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Research Article

Extreme Rainfall Incidents over Sindh Province, against Different Return Periods

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Abstract: High frequency and intensity of rainfall events are the major causes of flooding in Pakistan. Statistical distributions, i.e., Gumbell Maximum (GM) distribution & Generalized Extreme Value distribution (GEV), were used to analyze annual extreme rainfalls in different cities of Sindh (Karachi, Badin, Chor, Rohri). Different return periods of extreme annual rainfall were calculated by using these distributions. This article also indicated that GM distribution better explains the yearly incidence of rainfall in Karachi and Badin while the GEV distribution better explains that in Chor and Rohri. We also estimated 200 mm return level for Sindh province in the next 100-year period.

Keywords: Return period, return level, yearly rainfall, Gumbel Maximun Distribution

1. INTRODUCTION

The enhanced greenhouse effect is causing global climate change at a much faster rate compared to its natural pace. There is an increasing change in ambient temperature and rainfall which results an increase in the frequency and intensity of droughts and floods. Pakistan is now experiencing heavy rainfalls and floods due to global warming. This asks for analyzing the pattern of extreme rainfalls all over Pakistan. This study analyzed the extreme rainfalls and its variability over Sindh province of Pakistan, by selecting Karachi (24.54° N, 67.08° E), Badin (24.38° N, 68.58° E), Chor (26.58° N, 69.47° E) and Rohri (27.40° N, 68.90° E) (Fig. 1). Two approaches have been suggested to analyze extreme rainfall [1, 2]. The first step used a quartile method to evaluate the extreme rainfall [2, 3], while in second step statistical distributions have been used to demonstrate yearly extremes with their return periods [4, 5, 6]. Several literatures also depict that log-Pearson type III

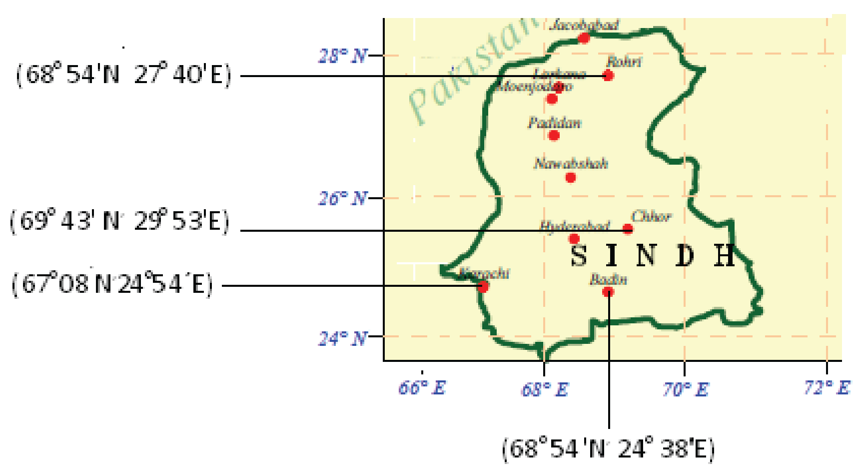

Fig. 1. Geographical location of Rohri, Chor, Karachi & Badin.

