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Research Article

A Neuro-Fuzzy based Non-linear Control Technique for Steam Boiler using Levenberg-Marquardt Algorithm

Waqar Uddin[1], Kamran Zeb[1], Zunaib Ali[1], Syed Riazul Hussnain[2], Shahid Qamar[1], Tariqullah Jan[3]*, and SanaulHaq[4]

[1]COMSATS Institute of Information & Technology, Abbottabad Campus, Abbotabad, Pakistan
[2]University of Engineering & Technology, Abbottabad Campus, Abbotabad, Pakistan
[3]University of Engineering & Technology, Peshawar, Main Campus, Peshawar, Pakistan
[4]Department of Electronics, University of Peshawar, Peshawar, Pakistan

Abstract: The objective of this paper is to design a nonlinear control system, to stabilize the drum level and steam pressure of the industrial boiler at desired values. It is difficult to maintain the accurate control performances and to achieve the desired estimated values by using conventional proportional integral derivative (PID) control system. Based on the dynamic behavior of the boiler an Adaptive Fuzzy Logic (AFL) control strategy is designed to stabilize the drum level and steam pressure at desired values. The proposed non-linear AFL strategy is robust to meet the control objectives and to handle the uncertainties faster than traditional controllers. The simulation results show that the proposed AFL has tracking ability with better steady state error and transient response than conventional PID controller.

Keywords: Adaptive Fuzzy Logic controller (AFLC), PID controller, Levenberg-Marquardt (LM)

1. INTRODUCTION

Steam boiler is one of the vital machinery in industries used for the purpose of generating steam. Steam is being used in many industrial processes including power generation, central heating, textiles, and cement industry. To achieve the desired performance, steam boiler needs to be efficient in order to provide best quality of steam. This necessitate for the selection of a valid boiler model and suitable control strategy to obtain the desired outputs. Therefore Pellegrinetti and Bentsman [1] model is suitable for representing the non-linear behavior of steam boiler which was developed from the Astrom and Bell [2-4] model. Steam generator is highly non-linear, complex and time varying system whose parameters change with operating conditions. The model has three inputs (fuel, feedback water, and air flow) and four outputs (drum pressure, excess oxygen, steam flow rate, and drum water level). Our interest lies in the control of three outputs: drum pressure, steam flow rate, and drum water level.

Boiler efficiency can be optimized by adopting a control strategy that provides desirable outputs. Poor control of drum water level may cause shutdown of steam generator plant. The water level in steam boiler must be maintained in allowable limits in order to operate the boiler efficiently and safely [5]. Violating the specified limits may cause either moist steam at the outlet that results rusting of turbine in case of steam-turbine unit, or overheating of drum material which will cause deterioration of boiler material. Similarly steam pressure is to be controlled to regulate steam at the outlet. Steam boiler control is difficult due to certain factors including non-linear characteristics, dynamic uncertainties and load disturbances. Efficient controller is needed to provide desired output to increase the efficiency of the plant. In recent years different controllers have been used for controlling the steam generator parameters including proportional integral (PI),

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*Corresponding author: Triqullah Jan; Email: tariqullahjan@uetpeshawar.edu.pk

proportional derivative (PD), proportional integral derivative (PID), state feedback controller (SFC), linear quadratic regulator (LQR), neural network (NN) and sliding mode predictive controller (SMPC).

As for as PID controller is concerned, the entire operating range of non-linear boiler model is divided in to three linear segments and multiple PIDs or any other linear controllers are used for controlling different segments [6]. The controllers are tuned heuristically to achieve best possible results. Estimation of internal states of the process increases the possibility of better and efficient control. PID lacks the property of estimating the internal states of multivariable MIMO process [7]. PID causes wastage of energy and decreases the plant efficiency. It requires human intervention and understanding for suitable resolutions and corrective actions. Conventional PID controller has demerits of inability to understand process, lack of identifying small drifts over interval of time from ideal response and is unable to follow the desired dynamic behavior over the entire non-linear operating region, resulting in decrease in the overall efficiency and economy of the plant [8].

