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Research Article

Solution of 7 Bar Tress Model Using Derivative Free Methods

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Abstract: The focus of this research is to formulate optimization model of 7-bar trusses along with stress, stability and deflection constraints. The derivative free methods are used for the optimization of engineering design problems. These methods are basically designed for unconstrained optimization problems. In formulated optimization truss problems the constraints are handled by using exterior penalty functions. The results of the truss optimization model are obtained by using MATLAB which demonstrate the effectiveness and applicability of these derivative free methods.

Keywords: Derivative free methods, penalty function, structural optimization, truss structure, unconstrained optimization

1. INTRODUCTION

The optimization phenomenon appears in very nearly all ranges of life like assembling, scheduling, engineering and business. Utilizing optimization procedures the best results of the problem are attempted to get by using least measure of restricted assets [1].

Two principle procedures of optimization, specifically, derivative based and derivative free are, no doubt utilized frequently. Among the direct search methods we concentrated on Hooke and Jeeves (HJ) strategy [2], Nelder and Mead (NM) strategy [3-5] and Multi-Directional search (MDS) technique [6]. These methods are intended for unconstrained optimization issues. They can additionally connected to constrained optimization problems by changing them into unconstrained optimization problems by utilizing the penalty function [7, 8]. The structures of the penalty function together with views for alter penalty parameters at the end of each one unconstrained minimization step describe specific scheme or strategy.

In the early years when the derivatives of functions were weigh down to calculate, the direct search methods were prevalent, yet as of late, we have various devices for strong and automatic differentiation [9] and additionally modeling languages[10] that cost derivatives consequently. In spite of this, direct search methods having their importance. Especially the development of simulation-based optimization [11] has made it hard to utilize derivative based methods. In addition, the objective function which is not numeric in nature can't be simplified by derivative based approaches.

For calculating different sorts of optimization issues a lot of direct search methods have been produced by the analysts. A definite investigation of these systems, with recorded foundation, might be found in [12].The consideration of this system is that change the constrained optimization problem to an unconstrained one by adding/subtracting the value of or from the objective function focused around constraint present in the result [13]. Specialist's effort to improve the preliminary structure of equipment and strive to upgrade the operation of that supply once it is introduced to understand the biggest generation, the best benefit, the base cost and the minimum energy utilization [14].

Structural Optimization Problem: The structural optimization problem [15] minimizing the objective function (expense, weight, volume)

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subject to demands on mechanical constraints. The aggregate structural volume (or weight) is typically allocated as the target capacity, in light of the fact that it is an elementary prerequisite to reduce the weight of the aeronautics and mechanical structures. For the structures in architectural engineering and civil engineering, reduction of weight toward oneself for the most part stimulates decreasing of the shape weights, and consequently encourages lower cost. Structural optimization may be sub-divided into shape optimization and topology optimization [16]. Structural optimization concerns could be attractively easy to figure, might be collected as, Find *x* to minimize subject to g*(x) ≤ 0*. Here f is the objective function and g is the constraints. Problem of this sort are called numerical programming problems

$$Min\ f(x)$$

$$Subject\ to\ g(x) \leq 0$$

Structural Design & Size: Derivative free strategies inspect instruments to make structural optimization that is prepared for size and shape streamlining of truss and edge structures. Limit is extended by including graphical overview utilities for structure visualization and enhancement process. The objective of the structural streamlining is the minimization of volume with stress and displacement soft-constraints.

Truss Structures: Truss parts are one dimensional in their close-by encourages structure and passes on simply axial loads in view of their pin relationship at nodes. This moreover infers that a truss node is simply allowed translational degrees of freedom. A truss segment needs simply a cross sectional region (A) to expose its geometry as a result of the critical load limit, and its length is controlled by the range of its end nodes. A three-dimensional truss segment has two nearby degrees of freedom and six global degrees of freedom, with three translational degrees of inflexibility at every one end of the components [17].

