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Research Article

Solving Wave and Diffusion Equations on Cantor Sets

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Abstract: In this paper, we apply the proposed local fractional Adomain's decomposition method to obtain the analytic solutions of wave and heat equations within local fractional derivative operators. The iteration procedure is based on local fractional derivative. The obtained results reveal that the methodology is very efficient and simple tool for solving fractal phenomena arising in mathematical physics and engineering.

Keywords: Local fractional calculus; Fractional differential equation; local fractional Adomian's decomposition method.

AMS Mathematics Subject Classification: 74H10; 35L05

1. INTRODUCTION

The local fractional calculus theory was applied to model and process the non-differentiable phenomena in fractal physical phenomena [1–12]. Here are some local fractional models, such as the local fractional Fokker-Planck equation [1], the local fractional stress-strain relations [2], the local fractional heat conduction equation [9], wave equations on the Cantor sets [11], local fractional Laplace equation [12], Newtonian mechanics on fractals subset of real-line [13], and the local fractional Helmholtz equation [14]. There are exist some analytical methods widely applied to solve non-linear problems includes fractional adomian decomposition method [15], the homotopy perturbation method [16], the heat-balance integral method [17], the complex transform method [18], the homotopy analysis method [19], the fractional sub-equation method [20] and the fractional variational iteration method [21] and more details seen in [22].

Recently, the application of Adomian decomposition method for solving the linear and nonlinear fractional partial differential equations in the fields of the physics and engineering had been established in [23, 24]. Adomian decomposition method was applied to handle the time- fractional Navier-Stokes equation [25], fractional space diffusion equation [26], fractional KdV-Burgers equation [27], linear and nonlinear fractional diffusion and wave equations [28], fractional Burgers' equation [29]. The Adomian decomposition method, as one of efficient tools for solving the linear and nonlinear differential equations, was extended to find the solutions for local fractional differential equations [30-33] and non-differentiable solutions were obtained.

In this paper, our aim is to apply the local fractional Adomain's decomposition method [34, 35] for solving fractional partial differential equations in the sense of local fractional derivative. To illustrate the validity and advantages of the method, we will apply it to the space-time fractional wave and heat equations.

2. PRELIMINARY RESULTS AND DEFINITIONS

In this section, we present few mathematical fundamentals of local fractional calculus and introduce the basic notions of local fractional

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continuity, local fractional derivative, and local fractional integral of non-differential functions.

Definition 2.1. If there exists the relation [7, 36]

$$\left|f(x)-f(x_0)\right|<\varepsilon^{\alpha},\ 0<\alpha\le 1, \tag{1}$$

with $\left|x-x_0\right|<\delta$, for $\varepsilon,\ \delta>0$ and $\varepsilon,\ \delta\in\Re$. Now $f(x)$ is called local fractional continuous at $x=x_0$, denoted by $\lim_{x\to x_0} f(x)=f(x_0)$. Then $f(x)$ is called local fractional continuous on the interval (a,b), denoted by

$$f(x)\in C_\alpha(a,b). \tag{2}$$

Definition 2.2. A function $f(x)$ is called a non-differentiable function of exponent α, $0<\alpha\le 1$, which satisfy Hölder function of exponent α, then for, $x,y\in X$ such that [7,36]

$$\left|f(x)-f(y)\right|<C\left|x-y\right|^{\alpha}. \tag{3}$$

Definition 2.3. A function $f(x)$ is called to be continuous of $\alpha, 0<\alpha\le 1$, or shortly α continuous, when we have the following relation [7,36] $\left|f(x)-f(x_0)\right|<\varepsilon^{\alpha}$,

$$f(x)-f(x_0)=o\left((x-x_0)^{\alpha}\right). \tag{4}$$

Compared with (4), Eq. (1) is standard definition of local fractional continuity. Here (3) is unified local fractional continuity.