Alternately, state feedback controller could be a suitable technique to achieve these goals. State feedback controller has the ability of understanding process and therefore provides a good control of manipulated variables. Its implementation for industrial boiler is difficult because of non-availability of methodology of right pole placement. LQR has a good aspect of reducing controller energy and avoid saturation of actuator. But LQR and state feedback controller techniques are used for linear models [9].

Sliding mode predictive controller (SMPC) has better approach to control drum pressure than PID and Smith predictor in terms of oscillations and speed of the response, but it fails if the settling time is considered [8]. However none of these controllers can match the desired performance of a real time industrial boiler. Industrial boiler requires adaptive controller that has properties of monitoring and updating its parameters accordingly. Adaptive control strategy is used for regulating different plant parameters according to the desired time domain specification. The control process becomes more complex when considering both drum level and drum pressure control within the same system [10]. Changing reference points in such system causes change in dynamics of the entire plant.

Among the adaptive controllers, adaptive neuro-fuzzy inference system has some limitations for different applications including boilers. It needs linear model for different load conditions, i.e., 50%, 70%, and 100%. For this reason, the design of controller should be modified for each operating load in order to achieve optimal performance [6].

Overcoming the above problems require adaptation of parameters. The overall adaptability is compensated using adaptive fuzzy logic controller (AFLC). Adaptive fuzzy controller updates its parameters and organizes their values itself. It does not require to understand the physics or modeling of plant [11]. Levenberg-Marquardt (LM) technique is used to update the AFLC parameters. This technique is used to minimize quadratic, linear and nonlinear error functions and is comparatively faster and convergent.

2. MODELING OF PLANT

This paper is based on the simulation model of Pellegrinetti and Bentsman [1]. The model is obtained from steam plant at Abbott power plant in Champaign, Illinois. This is a multivariable MIMO plant having three inputs (fuel flow, air flow, and feed water flow) and four outputs (drum pressure, oxygen level, drum level, and steam flow rate). The fuel flow has influence on the steam flow rate and drum pressure. The second input air flow affects the oxygen level whereas the drum level is effected by feed water flow and steam flow rate. Respective inputs should be controlled to obtain the desired output. The model has internal perturbation and measurement noises. Perturbation effect changes with time. The schematic and block diagrams of model are shown in Fig. 1 and 2, respectively. The model can be represented mathematically by the following system of equations:

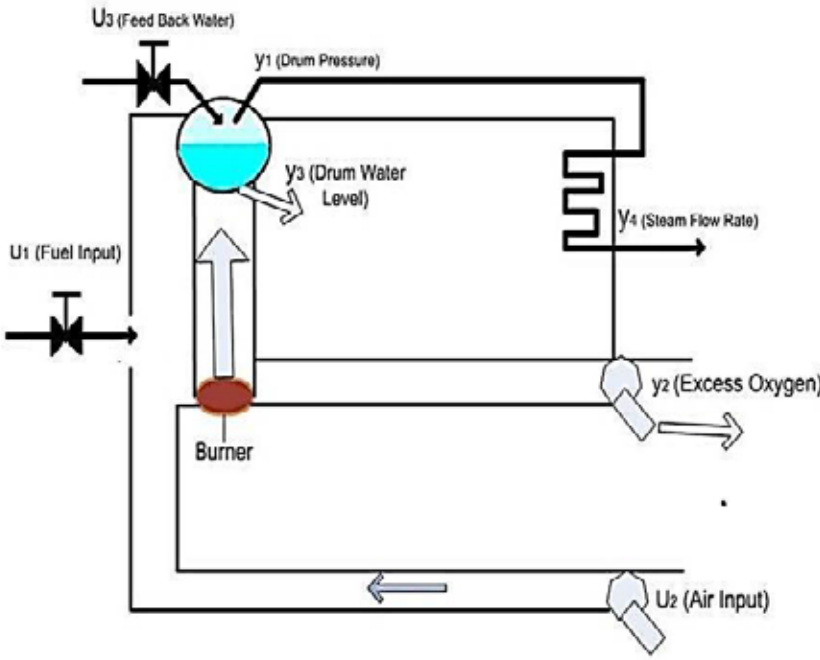

Fig. 1. Steam boiler schematic Diagram.