2. MATERIALS AND METHODS

2.1. Development of N Bar Truss Model

Consider N bar trusses, in these trusses we try to optimize the weight under stress constraint. Cross sectional area is considered as design variables.

Objective Function

In this problem, the objective function we have considered is weight of the general truss. The ρ is the parameters of material thickness and Li is the parameters of length of i^{th} part, respectively.

$$f(A) = \sum_{i}^{n} \rho A_i L_i$$

$$A_i \geq 0, \qquad i = 1, \ldots, n$$

Constraints

Firstly, points out the area and the amount of fundamental nodes for supports and loads. Accordingly, a feasible truss must have all the fundamental nodes.

Secondly, the truss must not deflect more than the allowable limit due to the application of loads.

$G_2 = \sum \delta_k^{max} - \sum \delta_k(A)$, k = 1,2,…,n

Thirdly, in a feasible truss, all parts must have focuses inside the suitable quality of the material. Since, typically a truss is subjected to different loading conditions connected independently; these demands must be utilized for each one loading condition. Since the trusses of different topologies are made on the fly, some of them may be statically determinate and some of them may be statically uncertain. Hence, we have utilized derivative free strategies to compute the stress and deflection.

$G_3 = \sum S_j - \sum \sigma_j(A) \geq 0$, j = 1,2,…,m

Finally, in a feasible truss all members must have stresses within the allowable strength of the material. Some bar trusses have compressive force and these become compressive stress constraint and some have tensile force and these become tensile stress constraint.

$G_4 = \sum T_j - \sum \sigma_j(A) \geq 0$, j = 1,2,…,m

$G_5 = \sum C_j - \sum \sigma_j(A) \geq 0$, j = 1,2,…,m

In the above NLP problem where ρ is the density of the material, it may be focused that this specific objective function does not depend on any state variable, as design constraints. The parameter S_j is the allowable strength of the material, T_j is the allowable tensile of the material, δ_k^{max} is the allowable deflection in the truss and C_j is the allowable compressive strength of the material. We recommended that the cross sectional areas must, for obvious physical reason, be non-negative $A_i \geq 0, i = 1, \ldots, n$.

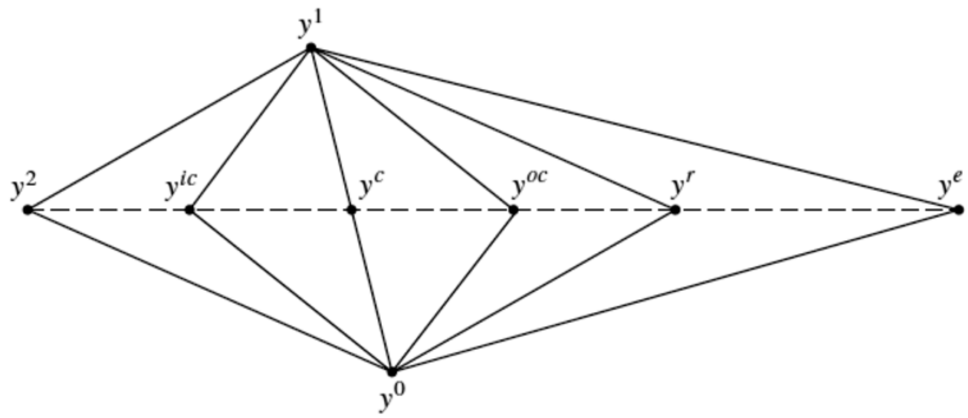

Fig. 1. Algorithm of Nelder–Mead method.

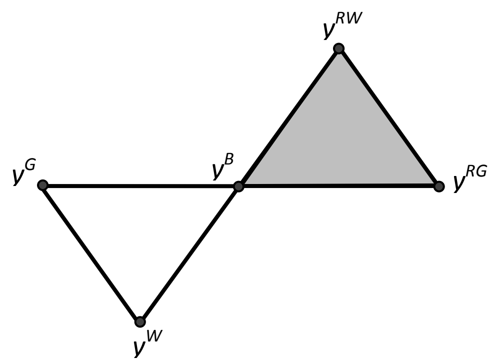

Fig. 2. Reflection.