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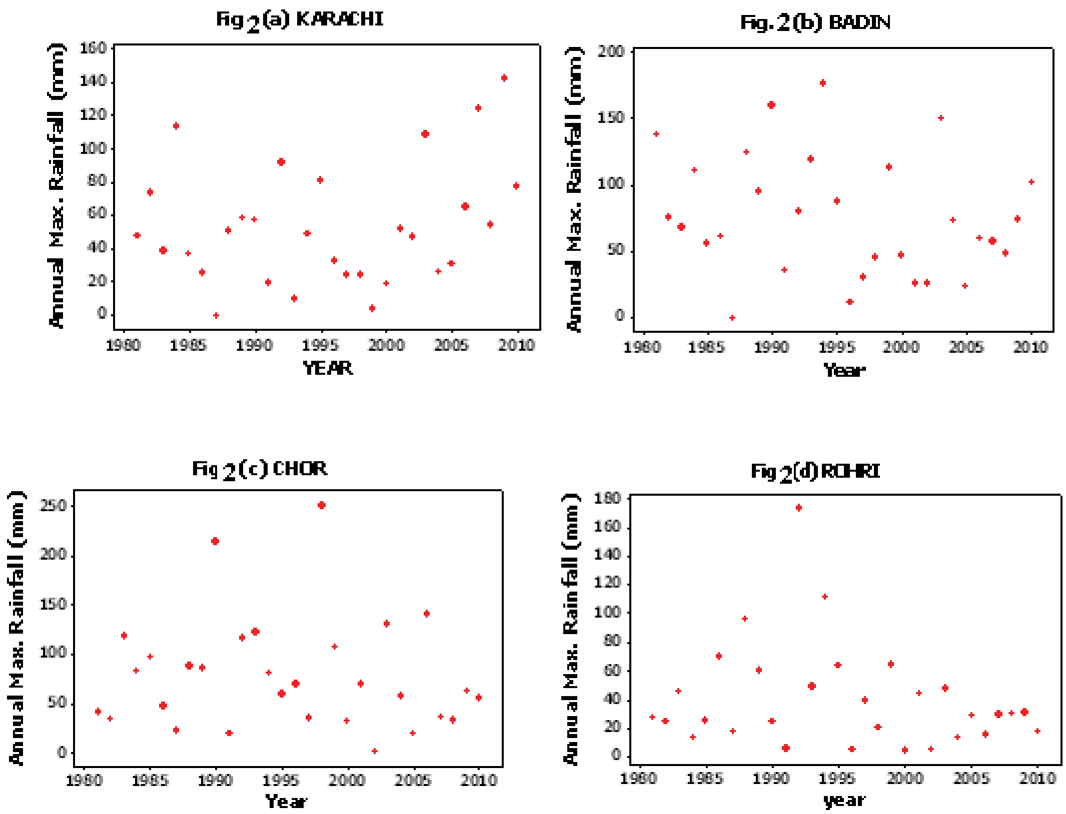

Fig. 2(A). Scatter plots of yearly maximum rainfall.

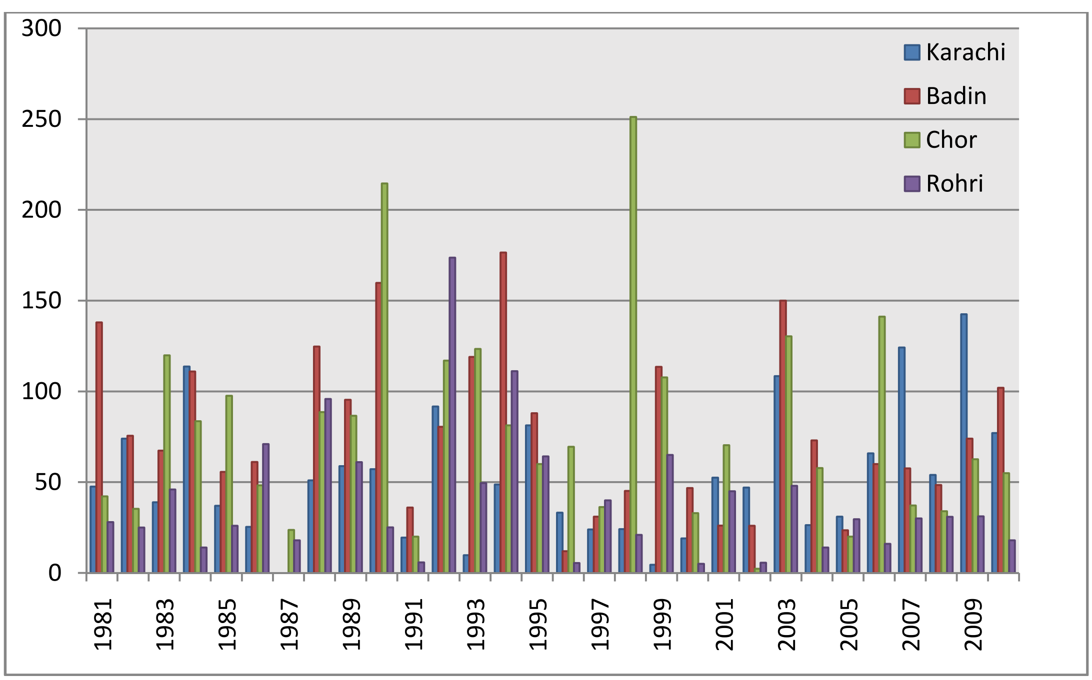

Fig.2(B). Multiple bar diagram of annual extreme rainfall (mm).

