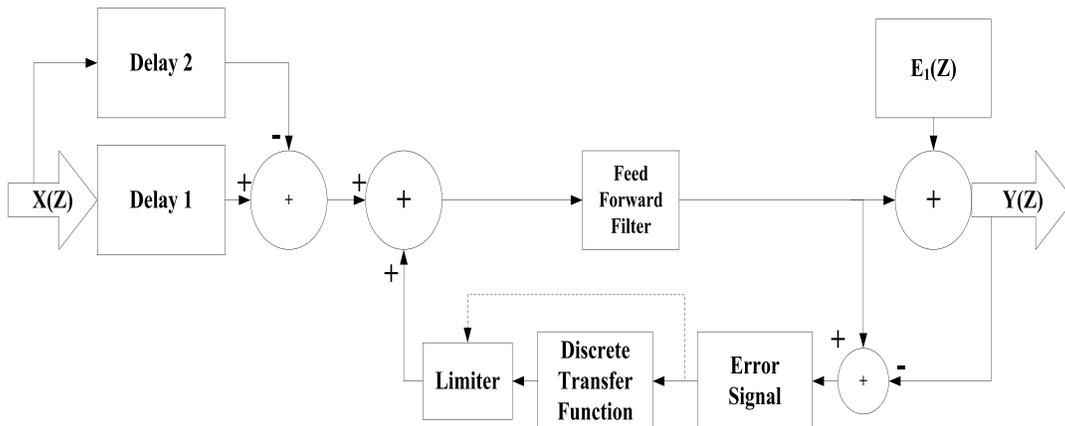


Fig. 5. Block diagram of 1st order hybrid MASH-EFM architecture.

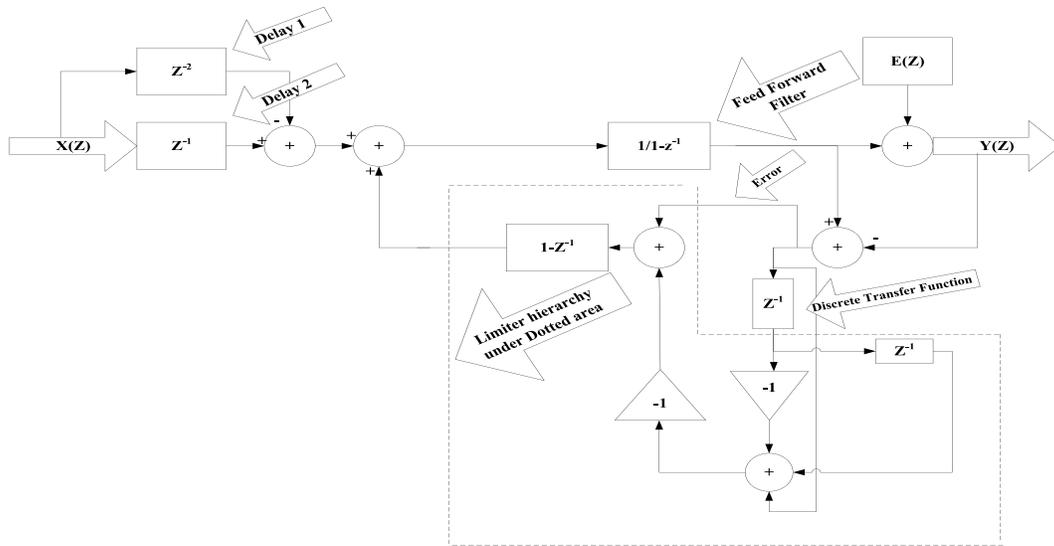


Fig. 6. Internal structure of 1st order hybrid MASH-EFM architecture.

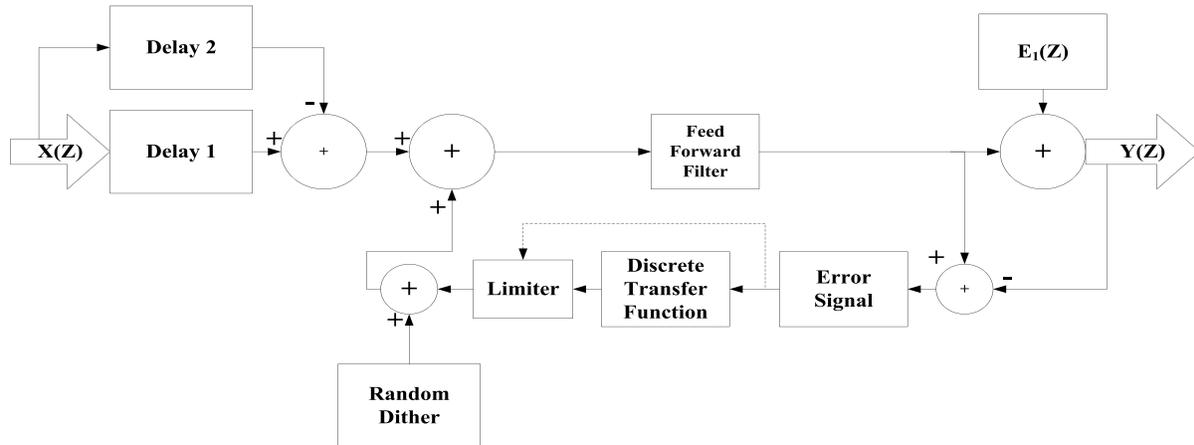


Fig. 7. Block diagram of 1st order dithered hybrid MASH-EFM architecture.

