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THE MECHANICS FOR PREDICTING DRAFT OF TILLAGE EQUIPMENT

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The importance of mechanics in tillage has been known for many years. In this study, an effort was made to develop the mechanics for predicting the draft of tillage equipment. The prediction equation thus developed was tested by experimentally measuring the draft forces under different soil and operating conditions. The percentage differences between the measured and predicted values of draft varied from 0.112% to 113.5%, with a maximum mean value of 21% of the measured values, over the range of data studied.

INTRODUCTION

The problems related to the design, construction and operation of tillage machinery, involve the use of mechanics which provide a method for describing the application of forces to the soil and its consequent reaction. The importance of developing the accurate mechanics for tillage implements has, therefore, been greatly recognized by engineers, as it provides a means by which the forces applied to the equipment can be predicted and controlled by their design and construction.

In this study, an effort was made to develop and test the mechanics of tillage equipment under different soil and operating conditions.

MATERIALS AND METHODS

(a) *Development of mechanics.* A tillage tool such as a plow shown in Figure 1a, when operated and fixed in soil resembles a retaining wall. The theory of retaining walls was, therefore, used in order to develop an equation for the draft of a tillage tool. Before developing the equation, the inertia force associated with the tillage tool was first evaluated.

Inertia force. In order to derive the equation for the inertia force (Q) associated with the inclined plane tillage tool, theorems of conservation of mass and momentum applied to matter within a control volume were utilized.

As the tillage tool moves forward with velocity V, the soil mass undergoes a continuous movement attaining a velocity V_e at the tool interface as shown

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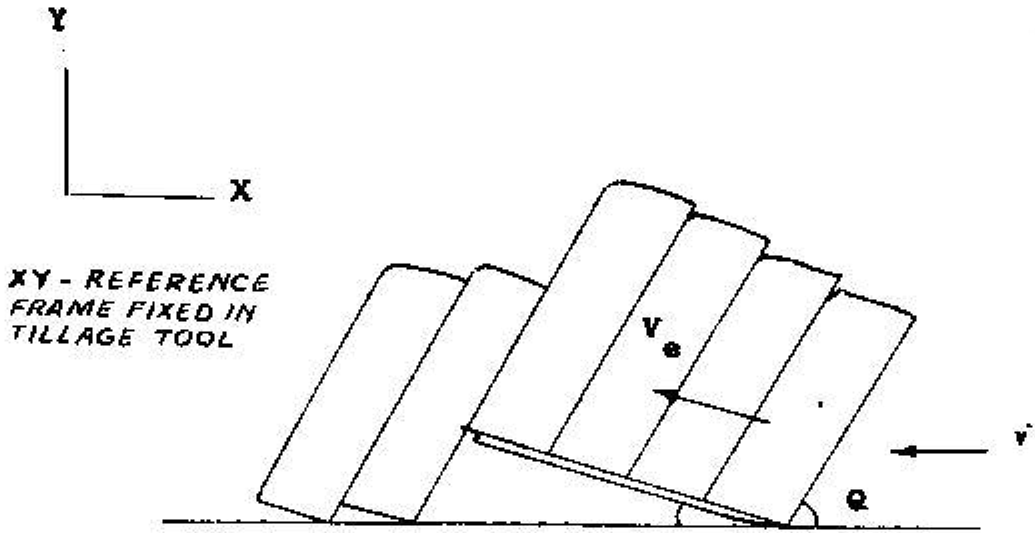


Fig. 1—a. Soil reacting to tillage tool.

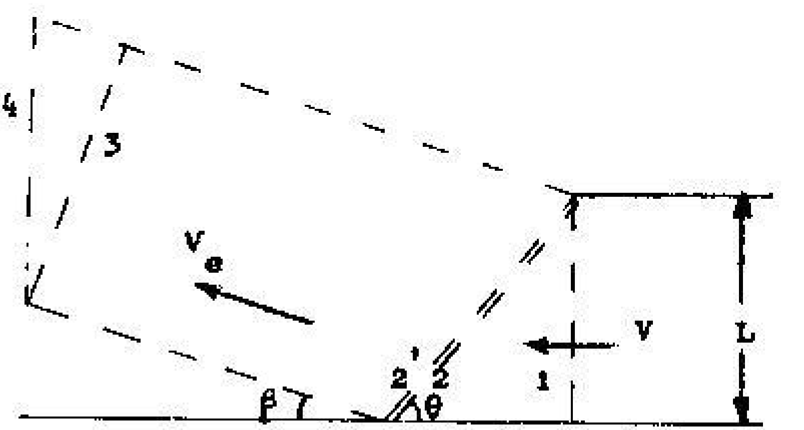


Fig. 1—b. Control surface of soil bounded by regions 1 and 4.