Definition 2.4. Setting $f(x)\in C_\alpha(a,b)$, local fractional derivative of $f(x)$ order α at $x=x_0$, is defined [7,36]

$$f^{(\alpha)}(x_0)=\left.\frac{d^{\alpha}f(x)}{dx^{\alpha}}\right|_{x=x_o}=\lim_{x\to x_0}\frac{\Delta^{\alpha}\left(f(x)-f(x_0)\right)}{(x-x_0)^{\alpha}}, \tag{5}$$

where $0<\alpha\le 1$,

$\Delta^{\alpha}\left(f(x)-f(x_0)\right)\cong\Gamma(1+\alpha)\Delta\left(f(x)-f(x_0)\right)$.

For any $x\in(a,b)$, there exists

$$f^{(\alpha)}(x)=D_x^{\alpha}f(x),$$

denoted by $f(x)=D_x^{\alpha}f(a,b)$.

Local fractional derivative of high order is written in the form

$$f^{(k\alpha)}(x)=\overbrace{D_x^{\alpha}\dots D_x^{\alpha}}^{k\ times}f(x),$$

and local fractional partial derivative of high order

$$\frac{\partial^{k\alpha}f(x)}{dx^{k\alpha}}=\overbrace{\frac{\partial^{\alpha}}{\partial x^{\alpha}}\cdots\frac{\partial^{\alpha}}{\partial x^{\alpha}}}^{k\ times}f(x).$$

Definition 2.5 Setting $f(x)\in C_\alpha(a,b)$, local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is defined [36]

$$\begin{aligned} {}_aI_b^{\alpha}f(x)&=\frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^{\alpha}\\ &=\frac{1}{\Gamma(1+\alpha)}\lim_{\Delta t\to 0}\sum_{j=0}^{N-1}f(t_j)(\Delta t_j)^{\alpha},\ 0<\alpha\le 1, \end{aligned} \tag{6}$$

where

$\Delta t_j=t_{j+1}-t_j,\ \Delta t=\max\{\Delta t_1,\Delta t_2,\Delta t_3,\dots\}$ and $\left[\Delta t_j,\Delta t_{j+1}\right],\ j=0,1,\dots,N-1,\ t_0=a, t_N=b,$

is a partition of the interval $[a,b]$. For any $x\in(a,b)$, there exists ${}_aI_x^{\alpha}f(x)$, denoted by $f(x)\in I_x^{(\alpha)}(a,b)$.

If $f(x)=D_x^{\alpha}f(a,b)$, or $I_x^{(\alpha)}(a,b)$, we have $f(x)\in C_\alpha(a,b)$.

Here, it follows that

$${}_aI_a^{\alpha}f(x)=0, \text{if } a=b.$$

$${}_aI_b^{\alpha}f(x)=-{}_bI_a^{\alpha}f(x), \text{ if } a<b.$$

$${}_aI_a^{0}f(x)=f(x).$$

For any $f(x)\in C_\alpha(a,b)$, $0<\alpha\le 1$, we have local fractional multiple integrals

$${}_{x_0}I_x^{(k\alpha)}f(x)=\overbrace{{}_{x_0}I_x^{(\alpha)}\dots{}_{x_0}I_x^{(\alpha)}}^{k\ times}f(x),$$

For $0<\alpha\le 1$, $f^{((k\alpha))}(x)\in C^k{}_\alpha(a,b)$, then we have

$$\left({}_{x_0}I_x^{(k\alpha)}f(x)\right)^{(k\alpha)}=f(x),$$

where $\quad {}_{x_0}I_x^{(k\alpha)}f(x) = \overbrace{{}_{x_0}I_x^{(\alpha)} \ldots {}_{x_0}I_x^{(\alpha)}}^{k\ times} f(x)$, and $f^{(k\alpha)}(x) = \overbrace{D_x^{\alpha} \ldots D_x^{\alpha}}^{k\ times} f(x)$.