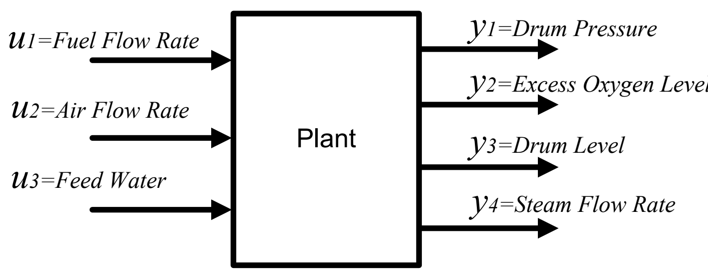

Fig. 2. Steam boiler block diagram.

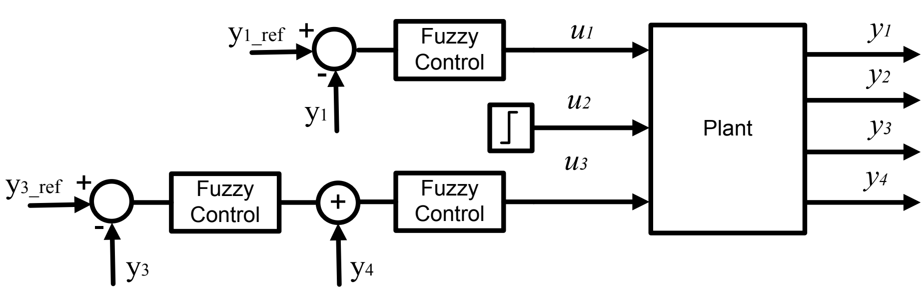

Fig. 3. Block diagram of proposed control.

$$x_1'(t) = -0.00478x_4(t)x_1^{\frac{9}{8}}(t) + 0.28u_1(t-2) - 0.01348u_3(t-3)$$

$$x_2'(t) = 0.1540357x_2(t) + \frac{\begin{bmatrix}103.5462u_2(t-2) - u_1(t-2)\\ \{107.4835 + 1.9515x_2(t)\}\end{bmatrix}}{29.04u_2(t-2) - 1.824u_1(t-2)}$$

$$x_3'(t) = -0.00533176x_1(t) - 0.02195x_4(t)x_1(t) + 0.7317058u_3(t-3)$$

$$x_4'(t) = -0.04x_4(t) + 0.029988u_1(t-2) + 0.018088$$

$$y_1(t) = -140214x_1(t-3)$$

$$y_2(t) = x_2(t-4)$$

$$y_3(t) = -0.1048569x_1(t-10) + 0.15479x_3(t-10) + 0.4954961x_4(t-10)x_1(t-10)$$

$$-0.20797u_3(t-13) + 1.272u_1(t-12)$$

$$+\left[\frac{\{-324212.7805x_1(t-10) - 99556.24778\}\{1 - 0.001185x_3(t-10)\}}{\{x_3(t-10)\}\{x_1(t-10) - 1704.50476\}}\right] - 103.7351$$

$$y_4(t) = [0.85663x_4(t-2) - 0.18128]x_1(t-2) \quad (1)$$

Where

x_1 is drum pressure state(kg/cm^2)

y_1 is measured drum pressure (PSI)

y_2 and x_2 are measured excess oxygen level and its state, respectively ($percent$)

x_3 is system fluid density (kg/m^3)

y_3 is drum water level ($inch$)

x_4 is exogenous variable related to load disturbances intensity $(0-1)$

y_4 is steam flow rate (kg/sec)

u_1, u_2, u_3 are the fuel, air, and feed water flow rate inputs, respectively, having range [0 1].