by Figure 1 a. Consider a control volume of soil bounded by regions 1 and 4, as represented by Figure 1 b.

According to the law of conservation of mass, for a control volume

$$\frac{\delta}{\delta t} \int_{c.v.} dm + \int_{c.s.} \rho \; \overline{V_r} . \overline{dA}^{*} = 0$$

Where ;

c.v. designates the control volume and c.s. the control surface

dm = elemental mass of soil inside the control volume

ρ = mass density of soil at a point inside the control volume

$\overline{V_r}$ = velocity of matter at a point on the control surface relative to the control surface

$\overline{dA}$ = element of area of control surface = vector $\hat{n}dA$, where $\hat{n}$ is the outwardly drawn unit vector

Since we have steady flow of soil, $\int_{c.v.} dm$ is constant and $\frac{\delta}{\delta t}\int_{c.v.} dm$ is zero, giving

$$\int_{c.s.} \rho \; \overline{V_r} . \overline{dA} = 0$$

For constant density of continuum of matter,

$$\int \overline{V_r} . \overline{dA} = 0$$

Considering a control surface in which area 1 is entrance and area 2 is exit,

$$\int_{c.s.} \overline{V_r} . \overline{dA} = \int_1 \overline{V_r} . \overline{dA} + \int_2 \overline{V_r} . \overline{dA} = V(-DL) + V \sin\theta \, A_2 = 0$$

$$A_2 = \frac{DL}{\sin\theta} \qquad \ldots \qquad (1)$$

Where :

D = width of tool, L = depth of soil disturbed

Considering a control surface in which area 2 is entrance and 2′ is exit

$$\int_{c.s.} \overline{V_r} . \overline{dA} = \int_2 \overline{V_r} . \overline{dA} + \int_{2'} \overline{V_r} . \overline{dA} = V \sin\theta \left(\frac{-DL}{\sin\theta}\right) + V_* \sin(\beta+\theta) A'_2 = 0$$

$$A'_2 = \frac{V D L}{V_* \sin(\beta + \theta)} \qquad \ldots \qquad (2)$$

* $\overline{dA}$ This denotes a Vector

Since $A_2 = A'_2$

$$\frac{D\,L}{\sin\theta} = \frac{V\,D\,L}{V_e \sin(\beta+\theta)}$$

$$\frac{V}{V_e} = \frac{\sin(\beta+\theta)}{\sin\theta}$$

$$V_e = \frac{V \sin\theta}{\sin(\beta+\theta)}$$

Considering a control surface in which area 2' is entrance and area 3 is exit

$$\int_{c.s.} \overline{V_r}\cdot\overline{dA} = \int_{2'} \overline{V_r}\cdot\overline{dA} + \int_{3} \overline{V_r}\cdot\overline{dA} = V_e \sin(\beta+\theta)\left\{\frac{-V\,D\,L}{V_e \sin(\beta+\theta)}\right\} + V_e\ A_3 = O$$

$$A_3 = \frac{V\,D\,L}{V_e} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Considering a control surface in which area 3 is entrance and area 4 is exit

$$\int_{c.s.} \overline{V_r}\cdot\overline{dA} = \int_{3} \overline{V_r}\cdot\overline{dA} + \int_{4} \overline{V_r}\cdot\overline{dA} = V_e\left(\frac{-V\,D\,L}{V_e}\right) + V_e\ (\cos\beta)\ A_4 = O$$

$$A_4 = \frac{V\,D\,L}{V_e \cos\beta} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

From the theorem of momentum, the net force ΣF on matter within control volume is given by Newton's law as

$$\Sigma F = \frac{\partial}{\partial t}\int_{c.v.} \overline{V}\,dm + \int_{c.s.} \overline{V}\,\rho\,V_r\cdot\overline{dA}$$

