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PREDICTING THE WATERTABLE ELEVATION USING WATER BALANCE MODELS

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The fluctuation of watertable in a shallow unconfined aquifer due to rainfall, evapotranspiration, tubewell and canal supplies were determined using the recursive least squares method. The parameters were predicted by minimizing the mean square error between the estimated and measured watertable levels. Watertable fluctuations were found to be dependent upon pumping rates and recharge source such as prior rainfall, and were simulated by a multiple linear regression model. Such models can be used for forecasting and controlling the ground watertables and once formulated are ideally suited to the management of groundwater systems where more costly and complex methods cannot be used.

INTRODUCTION

The process of variation of watertable level of an unconfined aquifer due to recharge and discharge parameters is a complex one. This involves flow through the unsaturated region of an aquifer. The rate of infiltration depends upon several parameters and variables. The most important of these are the soil moisture level of the unsaturated region and intensity and quantity of the surface water applied. The application of stepwise multiple linear regression techniques to the groundwater problem is a logical extension of the water balance approach, allowing a water balance equation to become dynamic and thus simulate a groundwater system in time. A system model essentially consists of input which is acted upon by a transfer function in order to produce the output. The input data may be represented by recharge and discharge and the output by water level predictions. Attia *et al.* (1986) presented a hydrologic budget analysis involving the inflow/outflow components in the whole Nile valley region. In addition, two-dimensional finite-element simulation model was adapted, to simulate the groundwater system for the determination of the groundwater/surface water interrelationship. The model was, thus, designed to adequately represent the groundwater system of the Nile valley. Eriksson (1970) analysed the water level variations using a simple first-order linear model. Houston (1983) used time series techniques for the groundwater system. Viswanathan (1983) obtained the rainfall/watertable level relationship of an unconfined aquifer using the recursive least squares method.

All other alternate methods do have some limitations and shortfalls. These factors emphasize that a study be carried out for the selection of a method suitable for the given circumstances.

MATERIALS AND METHODS

Groundwater balance equation: The general form of balance equation to forecast the behaviour of groundwater system can be given as follows:

$$Sgw = R - Etc + Qc - Tw$$

where

Sgw = Change in storage depth (cm)
R = Recharge by precipitation (cm)
Etc = Evapotranspiration (cm)
Qc = Recharge from canal (cm) and
Tw = Tubewell discharge (cm).

Development of linear prediction model: The recursive least squares method was used to develop a linear model which estimated the weekly water level given the recharge and discharge values during the current as well as preceding three weeks. The linear mathematical model that relates the watertable level variation with recharge and discharge on any given day "t" is assumed to be of the form as presented below:

$$WTt = A_0 + a_oR_t + a_1R_{t-1} + a_2R_{t-2} + a_3R_{t-3} + b_oTW_t + b_1R_{t-1} + b_2R_{t-2} + b_3R_{t-3} + c_oQc_t + c_1Qc_{t-1} + c_2Qc_{t-2} + c_3Qc_{t-3} + d_oEtc_t + d_1ETc_{t-1} + d_2Etc_{t-2} + d_3Etc_{t-3}$$

where

WT = Estimated depth of water level from the ground surface
R = Rainfall
Qc = Canal discharge
TW = Tubewell discharge and
Etc = Evapotranspiration.

Subscript "t" indicates the current week whereas the superscripts t-1, t-2 .... indicate consecutive preceding weeks. A_o is the constant term and other regression coefficients (a_0, a_1, a_2, ..., b_0, b_1, b_2, ..., c_0, c_1, c_2, ..., and d_0, d_1, d_2, ...) that indicate the positive and negative effects on watertable corresponding to the recharge and discharge sources, respectively.

Weekly watertable elevations were computed. Two forms of linear models were developed and investigated for Rabi and Kharif seasons. The models were calibrated to obtain a good match between the observed and predicted watertable elevations. By this process the coefficients introduced into various equations were statistically adjusted by the computer programme until a good fit was obtained. The accuracy of the model can be greatly improved by extending the time scale of the input data and including an autoregression term (Akbar, 1988).

RESULTS AND DISCUSSION

The linear model presented above was verified by using the real time data from the Mona Reclamation Project. This Mona Reclamation Project constitutes a part of SCARP-2 and lies in the north central part of the 'Chaj' Doab. The project area was selected due to the historical data availability. Ten years data of weekly mean rainfall, evapotranspiration, canal and tubewell discharges were used as depicted in Fig. 1.