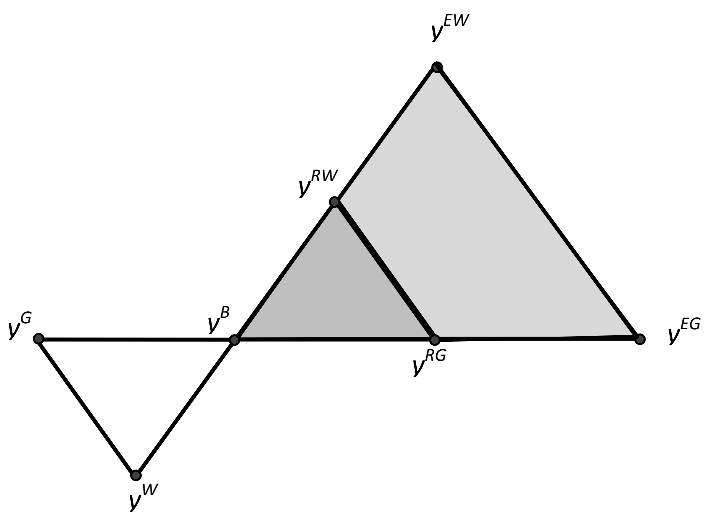

Fig. 3. Expansion.

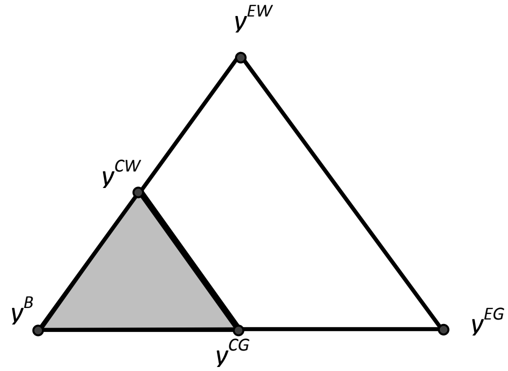

Fig. 4. Contraction.

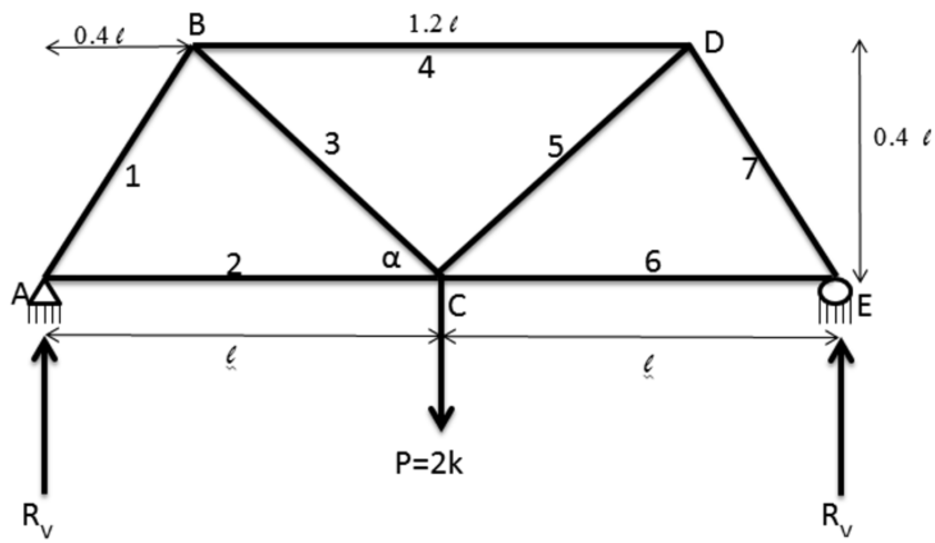

Fig. 5. Seven bar truss.