For steady state condition, $\frac{\partial}{\partial t}\int_{c.v.} \overline{V}\,dm = O$

$$\Sigma F = \int_{c.s.} \overline{V}\,\rho\,\overline{V_r}\cdot\overline{dA}$$

The Equation for $\overline{\Sigma F}$ can be expressed by its following two components :

$$\Sigma F_x = \int_{c.s.} V_x\ \rho\,\overline{V_r}\cdot\overline{dA}$$

$$\Sigma F_y = \int_{c.s.} V_y\ \rho\,\overline{V_r}\cdot\overline{dA}$$

Considering control surface in which area 2 is entrance and area 4 is exit

$$\Sigma F_x = \int_{c.s.} V_x \; \rho \; \overline{V_r} \cdot \overline{dA} = \int_2 V_x \; \rho \; \overline{V_r} \cdot \overline{dA} + \int_4 V_x \; \rho \; \overline{V_r} \cdot \overline{dA}$$

$$= - V \rho \sin\theta \left(\frac{-DL}{\sin\theta}\right) + (-V_e \cos\beta)\rho(V_e \cos\beta)\left(\frac{V D L}{V_e \cos\beta}\right)$$

$$= \rho DLV^2 - (V V_e \cos\beta)\rho DL$$

$$= \rho DLV^2 - V\left[\frac{V \sin\theta}{\sin(\beta+\theta)}\right]\cos\beta\,(PDL)$$

$$= \rho DLV^2 - \rho DL \frac{V^2 \sin\theta\cos\beta}{\sin(\beta+\theta)}$$

$$= \rho DLV^2 \left\{1 - \frac{\sin\theta\cos\beta}{\sin(\beta+\theta)}\right\}$$

$$= \rho DLV^2 \left(\frac{\sin(\beta+\;) - \sin\theta\cos\beta}{\sin(\beta+\theta)}\right)$$

$$= \frac{\rho DLV^2 \sin\beta\cos\theta}{\sin(\beta+\theta)}$$

$$\Sigma F_y = \int V_y \; \rho \; \overline{V_r} \cdot \overline{dA} = \int V_y \; \rho \; \overline{V_r} \cdot \overline{dA} + \int V_y \; \rho \; \overline{V_r} \cdot \overline{dA}$$

$$= 0 + (V_e \sin\beta)\rho(V_e \cos\beta)\left(\frac{V D L}{V_e \cos\beta}\right)$$

$$= \rho DLV V_e \sin\beta = \rho DLV \frac{V \sin\theta\sin\beta}{\sin(\beta+\theta)}$$

$$= \rho DLV^2 \frac{\sin\theta\sin\beta}{\sin(\beta+\theta)}$$

$$|\overline{\Sigma F}| = (\Sigma F_x^2 + \Sigma F_y^2)^{\frac{1}{2}}$$

$$= \rho DLV^2 \left[\frac{\sin^2\beta\cos^2\theta}{\sin^2(\beta+\theta} + \frac{\sin^2\theta\sin^2\beta}{\sin^2(\beta+\theta)}\right]^{\frac{1}{2}}$$

$$= \frac{\rho DLV^2 \sin\beta}{\sin(\beta\theta)}\left[\cos^2\theta + \sin^2\theta\right]^{\frac{1}{2}}$$

$$= \frac{\rho DLV^2 \sin\beta}{\sin(\beta+\theta)}$$

$$= \frac{w}{g} DLV^2 \frac{\sin\beta}{\sin(\beta+\theta)}$$

Where :

w = unit weight of soil

g = acceleration of gravity

The inertia force Q is, therefore, given by

$$Q = |\Sigma \overline{F}| = \frac{w}{g} DLV^2 \frac{\sin \beta}{\sin(\beta + \theta)} \quad \ldots \quad \ldots \quad (5)$$

The angle made by Q with the x-axis $= \tan^{-1} \left(\frac{\Sigma F_y}{\Sigma F_x} \right)$

$$= \tan^{-1}(\tan \theta) = \theta$$

The x and y components of Q are given as follows :

$$Q_x = Q \cos \theta$$

$$Q_y = Q \sin \theta$$

Draft.