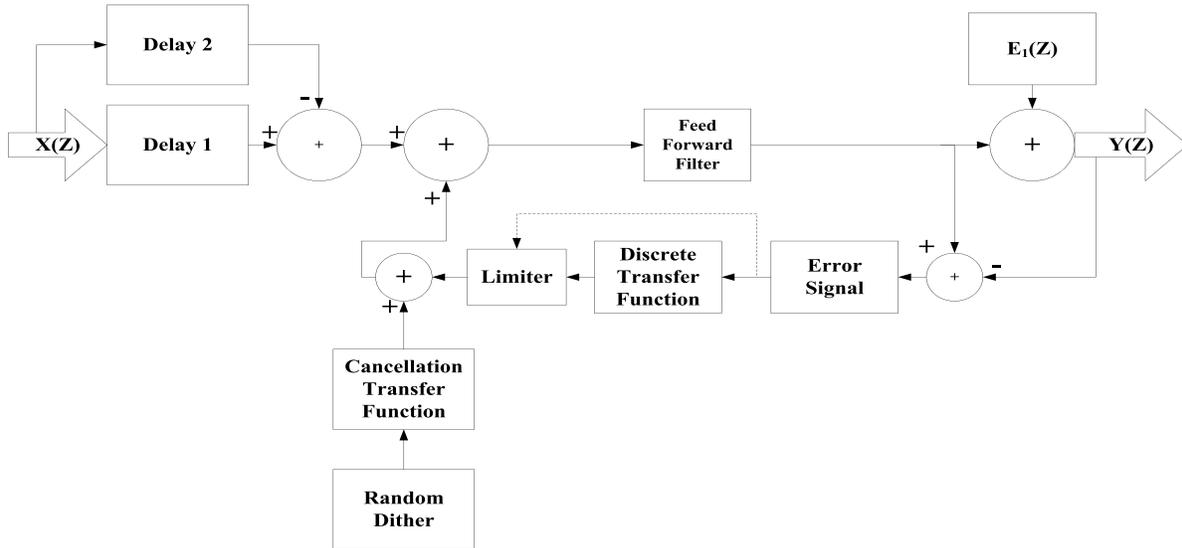


Fig. 8. Block diagram of 1st order dithered hybrid MASH-EFM architecture with cancellation Transfer Function (CTF).

In the modified version of this architecture i.e. 1st order Dithered hybrid MASH-EFM, Dithering is applied. It is used to uniformly distribute the noises and reduce the tones in the entire frequency domain [14]. The block diagram of 1st order Dithered hybrid MASH-EFM is delineated in Fig. 7.

If  $x[n]$  is the dithered signal with the same parameters as defined above for delay 1, delay 2 and feed forward filter, etc., then the output expression is analyzed as:

$$Y(Z) = X(Z)Z^{-1} + (1-Z^{-1})^2 E_1(Z) + \frac{D(Z)}{(1-Z^{-1})} \quad (9)$$

The transfer function  $(1-Z^{-1})$  in the denominator of  $D(Z)$  degrades the Dithered signal and more simplified form of the above mathematical expression suggests that it will deviate the STF from unity. This is due to the effect of Feed forward filter present in this architecture. So, in the final stage of this work, this proposed architecture is again modified by adding a cancellation transfer function (CTF),  $(1-Z^{-1})$  in it. This gives the innovation of new architecture named “first order Dithered hybrid MASH-EFM with CTF”. The block diagram of first order Dithered hybrid MASH-EFM with CTF is delineated in Fig. 8.

Now, the output expression will become:

$$Y(Z) = X(Z)Z^{-1} + (1-Z^{-1})^2 E_1(Z) + D(Z) \quad (10)$$

### 3. PERFORMANCE PARAMETERS OF PROPOSED DELTA-SIGMA MODULATOR

There are different performance parameters which determine the efficiency of Delta-sigma modulators. These are SQNR, SNR, SNDR, DR, ENOB, etc. This section defines these parameters.

#### 3.1 Signal-to-Quantization Noise Ratio (SQNR)

It is determined by dividing the original signal power with an in-band, shaped quantization noise power at the frequency of an input signal.

Assume that an input signal power and in-band shaped quantization noise power are represented by

$\sigma_x^2$  and  $\sigma_n^2$ , respectively. The quantization noise

power is denoted by .

$$\text{As, } \sigma_n^2 = \sigma_e^2 \cdot \frac{\pi^5}{5 \cdot OSR^5} \quad (11)$$

$$\text{And } \sigma_x^2 = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \quad (12)$$

$$\text{So, } SQNR = 10 \log_{10} \frac{\sigma_x^2}{\sigma_n^2} \quad (13)$$

#### 3.2 Signal-to-Noise Ratio (SNR)

It is obtained by dividing the signal power with an in-band noise power (IBN) at the frequency of an input signal. In this case, the above mentioned noise power associated with the harmonics will not be considered. SNR is mathematically expresses as:

$$SNR = 10 \log_{10} \frac{\sigma_x^2}{IBN} \quad (14)$$

IBN= In-band quantization noise + all types of noises present/occurred in the input signal bandwidth except the noise due to distortion.

#### 3.3 Signal-to-Noise plus Distortion Ratio (SNDR)

It is calculated by dividing the signal power with an in-band noise power at the input signal frequency. In this case, the in band noise (IBN) power related to the harmonics is also considered.

$$SNDR = 10 \log_{10} \frac{\sigma_x^2}{IBN} \quad (15)$$

IBN= In-band quantization noise + all types of noises present/occurred in the input signal bandwidth plus the noise due to distortion.

#### 3.4 Effective Number of Bits (ENOB)

It is defined as the total bits required for an optimal Nyquist-rate DAC to conclude same dynamic range. Mathematically, expressed as:

$$ENOB = \frac{DR(dB) - 1.76}{6.02} \quad (16)$$

#### 4. SIMULATION METHODOLOGY

An audio signal which contains the maximum frequency component of 20 kHz along with some other harmonic components has been simulated in MATLAB. The noise has been added to this signal so that the results of the sigma-delta modulator also include noise effect. Then this signal has been sampled at over sampling ratio's (OSR's) 5, 64 and 128.