The results of backward elimination procedures for each of the models (Rabi and Kharif) are shown in Tables 1 and 2. At each step the least significant variable got deleted and as a result the multiple correlation coefficient declined, while the significant of the equation increased and sums of squares error also increased. The resulting equations for Rabi and Kharif seasons are:

For Rabi,

$$Wtt = 82.62 - 5.76R_{t-1} - 4.67R_{t-2} - 2.73R_{t-3} + 6.05Tw_{t-1} + 5.45Tw_{t-2} + 9.48Tw_{t-3} - 0.0082Qc_t - - 0.0091Qc_{t-3} + 12.70Etc_t$$

For Kharif,

$$Wtt = 145.85 - 3.54R_{t-1} - 3.35R_{t-2} - 2.77R_{t-3} + 10.78Tw_{t-1} + 16.30Tw_{t-3} - 0.018Qc_t - 0.0163Qc_{t-1} - - 0.015Qc_{t-2} + 6.10Etc_t$$

Cross correlation of watertable with rainfall, tubewell, canal discharge and evapotranspiration show significant positive and negative lags, indicating the direct dependence of watertable level on recharge and discharge parameters, on weekly basis.

Comparison between observed and predicted watertable depth for both the Rabi and Kharif seasons are shown in Fig. 2 a and 2 b, respectively. the figures show a good agreement between the observed and the predicted values. It can be seen that although the general trends and fluctuations are well represented but the amplitude and the location of some of the peaks are not well represented. It is believed that this is due to some inaccuracies in the input data.

Table 1. Results of regression model for Rabi season

Backward elimination of first set of independent variables used in the model for Rabi season					
Variable in the equation			Variable in the equation		
Variable	B	Sig T	Variable	B	Sig T
R_{t-1}	-5.677	0.053	R_t	-0.018	0.614
R_{t-2}	-4.676	0.000	Tw_t	0.037	0.473
R_{t-3}	-2.734	0.000	Qc_{t-1}	-0.072	0.158
Tw_{t-1}	6.050	0.000	Qc_{t-2}	-0.035	0.490
Tw_{t-2}	5.452	0.000	Etc_{t-1}	0.025	0.659
Tw_{t-3}	9.490	0.001	Etc_{t-2}	0.012	0.799
Qc_t	-0.008	0.000	Etc_{t-3}	-0.016	0.687
Qc_{t-3}	-0.009	0.000			
Etc_t	12.704	0.012			
Constant	82.617	0.000			

F = 68.72; Significant F = 0.000; Multiple R = 0.84; R^2 = 0.70; Adjusted R^2 = 0.69; Standard error = 17.3.