The equation for draft will be derived by considering a free body diagram (Figure 2 b) for the forces acting on soil mass in front of an inclined tool shown in Figure 2 a. Angle θ equal to $45° - \varnothing/2$ shown in Figure 2 b gives the orientation of the shear surface resulting from the passive failure of soil.

According to Newton's Law, we have

$$\Sigma F_y = \frac{\delta}{\delta t} \int_{c.v.} V_y \, dm + \int_{c.s.} V_y \, \rho \, \overline{V_r} \cdot \overline{dA}$$

or $W_y + P_y + F_y + (cA)_y = Q_y$

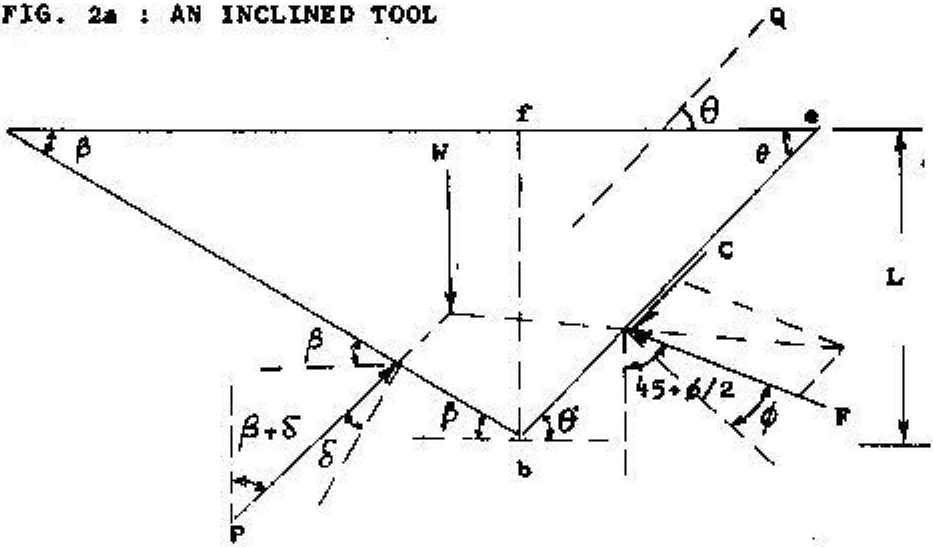
$$- W - cA \sin \theta + F \cos(45° + \varnothing/2) + P \cos(\beta + \delta) = Q \sin \theta$$

$$- W - (c\,D\,L \operatorname{cosec} \theta) \sin \theta + F \cos(45° + \varnothing/2) + P \cos(\beta\ \delta) = Q \sin \theta$$

$$W + c\,D\,L - F \cos(45° + \varnothing/2) - P \cos(\beta + \delta)$$

$$+ Q \sin(45° - \varnothing/2) = 0 \quad \ldots \quad (6)$$

FIG. 2a : AN INCLINED TOOL



$$\theta = 45^\circ - \phi/2$$

Where :

W = weight of soil mass shown in Figure 2 b

c = cohesion of soil

A = area of shear surface be = D L cosec θ

D = transverse width of tool

L = depth of soil disturbed

Q = inertia force given by Equation 5

∅ = angle of internal friction of soil

θ = angle of shear surface = 45° − ∅/2

β = angle of inclination of tool

δ = angle of soil-metal friction

$$\Sigma F_x = \frac{\partial}{\partial t}\int V_x \; dm + \int V_x \; \rho \; \overline{V_r} \; \overline{dA}$$

$$\text{or } W_x + P_x + F_x + (cA)_x = Q_x$$

$$P \sin(\beta + \delta) - F \sin(45^\circ + \varnothing/2) - cA \cos\theta = Q \cos\theta$$

$$P \sin(\beta + \delta) - F \sin(45^\circ + \varnothing/2) - c\,D\,L \operatorname{cosec}\theta \cos = Q \cos\theta$$

$$P \sin(\beta + \delta) - F \sin(45^\circ + \varnothing/2) - c\,D\,L \cot\theta = Q \cos\theta$$

$$P \sin(\beta + \delta) - F \sin(45^\circ + \varnothing/2) - c\,D\,L \cot(45^\circ - \varnothing/2) - Q \cos(45^\circ - \varnothing/2) = 0 \quad \ldots (7)$$