After that the discretized signal has been passed through the sigma-delta modulator. Sigma-delta modulator has been simulated in SIMULINK. The output signal of SDM will be represented in both time domain and frequency domain when OSR is 128 followed by the reconstruction of the signal through Sinc interpolation. Sinc interpolation can be described mathematically using interpolating formula as;

$$x(t) = \sum_{n=-\infty}^{+\infty} x(n) \text{sinc}[F_s(t - nT_s)] \quad (17)$$

The performance analysis has been recorded from the output signal of SDM in tabular form for over sampling ratio's 5, 64 and 128. An approach to simulation is shown in Fig.9.

An analog signal remains the same for all SDM analysis. Similarly, the Oversampling ratio and noise are also considered as the same for all types of analysis. The MATLAB simulation of an analog signal for different oversampling ratios is given in Fig.10.

##### 4.1 MATLAB/SIMULINK Model of SDM Architectures

All the two architectures are simulated in SIMULINK. After oversampling in MATLAB, the discretized data has been saved in discrete signal block and then quantized to make it digital. Then the digital data will pass through the SDM to remove quantization noises from a signal bandwidth. After

that the signal is exported to MATLAB script file and then reconstructed in MATLAB.

##### 4.1.1 SIMULINK Model of 1st and 2nd Order Traditional/Conventional Architectures

The 1st order traditional SDM has only one feed forward filter in its feed forward path while 2nd order has two feed forward filters. The NTF of 1st and 2nd order conventional SDM's are  $(1-Z^{-1})$  and  $(1-Z^{-1})^2$ , respectively. The both orders are simulated in SIMULINK and are shown in Fig.11. The power spectral density (PSD) of 1st and 2nd order simple SDM output when OSR is 128 is shown in Fig.12.

The Time domain output of 1st and 2nd order SDM consists of 10 cycles as shown in Fig.13. The difference in the Time domain output for above mentioned SDM orders is clearly observed by analyzing the first cycle. The signal is reconstructed using sinc interpolation as displayed in Fig.14. Reconstructed signal of 2nd order traditional SDM is closest to the original analog signal. The NTF of 1st order is written as:

$$NTF(z) = 1 - Z^{-1} \quad (18)$$

While the 2nd order NTF is mathematically expressed as:

$$NTF(z) = (1 - Z^{-1})^2 \quad (19)$$

The NTF of 1st and 2nd order existing architecture simulated in this work is graphically represented in Fig.15.

##### 4.1.2 SIMULINK Model of Proposed Architecture

The 1st order Dithered hybrid MASH-EFM architecture with CTF, simulated in SIMULINK is shown in Fig.16. The Power Spectral Density (PSD) of 1st order Dithered hybrid MASH-EFM with CTF reveals that noise is shifted to higher frequencies while the signal bandwidth is 22 kHz. The useful data lies in the bell shaped curve and is called as the region of interest. Next to this frequency region, noise is transferred due to noise shaping technique and the noise power is slightly reduced due to a Dithering effect in the area indicated by red rectangular box in Fig. 17. The time domain representation of output of proposed architecture

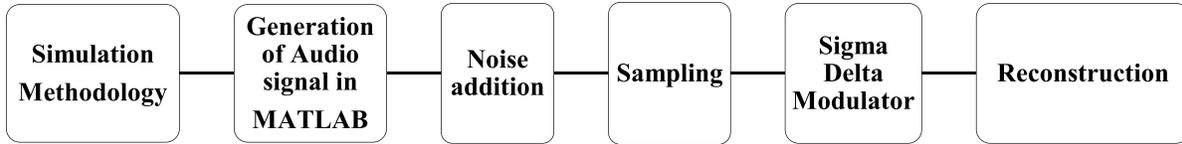


Fig. 9. Block diagram of Simulation methodology.