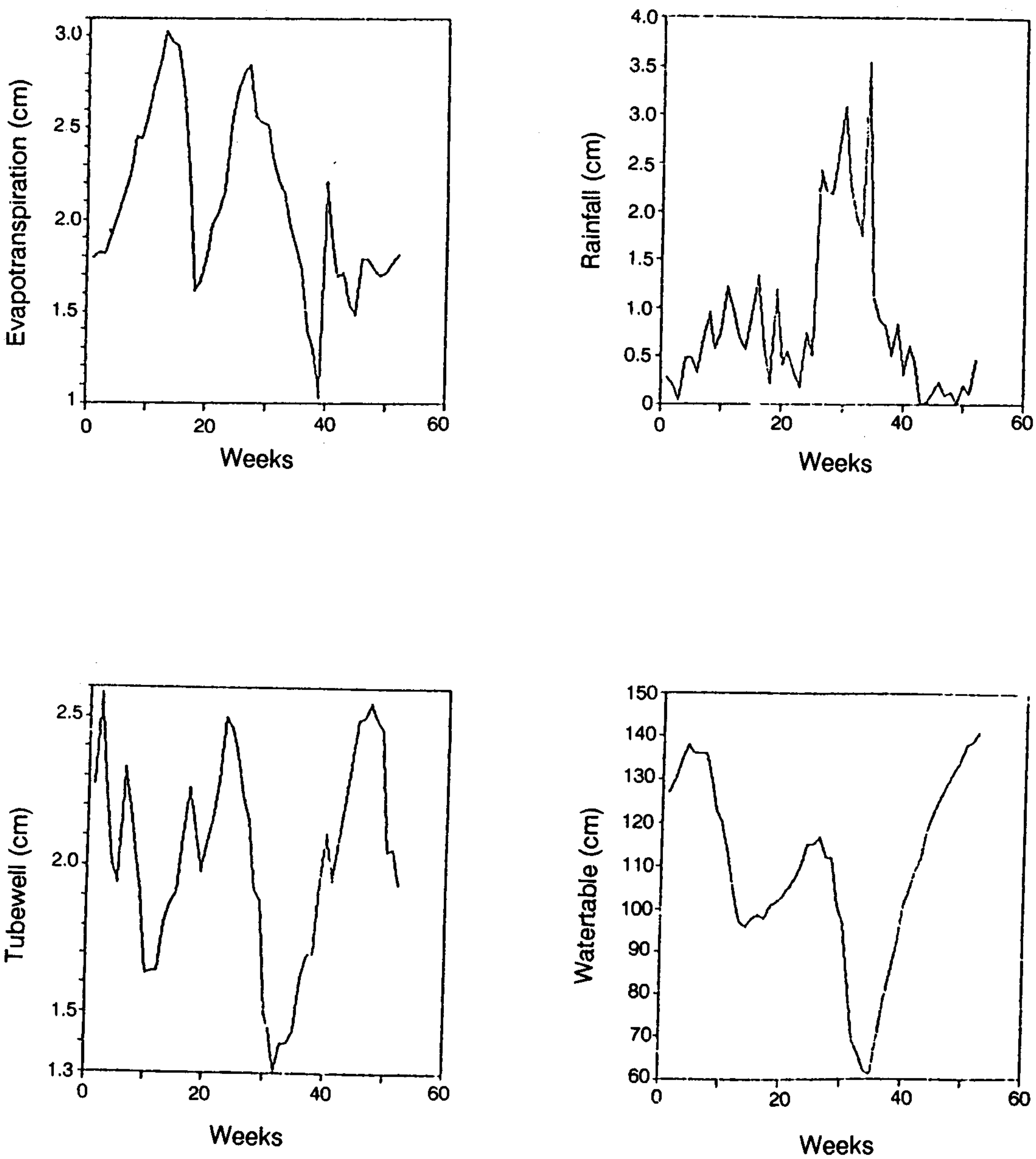

Fig. 1. Variation of rainfall, evapotranspiration, tubewell discharge and watertable depth over the year at the Mona Reclamation Project.

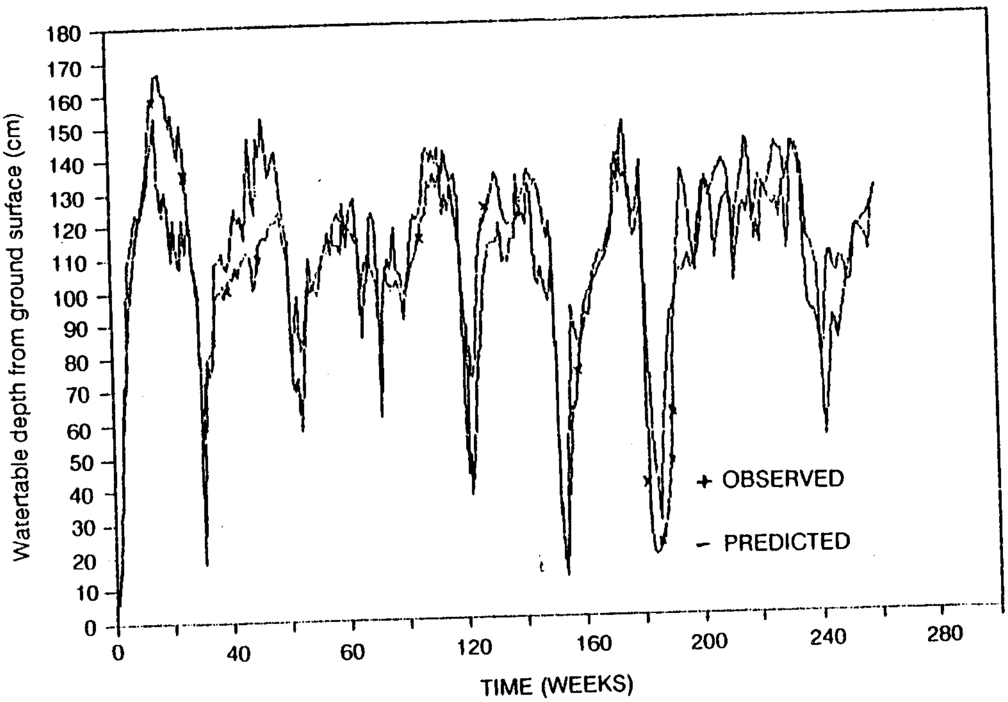

Fig. 2 a. Comparison between observed and predicted watertable level for Rabi season at the Mona Project.