The plant is linearized around the nominal operating points ($points\ at\ which\ x' = [x_1' x_2' x_3' x_4']^T = 0$)

$$x^0 = [22.5\ \ 1.5734733038535\ \ 621.17\ \ 0.8374]^T$$
$$y^0 = [320\ \ 2.5\ \ 1.6\ \ 12.05]^T \quad (2)$$
$$u^0 = [0.5138\ \ 0.5064\ \ 0.8127]^T$$

3. PROPOSED ADAPTIVE FUZZY LOGIC CONTROLLER

In this section the proposed fuzzy model based on Levenberg Marquardt (LM) technique is discussed. In recent years different controllers have been designed for benchmark non-linear model of steam boiler using conventional PID and state space approaches. Each approach has its own merits and demerits. None of these approaches can match the desired performance as required for the real plant. AFLC based on Levenberg-Marquardt technique is the best approach for the non-linear, complex, and poorly understandable plants. This is referred as direct adaptive control technique. The block diagram for proposed control is shown in Fig. 3. Fuzzy adaptive controller does not require a perfect model to achieve the optimal performance. The non-linear model uncertainty is handled by the knowledge based modifier that makes fuzzy controller adaptive by modifying the center of singleton membership function [12-14].

The training of steepest descent is very slow and smaller step size makes it convergent. Gauss-Newton method is faster to minimize the cost function but the probability of divergence increases. The technique fails if the Jacobian matrix's inverse does not exist. A second order method, Levenberg-Marquardt technique is faster and stable using Jacobian matrix [15-17]. Levenberg-Marquardt algorithm provides solution to the problem called *Non-linear least square minimization*. The technique minimizes the function of the following form [18]-[19]

$$f(x) = \frac{1}{2}\sum_{j=1}^{m} r_j^2(x) \tag{3}$$

where

$$x = [x_1 x_2 x_3 \ \cdots x_n]^T \in R^{n\times 1} \tag{4}$$

$x$represents a vector belong to $R^{n\times 1}$ and eachr_j is the function from $R^n \ to \ R$. The r_j is called the residuals and it is assumed that $m \geq n$. The function f is represented as a $residual \ vector$, i.e., $r: R^n \rightarrow R^m$ defined by

$$r(x) = [r_1(x), r_2(x), \cdots, r_m(x)] \tag{5}$$

The function f in Eq. (3) can be written as

$$f(x) = \frac{1}{2}\|r(x)\|^2 \tag{6}$$

The derivative of f can be written as the Jacobian matrix J of r with respect to x

$$J(x) = \frac{\partial r_j}{\partial x_i} \quad 1 \leq j \leq m\,, \quad 1 \leq i \leq n \tag{7}$$

The learning algorithm of Levenberg-Marquardt is given by

$$\Delta w = -\lambda(J^T J + \mu I)^{-1} J e \tag{8}$$

Where Δw is the update weight, $J$is the Jacobean matrix, e is the error defined as the difference between actual value and desired value. The other two parameters λ and μ are used to control the step size and the regularization term to make it invertible and to stabilize the algorithm [20].

The update rules for Levenberg-Marquardt is given by

$$w_{k+1} = w_k - \lambda\left(J_k^T J_k + \mu I\right)^{-1} J_k e_k \tag{9}$$

Eq. (9) is used for updating different parameters.w_{k+1}shows the updated value, $w_k$is the previous value, $J_k$is the Jacobean matrix, λ is combination coefficient, and $I$is identity matrix.

3.1 Controller Design

The controller based on Levenberg-Marquardt technique minimizes both linear and non-linear functions. The design emerges from the following cost equation

$$e_m = \frac{1}{2}\left(f_m - y_{ref}\right)^2 \tag{10}$$

The output equation of controller used for defuzzification is given by

$$f_m = \left(\sum_{i=1}^{R} b_i \mu_i\left(x_j^m, k\right)\right) \Big/ \left(\sum_{i=1}^{R} \mu_i\left(x_j^m, k\right)\right) \tag{11}$$

where

$$\mu_i\left(x_j^m, k\right) = \prod exp\left(-\frac{1}{2}\left(\frac{x_j^m - c_j^i}{\sigma_j^i}\right)\right) \tag{12}$$

The Eq. (12) is used for updating the input and output membership function of the fuzzy logic controller. $c_j^i$is the center of membership function, and σ_j^i represents variance.