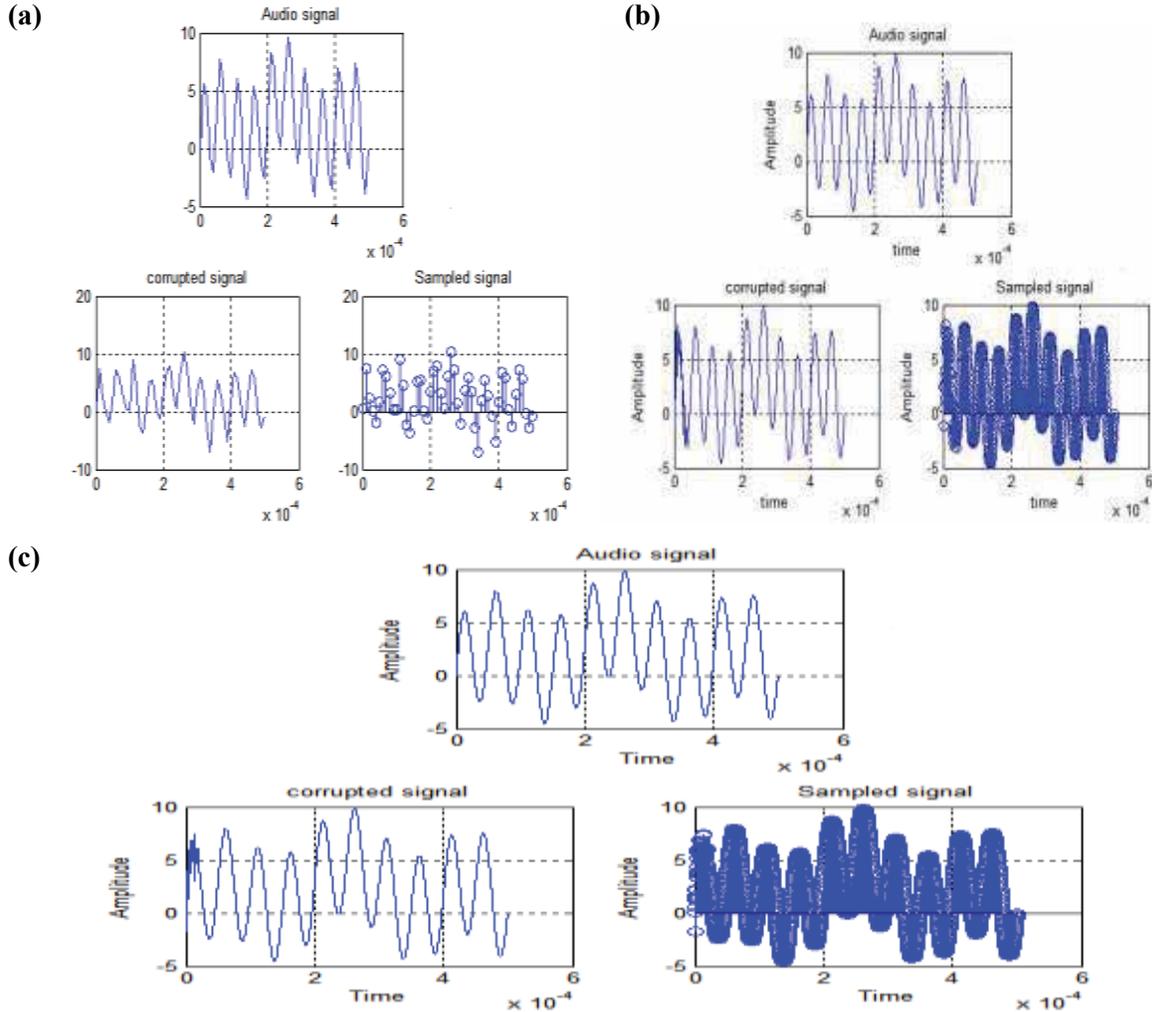


Fig. 10. Sampled analog signal at: (a) OSR=5; (b) OSR=64; (c) OSR=128.

output is shown in Fig.18.

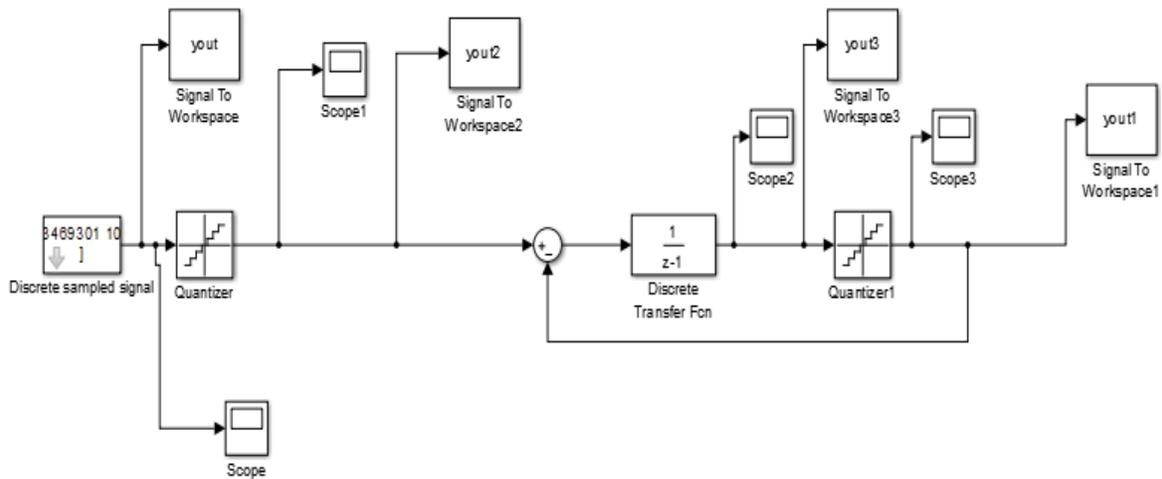
By comparing Fig. 13 and Fig. 18, Dithering effect is obvious in this proposed architecture. At the end the output signal is reconstructed as illustrated in Fig.19. Conventional Sinc interpolation will not properly reconstruct the original signal. So, a Dithering removed Sinc interpolation is used which removes the Dithering effect at Signal reconstruction.

## 5. RESULTS AND DISCUSSION

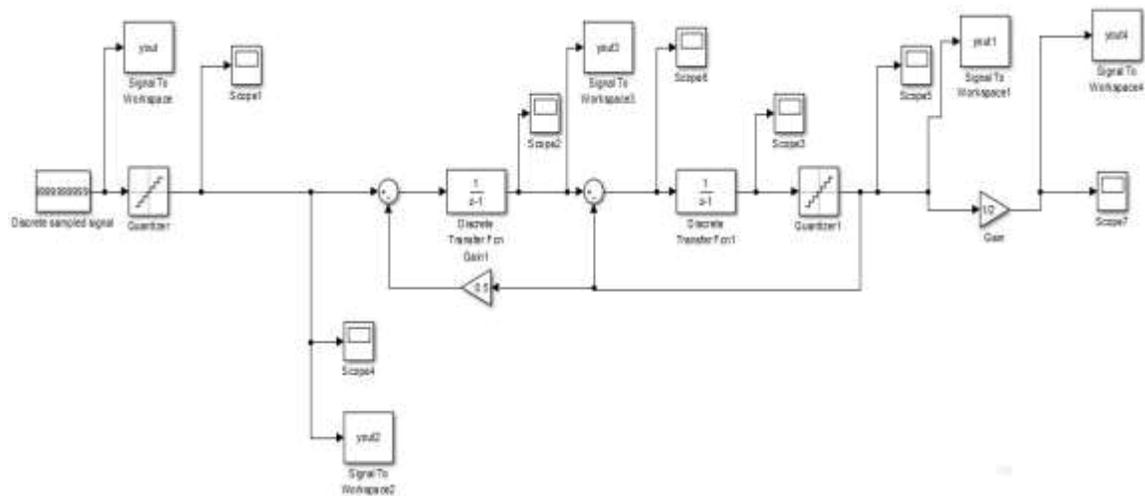
The above discussion reveals that as the order of modulator increases, the order of NTF also increases. Hence, the higher order SDM modulators transfer the quantization noise to a greater extent as compared to the low order modulators. As a result, the performance of SDM increases.