From Equation 6, $F = \dfrac{W + CDL - P\cos(\beta + \delta) + Q \sin(45^\circ \; \varnothing/2)}{\cos(45^\circ + \varnothing/2)}$

Substituting the value of F in Equation 7,

$$P \sin(\beta + \delta) - [W + c\,D\,L - P\cos(\beta + \delta) + Q \sin(45^\circ - \varnothing/2)] [\tan(45^\circ + \varnothing/2)] - Q \cos(45^\circ - \varnothing/2) - c\,D\,L \cot(45^\circ - \varnothing/2) = 0$$

$$P [\sin(\beta + \delta) + \cos(\beta + \delta) \tan(45^\circ + \varnothing/2)] = c\,D\,L \{\tan(45^\circ + \varnothing/2) + \cot(45^\circ - \varnothing/2)\} + Q \sin(45^\circ - \varnothing/2) \tan(45^\circ + \varnothing/2) + Q \cos(45^\circ - \varnothing/2) + W \tan(45^\circ + \varnothing/2)$$

$$P = \frac{1}{\sin(\beta + \delta) + \cos(\beta + \delta)\tan(45^\circ + \varnothing/2)} \Big\{ c\,D\,L [\tan(45^\circ + \varnothing/2) + \cot(45^\circ - \varnothing/2)] + Q [\sin(45^\circ - \varnothing/2) \tan(45^\circ + \varnothing/2) + \cos(45^\circ - \varnothing/2)] + W \tan(45^\circ + \varnothing/2) \Big\} \quad \ldots \quad \ldots \quad (8)$$

Where : Q is given by Equation 2.

The value of W is determined from the weight of soil mass enclosed by area abe having a transverse width D.

$$W = w\,D\,(\text{area of triangle abf} + \text{area of triangle bfe})$$

$$= w\,D\,[\tfrac{1}{2}\,L\,L \cot\beta + \tfrac{1}{2}\,L\,L \cot(45^\circ - \varnothing/2)]$$

$$W = \frac{W\,D\,L^2}{2} [\cot\beta + \cot(45^\circ - \varnothing/2)] \quad \ldots \quad \ldots \quad (9)$$

Where :

w = weight of soil per unit volume

D = width of tool

L = depth of tool

Draft = horizontal component of P = P sin $(\beta + \delta)$ ... ... (10)

where : P is given by Equation 8.

(b) *Measurement and prediction of draft.* The data on measured draft for tillage equipment was obtained for testing the mechanics for tillage equipment was obtained for testing the mechanics for tillage equipment. The measured draft was compared with the value predicted from Equation 10.

Figure 3 makes a graphical comparison of some of the results on the measured and calculated (predicted) drafts for a three-inch moldboard plow. The values of drafts correspond to the tool velocity of about 0.88 ft/Sec. Since the range of soil density in the experiments was limited, a variable $\bar{\gamma}$ = w-62.4 (where w = unit weight of soil, 62.4 = unit weight of water, and w—62.4 = submerged unit weight of soil) was selected so as draw the data close to the origin and facilitate its regression analysis. The effect of density on the measured and calculated (predicted) drafts may be observed from the figure.

Figure 4 shows a comparison of the drafts for a four-inch moldboard plow at different velocities on clay loam soil having a density of 87 lb/cft.

The differences between the measured and calculated (predicted) drafts for different types of plows as determined from the data considered in this study are given in Table 1.

TABLE 1. *Differences between measured and calculated drafts for different types of plows.*

Plow	Difference (%) From	Difference (%) To	Mean difference (%)	Mean absolute difference (lb)
1. 4-inch, 25 (moldboard)	0.46	40.62	— 2.2	1.13
2. 3-inch, 20 (moldboard)	1.28	41.04	21.22	14.75
3. 4-inch, 20 (moldboard)	1.47	46.00	3.62	19.68
4. 1.375 inch (disk)	0.11	113.6	—20.95	0.96

The differences shown in Table 1 may be referred to the following :

(i) Two-dimensional rather than three-dimensional stress approach was undertaken while developing the prediction equation.