analysis, Generalized Extreme Value distribution and Gumbel distribution have been commonly used to examine extreme rainfall events [7]. This study used the second approach to calculate the long return periods and return levels by using the best fit to the extreme value distributions.

2. EXPLORING DAILY RAIN DATA

Daily data from 1981 to the end of 2010, acuired from Pakistan Meteorological Department, has been used for the analysis of annual extreme rainfall over Sindh province. Scatter plots and bar diagram of yearly extreme rainfall values are plotted in Fig. 2(A) and 2(B) respectively. Fig. 2 (A) shows that there is no any observed trend. From Fig.2 (a) we get twelve values more than normal (53 mm) value and yearly maximum rain (142.5 mm) occurred on 19th July 2009, at Karachi station, while it also receives rainfall more than 100 mm i.e. (113.0 mm, 108.4 mm, 124.2 mm and 142.5 mm) at different time intervals, i.e., on 7 Aug, 1984, 29 July 2003,10 Aug 2007 and 19 July 2009, respectively. Fig. 2(b) depicts 12 values greater than normal value (i.e., 76 mm) and Badin receives annual maximum rainfall of 176.5 mm on 3rd Aug 1994, while the other two higher rainfall are above 150 mm per day (i.e., 159.8 mm and 176.5 mm) which occurred on 3 Sep. 1990 and 3 Aug 1994. Fig. 2(c) illustrates the highest frequency of annual rainfall, i.e., it has 13 rainfall incidences of greater than normal value (i.e., 78 mm) and the heaviest rainfall of 251.2 mm was received on 20 Sep 1998; while 2nd, 3rd and 4th highest rainfalls were of 214.6 mm, 141.2 mm and 130.4 mm which occurred on 7 Aug, 1990, 23Aug, 2006 and 26 July, 2003, respectively. Fig. 2 (d) exhibits 11 rainfall incidents of greater than the mean value and the highest rainfall was 173.7 mm on 3 July, 1992, while 2nd, 3rd and 4th highest were 111.2 mm, 95.8 mm and 71 mm which occurred on 3 July 1994, 7 Aug 1988 and 9 Aug 1986. It is obvious that the highest amount of rainfall (251.2 mm per day) occured at Chor, compare to the other stations, during the study period [Fig. 2(B)].

2.1. Extreme Value Distribution

Let R_1, R_2,…,R_n represent daily rainfall within a year. An appropriate model for extremes can be obtained by taking R = max { R_1, R_2,…,R_n }. Where n = 365 and R shows annual maxima. Asymptotic considerations recommend the GEV family [6], who's PDF, is

$$f(R) = \frac{1}{\beta}[1+S]^{1-\frac{1}{k}} \exp\{-[1+S]\}$$

$$1+S > 0 \quad (1)$$

where

$$S = \frac{k(R-\mu)}{\beta} \quad (2)$$

where μ, β and k are Location parameter, scale parameter, and shape parameter, respectively. From analytically integrating Equation (1) we get CDF, i.e.,

$$F(R) = e^{[-(1+S)]^{-\frac{1}{k}}} \quad (3)$$

The inverted CDF gives us the below quintile function.

$$R = F^{-1}(p) = \mu + \frac{\beta}{k}\{[-\ln(P)]^{-k} - 1\} \quad (4)$$

Here F(R) = P shows cumulative probability.