By comparing Eq. 18 and Eq. 19, it is obvious

(a)



(b)



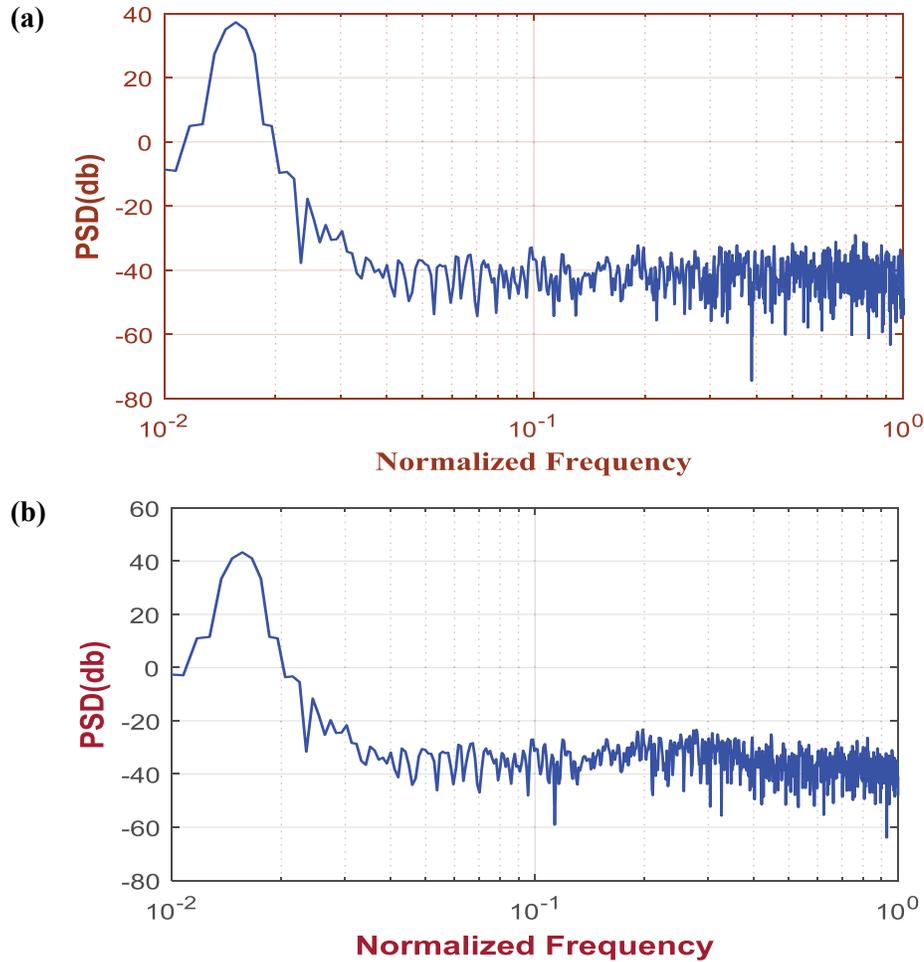
**Fig. 11.** Traditional SDM SIMULINK model: (a) 1st order; (b) 2nd order.

that the 2nd order traditional SDM provides better noise transfer function (NTF) than the 1st order. Fig. 15 illustrates that 2nd order conventional SDM shifts the noises (such as quantization noise) more efficiently than the 1st order. Thus, 2nd order will reconstruct the original signal properly.

The performance parameters such as SNR, SQNR, ENOB, DR and SNDR are also recorded in the tabular form in Table 1. All the performance parameters have been improved due to better noise shaping in 2nd order SDM as compared to 1st order SDM. 2nd order SDM achieves the SNR of 185.7 db when OSR is 128, which is significantly higher than the 1st order achieves, i.e., 138.9 db. The results present in Table 1 propose that the increase in modulator order will eventually

enhance the capability of DAC. The disadvantage of conventional SDM is that it does not prevent the output sequence (bits) from overflow when the quantizer saturates due to time varying inputs. This is because, this architecture (as shown in figure 3) does not possess any mechanism in the feedback path to avoid overflow. As a result, the PSD can experience idle tones in it which eventually degrades the performance of SDM.

The proposed architecture removes the above discrepancy by using limiter circuit in the feedback path. Hence, this architecture can effectively avoid overflow. The limiter is also responsible to improve the NTF. As a result, the proposed architecture is more robust than the conventional architecture.



**Fig. 12.** PSD (a) 1st order conventional SDM. (b) 2nd order conventional SDM.

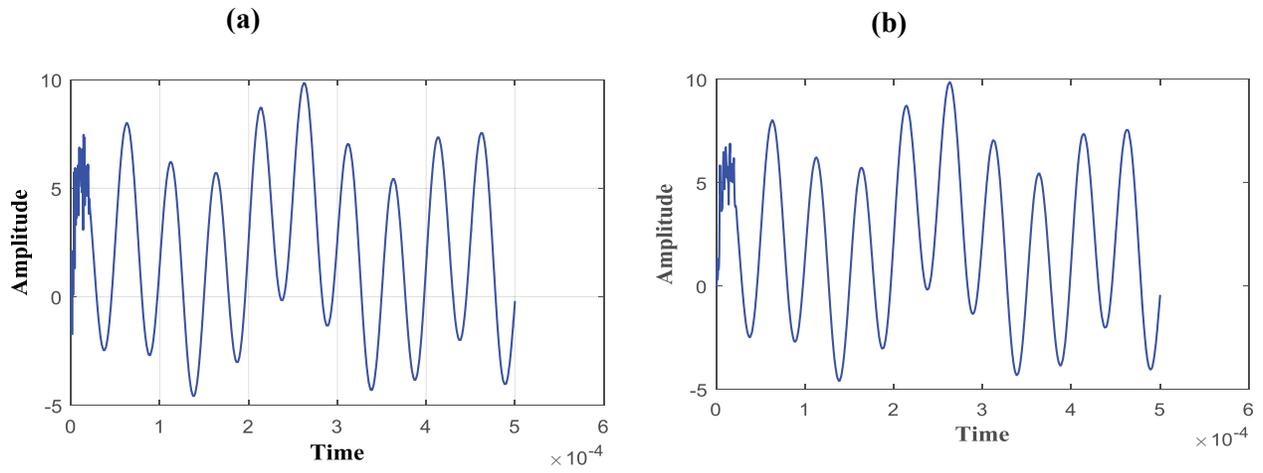
Eq. 8 shows that 1st order Dithered hybrid MASH-EFM with CTF (proposed architecture) obtains same NTF as 2nd order simple SDM achieves. Hence, the lower order proposed architecture removes the quantization noise as efficaciously as the conventional higher order modulators can. The NTF of 1st order Dithered hybrid MASH-EFM with CTF architecture is graphically represented in Fig. 20.