Hooke and Jeeves Method

This method starts with an initial point. In N-dimensional problem a Set of N linearly independent search directions generate 2N points.

Exploratory Move: Exploratory move is performed on the current point systematically to find the best point around the current point.

Pattern Move: When exploratory move success then pattern move is perform, a new point is found by jumping from the current base point along a direction connecting to the previous.

Nelder-Mead Simplex Method

The method uses the following operations

Reflection: Reflect the worst vertex over the centroid.

Expansion: If the function value at the reflect point is less than best point the expansion is performed.

Contraction: If the function value of the reflection point lies between the good and best vertex then

Inner Contraction: If function value greater than the best point then inner contraction is performed.

Outer Contraction: If the function values less than the best point then outer contraction is performed as shown in Fig. 1.

Shrink: If no one from the above conditions is satisfied then shrink produced.

Multi-directional Search Method

In N- dimensional problem method starts with a simplex of N+1 points. The method generates N points along N linearly independent search directions. The method uses the following operations

Reflection: The worst and good point is reflectede at the best point as shown in Fig. 2.

Expansion: If the value of the reflection points is less than the best point then expansion is performed as shown in Fig. 3.

Inner Contraction: If the values of the reflection points is not less than the best point then contraction is performed as shown in Fig. 4.

3. FORMULATION OF SEVEN BAR TRUSS MODEL

Consider a seven bar truss. The bars AB and DE, AC and CE , BC and DC have similar length but the bar BD have different length from the other bars and young modulus E. We are to minimize the weight under stress constraint. The design variables are the cross sectional areas$A_1, A_2, A_3, A_4, A_5, A_6, A_7$. Due to symmetry

$$A_1 = A_7, A_2 = A_6, A_3 = A_5 .$$

Thus there are particularly four design variables A_1, A_2, A_3, A_4 The objective function i.e the total weight of the truss becomes

$$f(A) = (A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7)\rho l$$

In this issue the amount of bars equivalent the amount of the level of flexibility, which infers that the bar constrains or burdens, may be gotten specifically from the harmony mathematical statements. We say that the truss is statically determinate.

Objective

In this problem we are interested to minimizing the weight of the truss structure.

Objective Function

Minimize $1.132A_1l+2A_2l+1.42A_3l+1.2A_4l$

Subject to Constraints

$\frac{PCsc\theta}{2A_1} \leq S_{c1}$ (Compressive Stress Constraint)

$\frac{P(Cot\theta + Cot\alpha)}{2A_4} \leq S_{c4}$ (Compressive Stress Constraint)

$\frac{PCot\theta}{2A_2} \leq S_{t2}$ (Tensile Stress Constraint)

$\frac{Pcsc\theta}{2A_3} \leq S_{t3}$ (Tensile Stress Constraint)

$\frac{P}{2Sin\theta} \leq \frac{\pi EA_1^2}{1.28l^2}$ (Stability Constraint)

$\frac{P(Cot\theta + Cot\alpha)}{2} \leq \frac{\pi EA_4^2}{5.76l^2}$ (Stability Constraint)

$\frac{Pl}{E}\left(\frac{0.566}{A_1} + \frac{0.500}{A_2} + \frac{2.236}{A_3} + \frac{2.700}{A_4}\right) \leq \delta_{max}$

$(Deflection\ constraint)$

$10mm^2 \leq A_1, A_2, A_3, A_4 \leq 500mm^2$

4. RESULTS AND DISCUSSION

The result which we get from seven bar truss by applying HJ method is that, when we take the initial guess in the range of 1 to 4. We have taken

Table 1. Parameters of seven bar truss problem.

Parameter	Description	Value
S_{ci}	Allowable Compressive strength in bar i	500 Mpa
S_{ti}	Allowable Tensile strength in bar i	500 Mpa
δ_{max}	Allowable deflection	2 mm
E	Modulus of Elasticity	200 Gpa

Table 2. The Result of seven bar problem by applying Hooke and Jeeves method.