In fitting the GEV distribution, either the method of L-moment is used or maximum likelihood (MLH) method is used [9]. L-moment fitting is favorable for mainly small data samples [8]. The modification of MLH is easy to include additional influences. The special case of GEV distribution which is independent of parameter k, is called Gumbel Distribution which is unbounded. Gumbel found the application of extreme value theory to solve engineering problems and meteorological modeling phenomenon. The PDF of Gumbel distribution is:

$$F(R) = \frac{1}{\beta}e[-e^{-S*} - S*] \quad (5)$$

where

$$S* = \frac{(R-\mu)}{\beta} \quad (6)$$

with location parameter 'μ', scale parameter '**β**' >0 and independent of shape parameters 'k'. Due to its huge utilization to represent the statistics of extremes it is sometimes called "the" extreme value distribution. The Gumbel PDF is rightly skewed and exhibits its maximum at x = μ. CDF of Gumbel distribution is;

$$F(R) = e^{-e^{-S*}} \quad (7)$$

If we invert this CDF, we get quintile function as;

$$R = F^{-1}(p) = \mu - \beta \ln\{-\ln(P)\} \quad (8)$$

The above parameters μ and β can be calculated by using the mean and standard deviation as;

$$\hat{\beta} = \frac{s\sqrt{6}}{\pi} \quad (9)$$

and

$$\mu = \bar{X} - \gamma\hat{\beta}, \quad (10)$$

where γ is Euler's constant and whose value is 0.5771.

2.2. Distribution Fitting

This section compares the GEV and Gumbel Maximum distribution for extreme annual rainfall and investigates that which distribution is the best fitted. Here three methods are used.

1) Histogram with fitted probability density function.
2) Chi square test and
3) Probability probability (p-p) plots.

The histogram plots shown in Fig. 3 depict that more area has been covered by GEV distribution as compared to Gumbel Maximum distribution. Thus GEV provides the best fit for yearly extreme rainfalls in Karachi and Badin while Fig. 3(c) and Fig 3(d) show that Gumbel Max provides the best fit for other two cities of Sindh.

Then Chi square test was applied to get the best fitted distribution for extreme annual rainfalls in Karachi, Badin, Chor, and Rohri; the result is summarized in Table 1. As for as Karachi is concerned, the estimated χ^2 = **0.17932** for Gumbel maximum distribution. Under the negative null hypothesis, the statistic is designed from a χ^2 distribution having degree of freedom ν = 6 − 2 − 1 = 3 for Gumbel maximum distribution. representing the ν = 3 of row chi-squared table, expected χ^2 = 0.17932 is smaller than the 95th percentile value i.e. 7.8, so the null hypothesis would be accepted even at 5% level. Thus, the extreme yearly rainfall of Karachi follows Gumbel maximum distribution. Likewise, Table 1 depicts that calculated chi squared values for remaining stations are also smaller than 95th percentile value of 7.8 with 5% significance level. Therefore, the yearly extreme rainfall of four stations follows the Gumbel maximum distribution.

Table 1 Review of goodness of fit test for yearly maximum rainfall in Karachi, Badin, Chor and Rohri.

	Station	Chi-Square Test	
		Generalized Extreme Value Distribution	Gumbel Maximum Distribution
1	Karachi	1.0522	0.17932
2	Badin	0.58497	0.29399
3	Chor	0.20884	0.3613
4	Rohri	0.31646	3.7228

Next, the best fitted test for GEV distribution for which χ^2 is 5.99 at 5% level, with df = ν = 6 − 3 − 1 = 2. Thus, from Table (1) it is clear that the estimated chi squared values for the above stations are smaller than 95th percentile value 5.99 with 5% level of significance. Hence, the yearly extreme values of rainfall of considered stations have GEV distribution. As, calculated values of chi square for Gumbel Maximum are smaller than that of GEV distribution for Karachi and Badin, thus Gumbel maximum is the best fitted distribution for Karachi and Badin, whereas the calculated values of chi square for GEV distribution are smaller that of Gumbel maximum distribution for Chor and Rohri. Therefore GEV distribution is the best fitted for Chor and Rohri.