Moreover, second order conventional SDM uses feed forward filters in both the stages which makes the designing of SDM more complex and increases hardware requirements. Despite of this, the proposed architecture is easy to design because it uses only one feed forward filter.

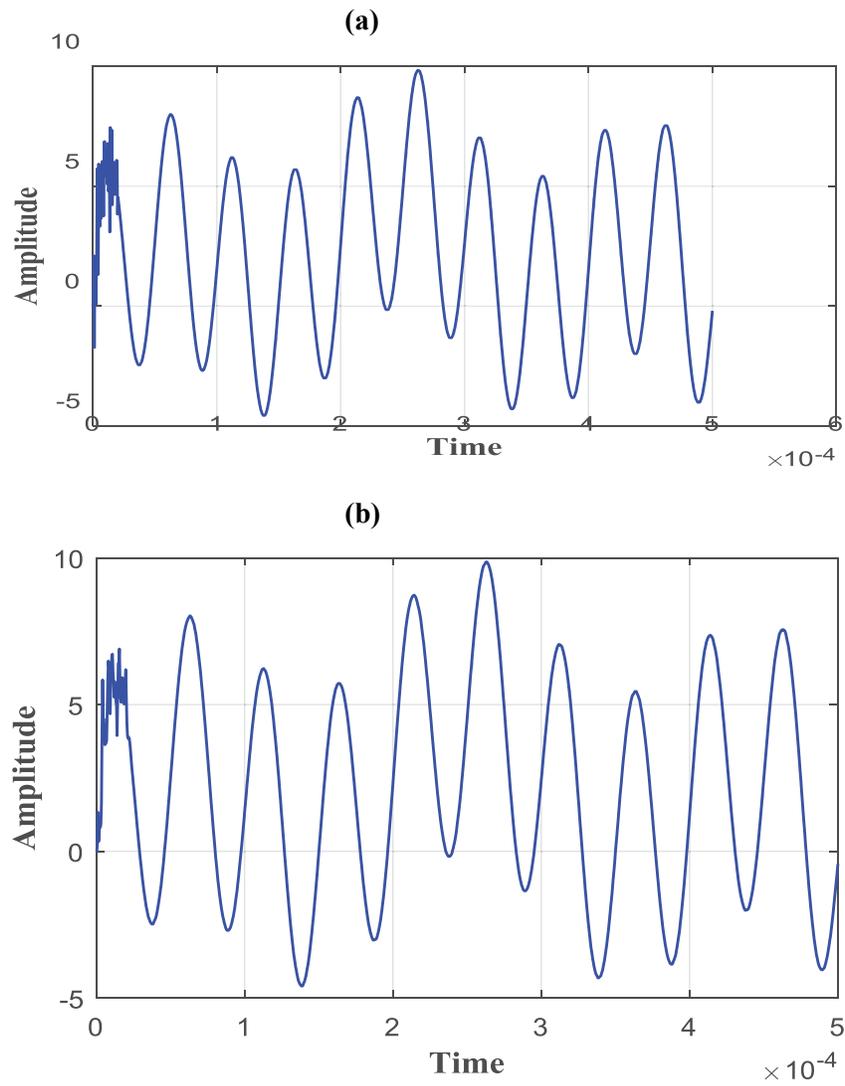
The above mentioned performance parameters are also calculated for a proposed architecture and compared with the existing architecture as

demonstrated in Table 2. The results show that proposed architecture obtains approximately same SNR value as 2nd order traditional SDM. Eventually, enhances the DAC capability. SNR values are plotted in MATLAB for 2nd order Traditional SDM and 1st order Dithered hybrid MASH-EFM with CTF as displayed in Fig. 21. Frequency is mapped from 0 to  $\pi$  rad/sec in this graph. The lowest frequency region shows maximum SNR because the noises are removed from this region and transferred to higher frequency region. The original signal frequency lies in the lowest frequency region.

Dithering is one of the main aspects which supersede the proposed architecture. Due to Dithering, all the noises are uniformly distributed over the entire spectrum. This ensures less probability of occurring spikes of noises in the signal bandwidth. This affect along with the better



**Fig. 13.** Time domain representation: (a) 1st order traditional SDM (b) 2nd order traditional SDM.



**Fig. 14.** Signal reconstruction: (a) 1st order traditional SDM (b) 2nd order traditional SDM.

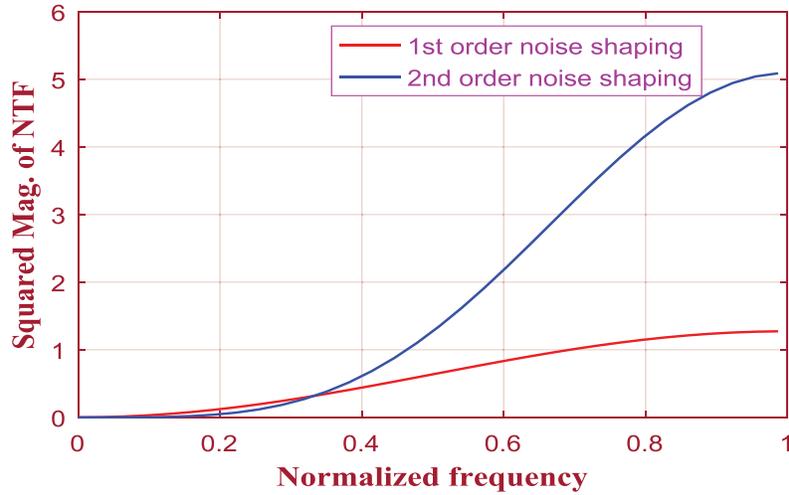


Fig. 15. NTF of 1st and 2nd order traditional SDM.