3.1.1 Derivation of Equation for Updating Parameters

The equation for updating the parameters is derived from the error defined by Eq. (10). The derivative of Eq. (10) results in Jacobian of each term, i.e., variance, center and output membership function.

Taking the derivative of Eq. (10) with respect to b_j

$$\frac{\partial e_m}{\partial b_j} = \frac{\partial}{\partial b_j}\left(\frac{1}{2}\left(f_m - y_{ref}\right)^2\right) \tag{13}$$

$$\frac{\partial e_m}{\partial b_j} = \left(f_m - y_{ref}\right)\frac{\partial \varepsilon}{\partial b_j} \tag{14}$$

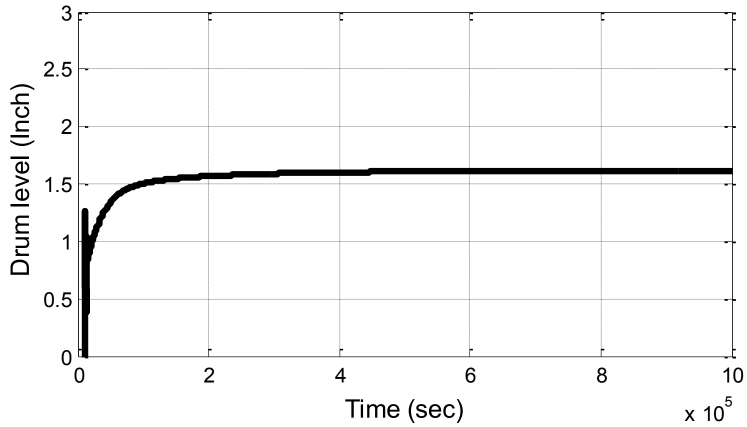

Fig. 4. Drum water level.

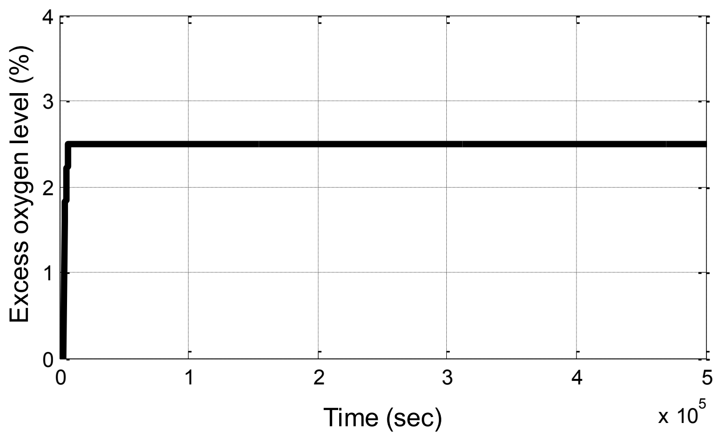

Fig. 5. Excess of oxygen.

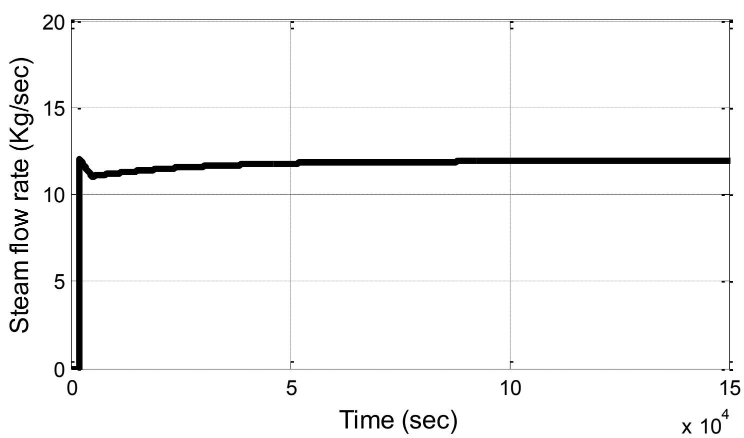

Fig. 6. Steam flow rate.