Definition 2.6 Mittag-Leffer function in fractal space is defined by

$$E_\alpha\left(x^\alpha\right) = \sum_{k=0}^{\infty} \frac{x^{k\alpha}}{\Gamma(1+k\alpha)},\ 0 < \alpha \le 1. \tag{7}$$

The following rules hold

$$\cos_\alpha\left(x^\alpha\right) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k\alpha}}{\Gamma(1+k2\alpha)}. \tag{8}$$

$$\sin_\alpha\left(x^\alpha\right) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)}. \tag{9}$$

3. LOCAL FRACTIONAL DERIVATIVES AND INTEGRALS

Some useful formulas and results of local fractional derivative were summarized [7,37].

$$\frac{d^\alpha x^{k\alpha}}{dx^\alpha} = \frac{\Gamma(1+k\alpha)x^{(k-1)\alpha}}{\Gamma(1+(k-1)\alpha)}. \tag{10}$$

$$\frac{d^\alpha E_\alpha\left(x^\alpha\right)}{dx^\alpha} = E_\alpha\left(x^\alpha\right). \tag{11}$$

$$\frac{d^\alpha E_\alpha\left(k x^\alpha\right)}{dx^\alpha} = kE_\alpha\left(k x^\alpha\right). \tag{12}$$

$$\frac{1}{\Gamma(1+\alpha)} \int_a^b E_\alpha\left(x^\alpha\right)(dx)^\alpha = E_\alpha\left(b^\alpha\right) - E_\alpha\left(a^\alpha\right). \tag{13}$$

$$\frac{1}{\Gamma(1+\alpha)} \int_a^b \sin_\alpha\left(x^\alpha\right)(dx)^\alpha = \cos_\alpha\left(a^\alpha\right) - \cos_\alpha\left(b^\alpha\right). \tag{14}$$

$$\frac{1}{\Gamma(1+\alpha)} \int_a^b x^{k\alpha}(dx)^\alpha = \frac{\Gamma(1+k\alpha)}{\Gamma(1+(k+1)\alpha)}\left(b^{(k+1)\alpha} - b^{(k+1)\alpha}\right). \tag{15}$$

4. ANALYSIS OF LOCAL FRACTIONAL ADOMIAN DECOMPOSITION METHOD

Consider the general local fractional differential equation in a local fractional differential operator form

$$L_{xx}^{(2\alpha)}u(x) + R_x^{(\alpha)}u(x) = f(x), \tag{16}$$

In Eq. (15) $L_x^{(2\alpha)}$ is local fractional $2\alpha^{th}$ order differential operator, which by the definition reads

$$L_x^{(2\alpha)}S(x) = \frac{d^\alpha}{dx^\alpha}\left[\frac{d^\alpha S(x)}{dx^\alpha}\right]$$

and

$$R_x^{(\alpha)}S(x) = \left.\frac{d^\alpha S(x)}{dx^\alpha}\right|_{x=x_0} = \lim_{x \to x_0} \frac{\Delta^\alpha\left(S(x) - S(x_0)\right)}{\left(x - x_0\right)^\alpha}$$

is local fractional α^{th} order differential operator $0 < \alpha < 1$, and $S(x)$ is local fractional continuous. Applying the inverse operator $L_x^{(-2\alpha)}$ to both sides of (16) yields

$$L_{xx}^{(-2\alpha)}L_x^{(2\alpha)}u(x) = -L_{xx}^{(-2\alpha)}R_x^{(\alpha)}u(x) + L_{xx}^{(-2\alpha)}f(x), \tag{17}$$

If the inverse differential operator $L_x^{(-2\alpha)}$ exists, according to the local fractional decomposition method mentioned above, we have

$$\begin{cases} u_0(x) = r(x), \\ u_{n+1}(x) = -L_{xx}^{(-2\alpha)}R_x^{(\alpha)}u_n(x), n \ge 0. \end{cases} \tag{18}$$

Finally, we can find a solution in the form

$$u(x) = \sum_{n=0}^{\infty} u_n(x). \tag{19}$$

Hence, we can obtain that the following condition

$$\left|f(x) - f(x_0)\right| < \varepsilon^\alpha,$$

where fractional dimension $f(x)$ is equal to α for any $x \in (a,b)$.