In order to display the goodness of fit test results p-p Plots are drawn for the yearly maximum rainfall of above cities in Fig. 4. Here we check only the deviation of observed data points from theoretical values. Fig. 4 indicates more deviation in GEV distribution while it is lesser in Gumbel Maximum distribution for Badin and Karachi, and vice versa for Chor and Rohri. So Gumbel Max. is best fitted for Badin and Karachi while GEV is best for Chor and Rohri. Hence, the p-p plots supported the above chi-squared test.

4. RESULTS AND DISCUSSION

This section calculates the annual return period of extreme rainfall of four mentioned cities of Sindh province. The average return period is calculated by the formula:

$$\text{Return } (R) = \frac{1}{\omega[1 - F(R)]} \quad (11)$$

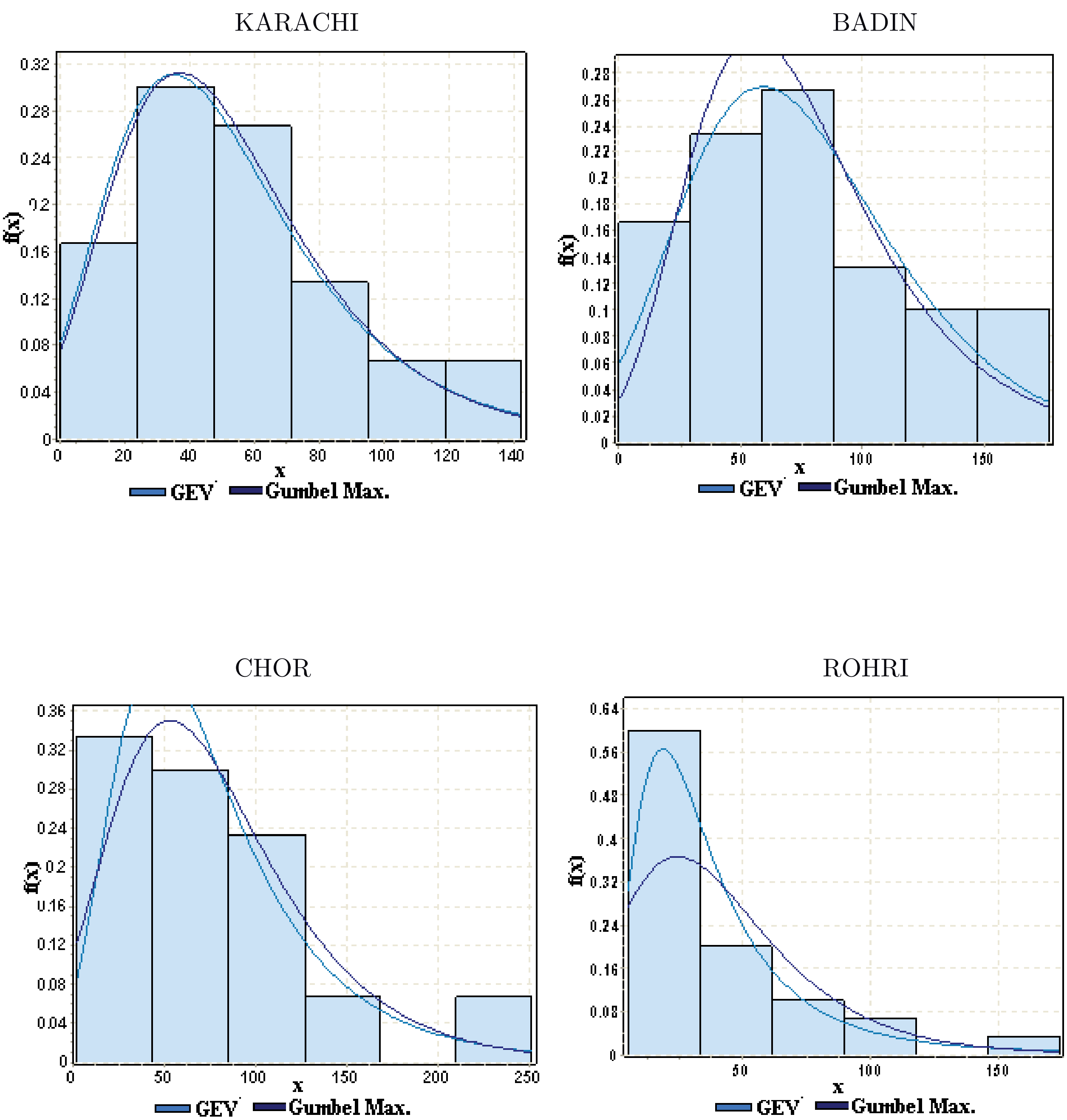

Fig. 3 Comparison of pdf for GEV and Gumbel max. distribution.