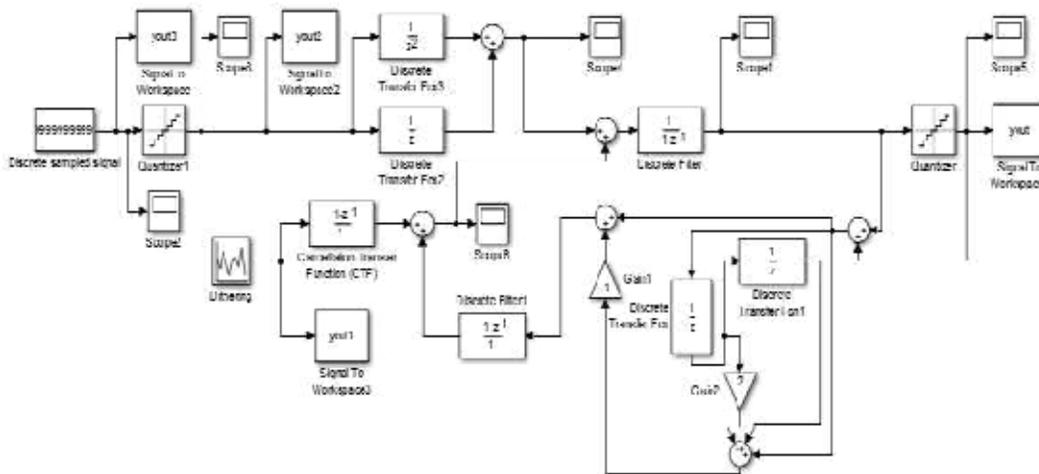


Fig. 16. SIMULINK model of 1st order dithered hybrid MASH-EFM with CTF.

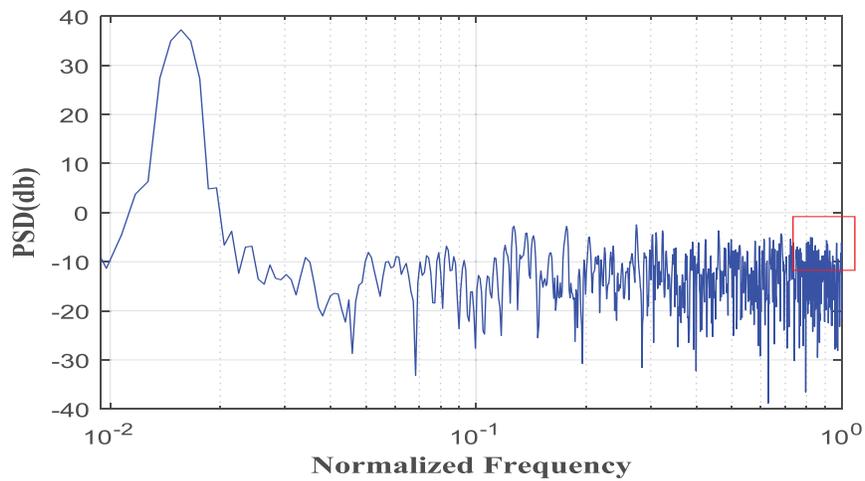
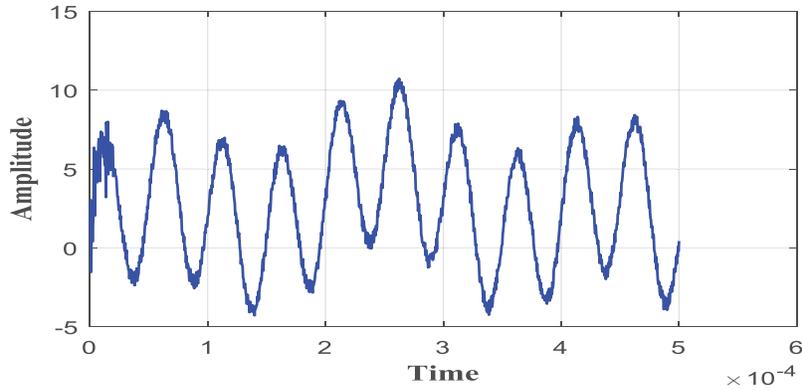
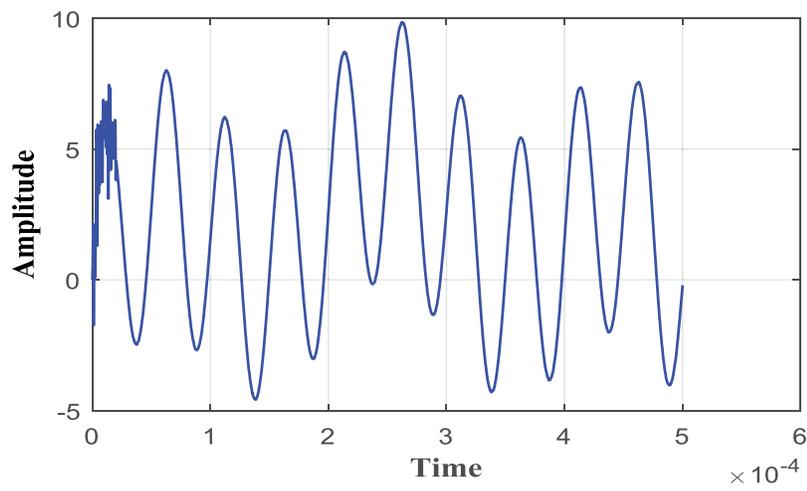