By putting the value of f_m from Eq. (10) and $\varepsilon = (f_m - y_{ref})$, we obtain

$$\frac{\partial e_m}{\partial b_j} = \varepsilon \frac{\partial}{\partial b_j}\left[\left(\sum_{i=1}^{R} b_i \mu_i(x_j^m, k)\right) \Big/ \left(\sum_{i=1}^{R} \mu_i(x_j^m, k)\right)\right] \tag{15}$$

$$\frac{\partial e_m}{\partial b_j} = \varepsilon \frac{\partial}{\partial b_j}\left[\left(\sum_{i=1}^{R} b_i \prod exp\left(-\frac{1}{2}\left(\frac{x_j^m - c_j^i}{\sigma_j^i}\right)\right)\right) \Big/ \left(\sum_{i=1}^{R} \prod exp\left(-\frac{1}{2}\left(\frac{x_j^m - c_j^i}{\sigma_j^i}\right)\right)\right)\right] \tag{16}$$

The above Eq. (15) shows the Jacobian of output membership function b_j. Similarly by taking derivatives with respect to σ_j and c_j will result in Jacobian of variance and center of membership function, respectively.

3.1.2 Update Equation for Output Membership Function

This equation is used for updating the output membership function, i.e., control-output to the plant. The variable b_j represents the center of output membership function. The center of output membership function updates according to the output of the plant.

$$b_j(k+1) = b_j(k) - \lambda \left[\left[\varepsilon\left[\frac{\mu_i(x_j^m, k)}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\right]\left[\left[\varepsilon\left[\frac{\mu_i(x_j^m, k)}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\right]\right]^T + \mu I\right]^{-1}\left[\varepsilon\left[\frac{\mu_i(x_j^m, k)}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\right]\varepsilon \tag{17}$$

3.1.3 Update Equation for Variance

From Eq. (12) the magnitude of membership function is inversely proportional to the variance. The higher value of variance results in lower magnitude and vice versa. Variance defines the spread of the membership function which is updated by the equation given below.

$$\sigma_i(k+1) = \sigma_i(k) - \lambda \left[\left[\varepsilon\left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)^2}{(\sigma_j^i)^3}\right)\mu_i(x_j^m, k)\right]\left[\varepsilon\left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)^2}{(\sigma_j^i)^3}\right)\mu_i(x_j^m, k)\right]^T + \mu I\right]^{-1}\left[\varepsilon\left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)^2}{(\sigma_j^i)^3}\right)\mu_i(x_j^m, k)\right]\varepsilon \tag{18}$$

3.1.4 Update Equation for Center

The Eq. (19) updates the center of membership function. The center acquires different values according to the crisp input to controller.

$$c_i(k+1) = c_i(k) - \lambda \left[\left[\varepsilon\left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)}{(\sigma_j^i)^2}\right)\mu_i(x_j^m, k)\right]\left[\varepsilon\left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)}{(\sigma_j^i)^2}\right)\mu_i(x_j^m, k)\right]^T + \mu I\right]^{-1}\left[\varepsilon\left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)}{(\sigma_j^i)^2}\right)\mu_i(x_j^m, k)\right]\varepsilon \tag{19}$$

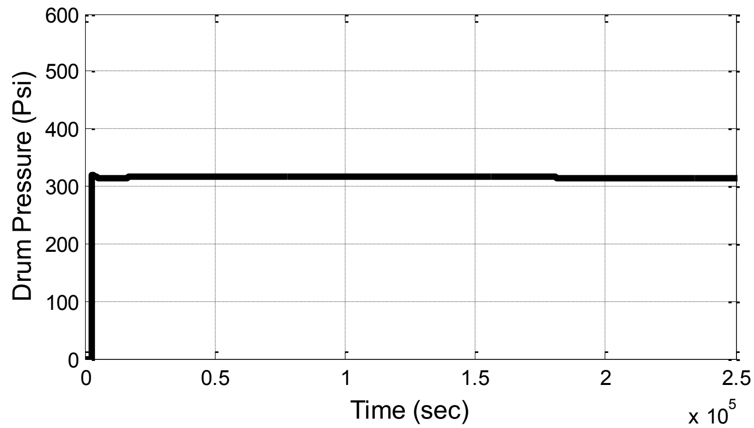

Fig. 7. Drum pressure.