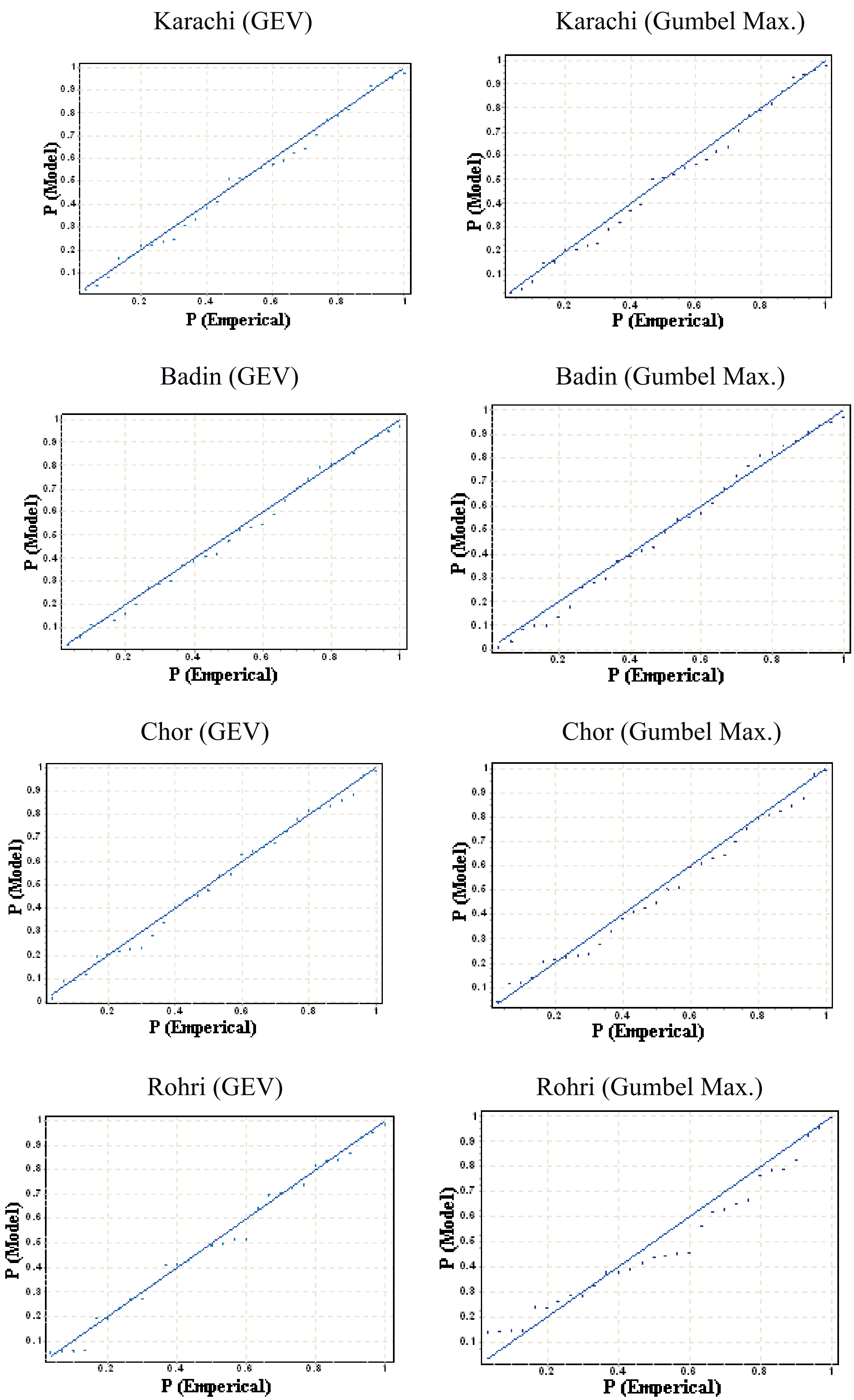

Fig. 4 P-P plot of GEV & Gumbel maximum distribution of four major cities of Sindh.