Initial guess	Function value	Final point	Function value	No. of Function Evaluations
1,3,2,2	22.856	0.0724, 0.0110, 0.0174, 0.1528	0.3120	232

Table 3. The Result of seven bar problem by applying Nelder-Mead method.

Initial guess	Function value	Final point	Function value	No. of function Evaluations
2,2,1,1,1,3 ,3,1,2,1,2,2, 3,3,3,1,1,3, 2,2	235.70	(0.0722, 0.0717, 0.0722, 0.0722, 0.0722, 0.0063, 0.0063, 0.0057, 0.0063, 0.0063, 0.0237, 0.0237, 0.0237, 0.0231, 0.0237, 0.1518, 0.1518, 0.1518, 0.1518, 0.1523)	0.3100	217

Table 4. The Result of seven bar problem by applying Multi-directional search method.

Initial guess	Function value	Final point	Function value	No. of Function Evaluations
2,2,1,1,1,3 ,3,1,2,1,2,2, 3,3,3,1,1,3, 2,2	237.70	0.0712, 0.0712, 0.0622, 0.0522, 0.0222, 0.0043, 0.0043, 0.0047, 0.0033, 0.0033, 0.0235, 0.0235, 0.0235, 0.0232, 0.0237, 0.1518, 0.1518, 0.1518, 0.1518, 0.1523	0.2522	325

lot of points between this range and apply this method the result does not show the consistent performance. And the final solution which we get is feasible because it satisfied all the constraints and no constraint is active at this solution. The function value is 0.3120 at the points (0.0724, 0.0110, 0.0174, 0.1528).

The best result which we get from seven bar truss by applying NM method is that, when we take the initial guess in the range of 1 to 10. We have taken lot of points between this range and apply this method the result does not show the consistent performance. And we do not take the better point in this range. But when we take the initial guess in the range 1 to 5, it also show not consistent performance but we got a point which is converges and the solution which we get from this is feasible and satisfied all the constraint and no active constraint at this solution. The function value is 0.3100 at the points (0.0722, 0.0717, 0.0722, 0.0722, 0.0722, 0.0063, 0.0063, 0.0057, 0.0063, 0.0063, 0.0237, 0.0237, 0.0237, 0.0231, 0.0237, 0.1518, 0.1518, 0.1518, 0.1518, 0.1523)

The best result which we get from seven bar truss by applying MDS method is that, when we take the initial guess in the range of 1 to 10. We have taken lot of points between this range and applied this method the result does not show the consistent performance. And we do not take the better point in this range. But when we take the initial guess in the range 1 to 5, it also show not consistent performance but we got a point which is convergent and the solution which we get from this is feasible and satisfied all the constraint and there is no active constraint at this solution. The function value is 0.2522 at the points (0.0712, 0.0712, 0.0622, 0.0522, 0.0222, 0.0043, 0.0043, 0.0047, 0.0033, 0.0033, 0.0235, 0.0235, 0.0235, 0.0232, 0.0237, 0.1518, 0.1518, 0.1518, 0.1518, 0.1523)

5. CONCLUSIONS

We applied Hooke and Jeeves method, Nelder-Mead method and Multi-directional search method on seven bar truss optimization problems. We implemented these three methods in MATLAB on the formulated problems for many times at various initial guesses and for a number of step sizes. Observing all tables we can conclude that result of Nelder and Mead is not acceptable due to its far away convergence even its number of function evaluations is smaller than number of function evaluations of MDS method. Function value of MDS method is comparatively much better than function value of Nelder and Mead method. By comparing the function values obtained by these three methods we conclude that the performance of N&M method is worse than the other two methods. From these tables we conclude that the performance of Multi-directional Search method is better than the other two methods because the function value is smaller than the function value of other two methods.

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