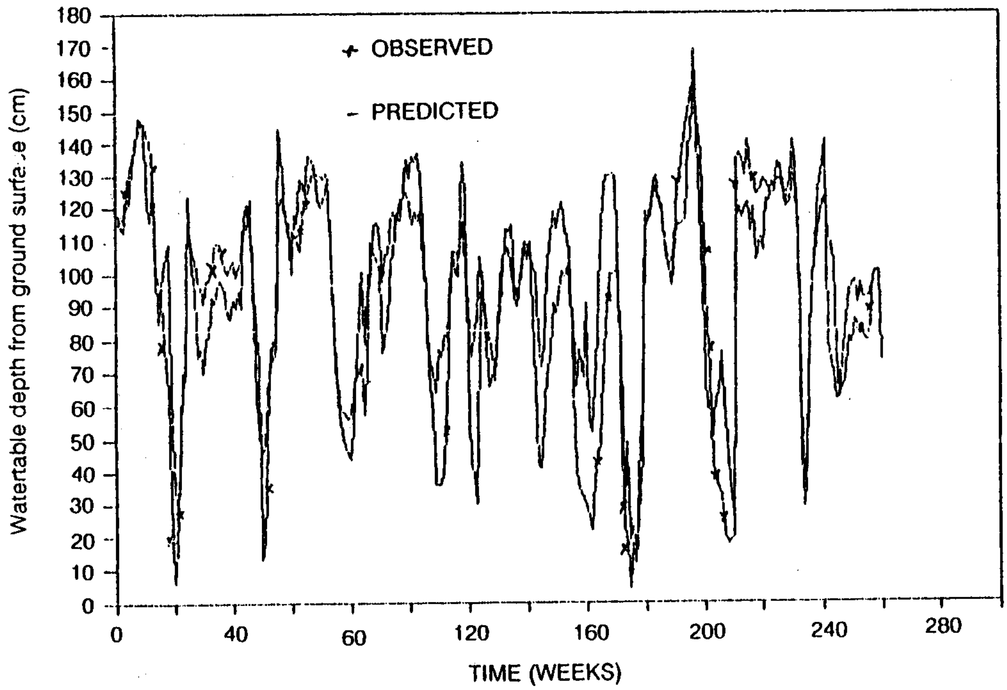

Fig. 2 b. Comparison between observed and predicted watertable level for Kharif season at the Mona Project.

Table 2. Results of regression model for Kharif season

Backward elimination of first set of independent variables used in the model for Rabi season					
Variable in the equation			Variable in the equation		
Variable	B	Sig T	Variable	B	Sig T
R_{t-1}	-3.541	0.000	R_t	-0.027	0.452
R_{t-2}	-3.352	0.000	Tw_t	0.098	0.104
R_{t-3}	-2.774	0.000	Qc_{t-1}	0.112	0.156
Tw_{t-1}	10.782	0.000	Qc_{t-2}	0.010	0.790
Tw_{t-2}	16.304	0.000	Etc_{t-1}	0.004	0.967
Tw_{t-3}	-0.018	0.006	Etc_{t-2}	0.000	0.992
Qc_t	-0.016	0.005	Etc_{t-3}	-0.011	0.784
Qc_{t-3}	-0.015	0.014			
Etc_t	6.090	0.020			
Constant	145.850	0.000			

F = 67.27; Significant F = 0.000; Multiple R = 0.82; R^2 = 0.67; Adjusted R^2 = 0.66; Standard error = 20.30.

Analysis of the residuals of both the equations show that the means are not significantly different from zero, though the standard deviations are rather high. It is demonstrated that the regression techniques can be applied to relate the watertable fluctuation with various hydrologic parameters. However, the models presented here can only be applied for specific locations for which they have been developed.

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