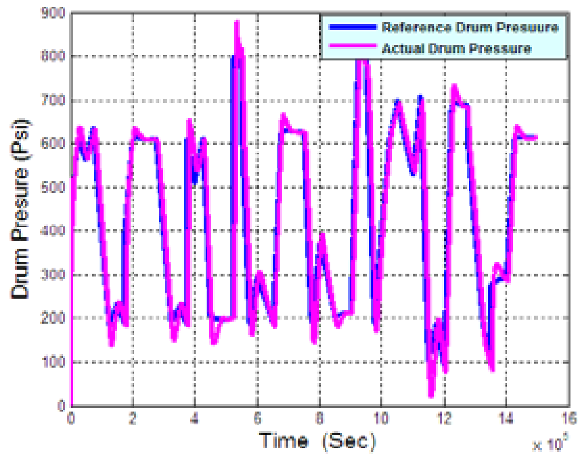

Fig. 8. Controlled drum pressure.

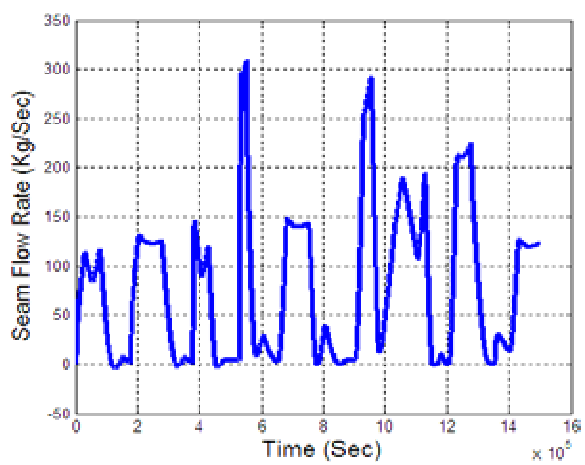

Fig. 9. Controlled steam flow rate.

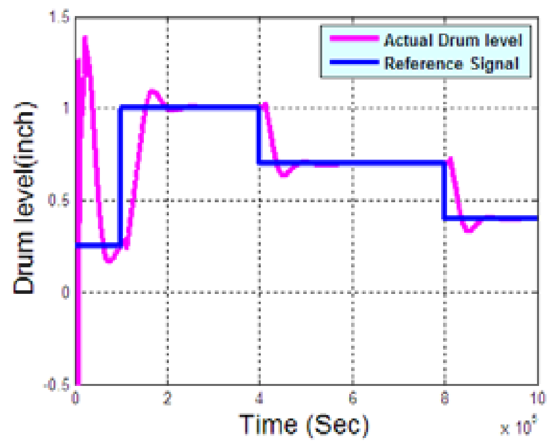

Fig. 10. Controlled drum level.

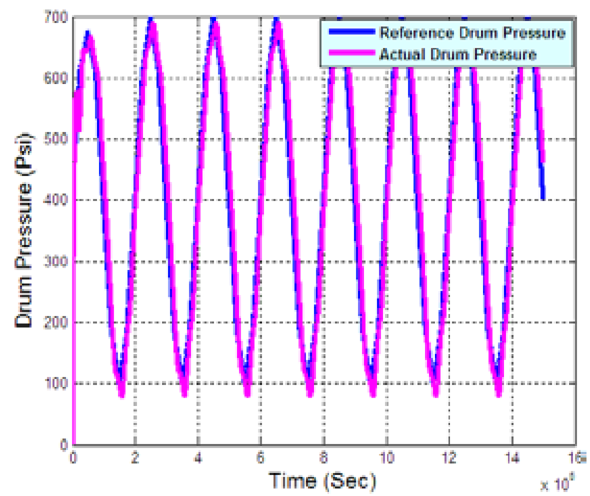

Fig. 11. Controlled drum pressure.