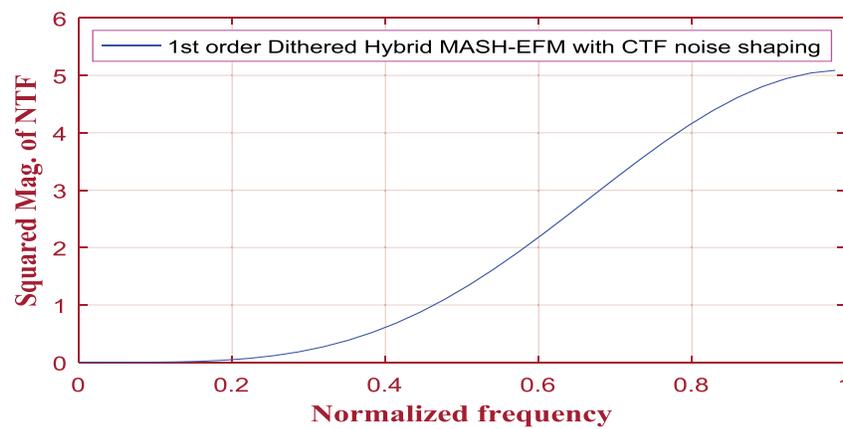
Fig. 17. PSD of 1st order dithered hybrid MASH-EFM with CTF.



**Fig. 18.** Time domain representation of 1st order Dithered hybrid MASH-EFM with CTF.



**Fig. 19.** Signal reconstruction of proposed architecture.



**Fig. 20.** NTF graph of 1st order Dithered hybrid MASH-EFM with CTF.

Fig. 21.(a)

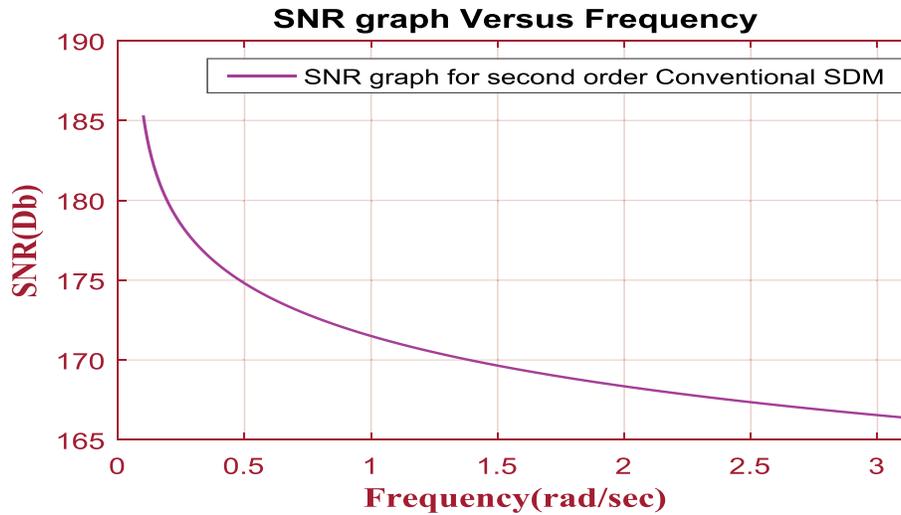
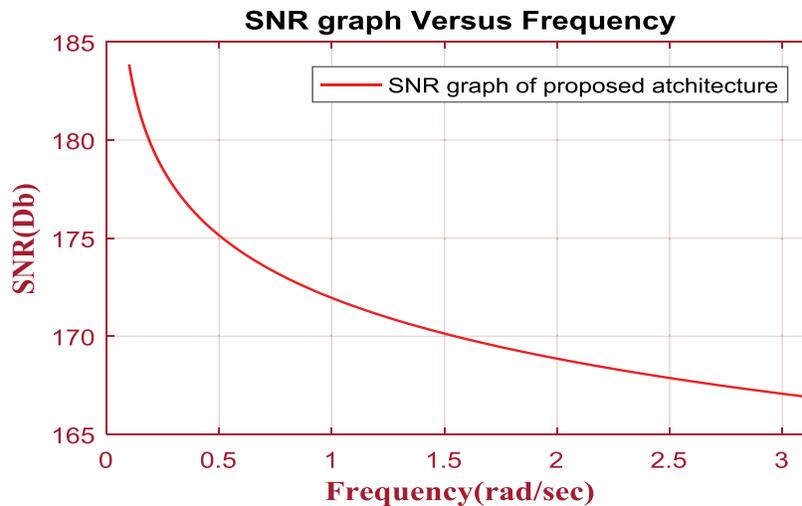


Fig. 21.(b)



**Fig. 21.** SNR vs. Frequency graph: (a) 2nd order Traditional SDM (b) 1st order Dithered hybrid MASH-EFM with CTF.

noise shaping will reconstruct the original signal more precisely than the traditional SDM.

## 6. CONCLUSIONS

The proposed architecture obtains very high SNR, SQNR, ENOB and SNDR as compared to the existing architectures, because of efficient noise shaping. 1st order Dithered hybrid MASH-EFM with CTF obtains SNR equal to 184 db when OSR is 128 which is approximately same as 2nd order conventional SDM acquires. Main advantage of this architecture is that a lower order modulator obtains the same performance parameters as other higher order modulators. Similarly, other parameters are also significantly improved in this new architecture. Also, this architecture uses a Dithering phenomenon

which helps in the reconstruction of original signal more properly.

## 7. ACKNOWLEDGMENTS

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