Table 2 Return levels against given return periods.

Return period (years)	Karachi (mm)	Badin (mm)	Chor (mm)	Rohri (mm)	Average (mm)
2	47.06	68.48	65.27	30.17	52.75
5	78.73	108.55	113.4	57.91	81.13
10	99.7	135.08	149.4	81.55	103.04
20	119.81	160.53	187.39	109.21	130.33
30	134.37	178.96	217.21	132.89	155.05
50	145.85	193.48	242.16	154.04	174.87
100	165.36	218.16	287.77	195.88	200.34

This Eq. represents that the Return (R) is a function of the Cumulative Distribution Function calculated at R and the average sampling frequency ω, for yearly maximum value ω = 1/year.

4.1. Return (R) for Karachi

Our calculation reveals that the heaviest rainfall during the period 1981 to 2010, i.e., 142.5 mm, occurred on 19th July, 2009. Obviously, it is desirable to know that when such a high rainfall or even a more heavy rainfall can reoccur in Karachi. So by using Gumbel Distribution (being the best distribution), we calculated the parameter values as μ = 36.814 and β = 27.943.

The cumulative probability is computing using Eq. (6):

$$S^* = \frac{(R-\mu)}{\beta}$$

$$S^* = 3.782,$$

and using Eq. (7)

$$F(R) = P(R \leq 142.5) = e^{-e^{-S^*}}$$

$$F(R) = 0.98$$

Now Eq. (11) gives

$$\text{Return } (R) = \frac{1}{\omega[1 - F(R)]}$$

Return (R) = 44 years

This implies that, it will take 44 years to reoccur the rainfall equal to 142.5 mm or more per day. Similarly return period for Chor and Rohri was calculated by using GEV.

4.2. Return Levels against Given Return Periods

This section provides the calculation of return level for T = 2 years, with p = 0.5 and F(X) = P = 1 – p = 0.5. From Eq. (8), we have 47.06 mm which reveals that in the next two years Karachi may receive 47.06 mm rain approximately. Different return levels 78.73 mm, 99.7 mm, 119.81 mm, 134.37 mm, 145.85 mm and 165.36 mm for Karachi are calculated respectively for return periods, 5, 10, 20, 30, 50 and 100 years.

Hence, return levels are determined against different return period for Karachi and Badin by using Gumbel distribution (Eq. 6, 7 and 8) and for Chor and Rohri by using GEV (Eq. 2, 3 and 4). The calculations are summarized in Table 2. Our results demonstrate that above cities of Sindh have100 mm daily rainfall against the return period of 20-year and 140 mm rainfall against 50-year, suggesting the necessity of appropriate flood forecasting and for acquiring the requisite improvements in river structure in the country.

5. CONCLUSIONS

This study employed probability distributions for the calculation of return periods of yearly rainfall in four major cities of Sindh province of Pakistan, viz. Karachi, Badin, Chor and Rohri. Daily data of rainfall of these cities have been employed from firs Jan 1981 to31 Dec 2010. Analyses of time series of these cities shows no obvious trend.

Our calculations explain that Gumbel maximum distribution provides the best fit for coastal areas of the country, like Karachi and Badin, while the generalized extreme value (GEV) depicts the actual pattern of rainfall in Chor and

Rohri. Our results also revealed that the considered cities of Sindh will have 130 mm return levels against 20-year return period, while rainfall of 170 mm or more per day is expected in the next 50-year return period. So it suggests for upgrading the flood forecasting system, by using modern technology like GIS, improving the river flows, and constructing new dams in Pakistan.

6. ACKNOWLEDGEMENTS

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