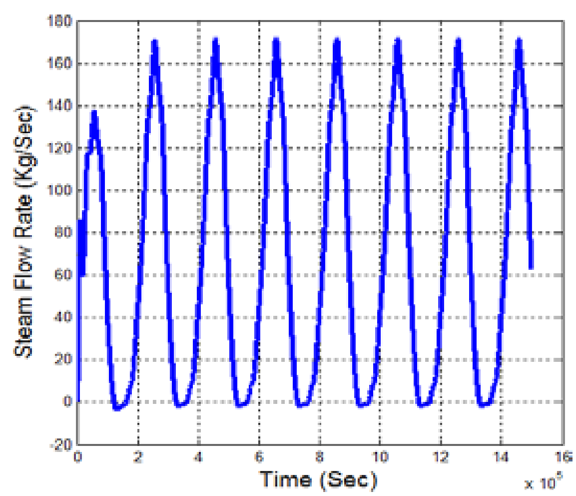

Fig. 12. Controlled steam flow rate.

4. SIMULATION RESULTS

4.1 Open Loop Responses

Fig. 4–7 show open loop responses of different parameters of steam boiler to unit step. It is observed that AFLC has the ability to understand the process and update its parameters accordingly to give desired controlled input to the plant. The drum level will rise to 1.5 inches by applying unit step at feed water input (u_3). Applying unit step at air flow input (u_2) will result the oxygen level output at 2.5%. Similarly, steam flow rate will be 12 kg/sec and drum pressure will acquire the value of 320 psi by applying step input at fuel flow (u_1).

4.2 Adaptive Fuzzy Logic Controller (AFLC) Closed Loop Responses

The simulation is performed using MATLAB, and keeping time $1500\ sec$ throughout the simulation. The Fig. 8 to 12 show controlling of different steam boiler parameters with AFLC. As the plant has dynamics of high order, as well as nonlinearities, instabilities, and time delays, for this reason multiple signals are given as reference to check different parameters responses. Fig. 8 and 9 show drum pressure and corresponding steam flow rate which approximately resemble with real time responses of Abbott power plant in Champaign, Illinois. Initially steam generator takes time to reach the required drum pressure due to burning of fuel and rise in temperature from cold start, but in fact the steam flow rate is associated with the drum pressure. Steam generator turbine needs constant steam flow to avoid fluctuation in connected load, i.e., electric generator in the case of steam generator-turbine unit. As AFLC results in oscillation free output of both drum pressure and steam flow rate, therefore AFLC is preferred over pervious controlled schemes. In addition, the overshoot in responses settles abruptly due to updating of various parameters of ALFC, resulting in smooth and controlled output.

Fig. 11 and Fig. 12 show drum pressure and corresponding steam flow rate with a sine wave as a reference signal. The output parameter, drum water level (Fig. 10) is a slow process, which is affected by fuel flow rate and steam flow rate directly. In real plant, drum level is kept constant at center of the drum throughout the operation [16]. Keeping in view the resemblance with real plant, a changing step within its limit is applied which is tracked by the output response of drum level. The actual drum level shows less settling time with AFLC. At the start of the simulation, the AFLC quickly updated its parameters and started to follow the desired drum level which was at 0.2 inch. At 400 sec the desired level of drum changes to 1 inch, the response of AFLC has 0.85% overshoot and settling time is 158 sec.

5. CONCLUSIONS

We proposed adaptive fuzzy logic controller (AFLC) for the control of multivariable steam boiler. The results of AFLC for different controlling parameters show that percentage overshoot and settling time is within allowable limits, and its response has improved both in transient and steady state region, because ALFC does not require a perfect model for its optimal performance. The self-learning and updating mechanism of adaptive fuzzy controller reduces the problem of estimating the internal states of MIMO system. Fuzzy controller removes the fluctuations from the actual response as it occur in drum pressure and steam flow rate. Throughout the simulation AFLC keeps the drum level within its allowable limit. The Gauss Newton based Levenberg-Marquardt technique enhances the process of parameter updating, thereby minimizing the computational time.

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