5. NUMERICAL APPLICATIONS

Example 5.1. Consider the diffusion equation involving local fractional derivative

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}},\ 0 < \alpha \le 1, \tag{20}$$

subject to the initial conditions

$$u(x,0) = \frac{x^\alpha}{\Gamma(1+\alpha)}. \tag{21}$$

Making use of Eq. (18), the recurrence relation reads as

$$u_0(x,t)=u(x,0),$$
$$u_{n+1}(x,t)=L_t^{(-\alpha)}\left\{L_{xx}^{(2\alpha)}u_n(x,t)\right\},\ n\ge 0. \quad (22)$$

The component of the solution can be determine from initial conditions as

$$u_0(x,t)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}, \quad (23)$$

Applying the recursive relation (22) and, we get the following results

$$\begin{aligned} u_1(x,t)&=L_t^{(-\alpha)}\left\{L_{xx}^{(2\alpha)}u_0(x,t)\right\},\\ &=0,\\ u_2(x,t)&=L_t^{(-\alpha)}\left\{L_{xx}^{(2\alpha)}u_1(x,t)\right\},\\ &=0,\\ u_3(x,t)&=L_t^{(-\alpha)}\left\{L_{xx}^{(2\alpha)}u_2(x,t)\right\}\\ &=0,\\ &\vdots \end{aligned}$$

and so on.

Thus the approximate solution of (20) in the form (7) is given by

$$u(x,y)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}.$$

The result is the same as the one which is obtained in [38].

Example 5.2 Consider the following diffusion equation on cantor set

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}=0,\ 0<\alpha\le 1, \quad (24)$$

subject to the initial conditions

$$u(x,0)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \quad (25)$$

According to local fractional Adomain's decomposition method, the recurrence relation reads as

$$u_0(x,t)=u(x,0),$$
$$u_{n+1}(x,t)=L_t^{(-\alpha)}\left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}L_{xx}^{(2\alpha)}u_n(x,t)\right\},\ n\ge 0. \quad (26)$$

Applying the recursive relation (26) and the initial conditions (25), we get the following results

$$u_0(x,t)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \quad (27)$$

$$\begin{aligned} u_1(x,t)&=L_t^{(-\alpha)}\left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}L_{xx}^{(2\alpha)}u_0(x,t)\right\}\\ &=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{\alpha}}{\Gamma(1+\alpha)}, \end{aligned} \quad (28)$$

$$\begin{aligned} u_2(x,t)&=L_t^{(-\alpha)}\left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}L_{xx}^{(2\alpha)}u_1(x,t)\right\}\\ &=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}, \end{aligned} \quad (29)$$

$$\begin{aligned} u_3(x,t)&=L_t^{(-\alpha)}\left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}L_{xx}^{(2\alpha)}u_2(x,t)\right\}\\ &=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}, \end{aligned} \quad (30)$$

$\vdots$,

and so on.

Thus, the final series solution is reads as

$$u(x,t)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(\begin{aligned}&1+\frac{t^{\alpha}}{\Gamma(1+\alpha)}+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\\ &+\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}+\dots\end{aligned}\right),$$

The closed form solution is

$$u(x,t)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}E_{\alpha}(x^{\alpha}). \quad (31)$$

This result is the same as obtained by Yang [38].

Example 5.3 Consider the following wave equation on cantor set

$$\frac{\partial^{2\alpha}u(x,t)}{\partial t^{2\alpha}}-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}=0,\ 0<\alpha\le 1, \quad (32)$$

with the fractal value conditions given by

$$u(x,0)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}. \quad (33)$$

According to local fractional Adomain's decomposition method, the recurrence relation reads as

$$u_0(x,t) = u(x,0) + \frac{t^\alpha}{\Gamma(1+\alpha)} \frac{\partial^\alpha u(x,0)}{\partial x^\alpha},$$

$$u_{n+1}(x,t) = L_{tt}^{(-2\alpha)} \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} L_{xx}^{(2\alpha)} u_n(x,t) \right\}, \; n \geq 0. \quad (34)$$

Applying the recursive relation (34) and the initial conditions (33), we get the following results

$$u_0(x,t) = \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{t^\alpha}{\Gamma(1+\alpha)}, \quad (35)$$

$$u_1(x,t) = L_{tt}^{(-2\alpha)} \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} L_{xx}^{(2\alpha)} u_0(x,t) \right\} = 0, \quad (36)$$

$$u_2(x,t) = L_{tt}^{(-2\alpha)} \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} L_{xx}^{(2\alpha)} u_1(x,t) \right\} = 0, \quad (37)$$

$$u_3(x,t) = L_{tt}^{(-2\alpha)} \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} L_{xx}^{(2\alpha)} u_2(x,t) \right\} = 0, \quad (38)$$

$\vdots$,

and so on.

Finally, we obtained

$$u(x,t) = \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{t^\alpha}{\Gamma(1+\alpha)}. \quad (39)$$

This is the same as obtained by Yang [38].

Example 5.4 Finally, consider the following wave equation on cantor set

$$\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} - c \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0, \; 0 < \alpha \leq 1, \quad (40)$$

with the fractal value conditions given by

$$u(x,0) = E_\alpha(x^\alpha), \quad (41)$$

where c is a constant.

According to local fractional Adomain's decomposition method, the recurrence relation reads as

$$u_0(x,t) = u(x,0) + \frac{t^\alpha}{\Gamma(1+\alpha)} \frac{\partial^\alpha u(x,0)}{\partial x^\alpha},$$

$$u_{n+1}(x,t) = L_{tt}^{(-2\alpha)} \left\{ \frac{x^\alpha}{\Gamma(1+\alpha)} L_{xx}^{(2\alpha)} u_n(x,t) \right\}, \; n \geq 0. \quad (42)$$

Applying the recursive relation (34) and the initial conditions (33), we get the following results

$$u_0(x,t) = E_\alpha(x^\alpha)\left(1 + \frac{t^\alpha}{\Gamma(1+\alpha)}\right), \quad (43)$$

$$u_1(x,t) = L_{tt}^{(-2\alpha)} \left\{ c L_{xx}^{(2\alpha)} u_0(x,t) \right\} = cE_\alpha(x^\alpha)\left(\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right), \quad (44)$$

$$u_2(x,t) = L_{tt}^{(-2\alpha)} \left\{ c L_{xx}^{(2\alpha)} u_1(x,t) \right\} = c^2 E_\alpha(x^\alpha)\left(\frac{t^{4\alpha}}{\Gamma(1+4\alpha)} + \frac{t^{5\alpha}}{\Gamma(1+5\alpha)}\right), \quad (45)$$

$$u_3(x,t) = L_{tt}^{(-2\alpha)} \left\{ c L_{xx}^{(2\alpha)} u_2(x,t) \right\} = c^3 E_\alpha(x^\alpha)\left(\frac{t^{6\alpha}}{\Gamma(1+6\alpha)} + \frac{t^{7\alpha}}{\Gamma(1+7\alpha)}\right), \quad (46)$$

$\vdots$

and so on.

Finally, we obtained

$$u(x,t) = E_\alpha(x^\alpha)\left(\cosh_\alpha(cx^\alpha) + \sinh_\alpha(cx^\alpha)\right). \quad (47)$$

This is the same as obtained by Yang [38].

6. CONCLUSION

In this paper, the non-differentiable solution for the heat and wave equations involving local fractional derivative operators in mathematical physics fractal value conditions are investigated by using the proposed local fractional Adomain's decomposition method. The obtained results demonstrate the reliability of the methodology and its wider applicability to local fractional differential equation arising in mathematical physics, engineering and hence can be extended to other problems of diversified nonlinear nature.

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