(ii) The prediction equation was developed from the considerations of steady and incompressible flow of soil. The condition of soil is, however, presumed to vary with the operation of the tillage tool.

(iii) The soil strength paremeters were not determined under the same stress conditions as occurring in front of the tool.

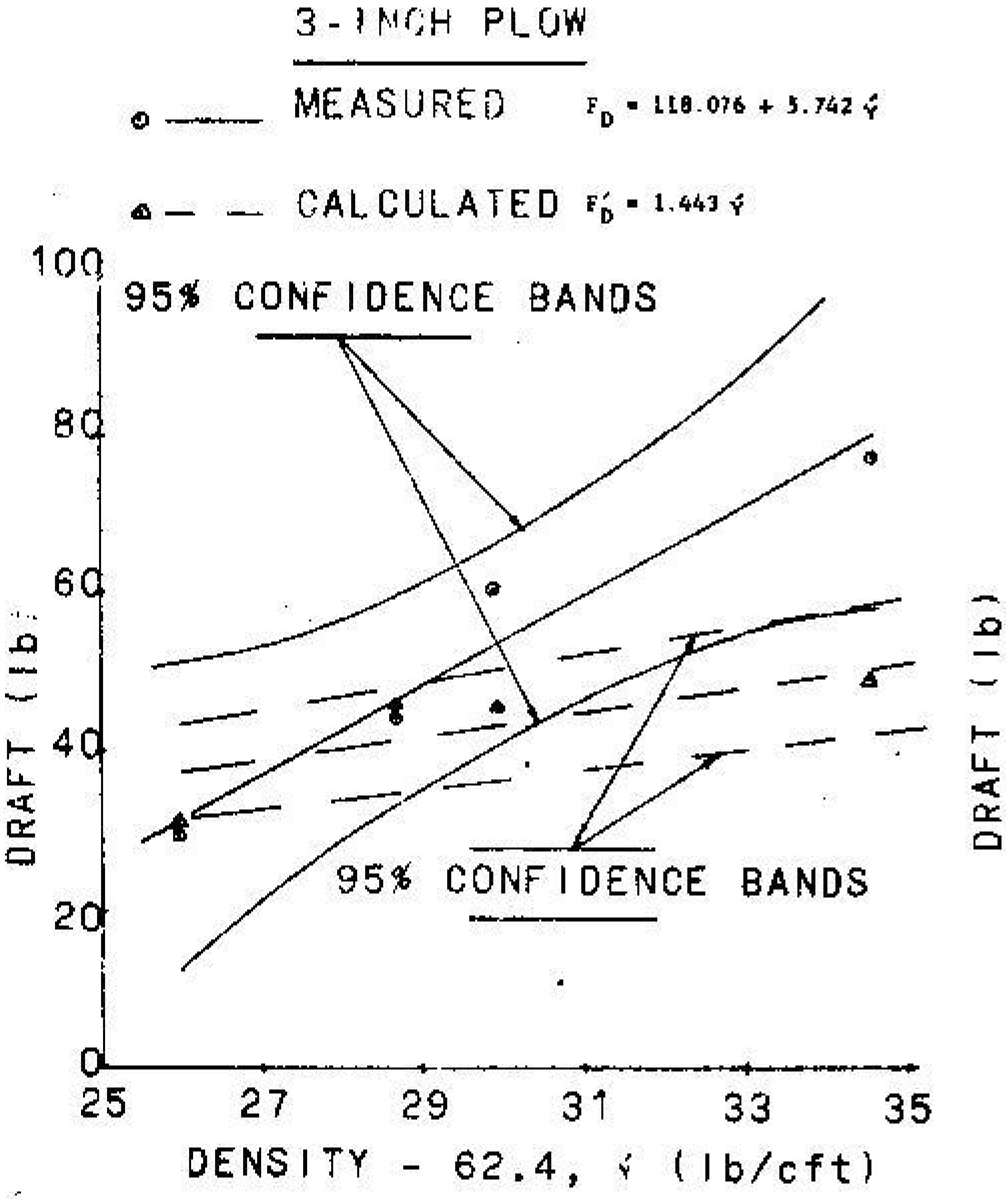


Figure 3. Comparison of measured and calculated drafts for three-inch plow

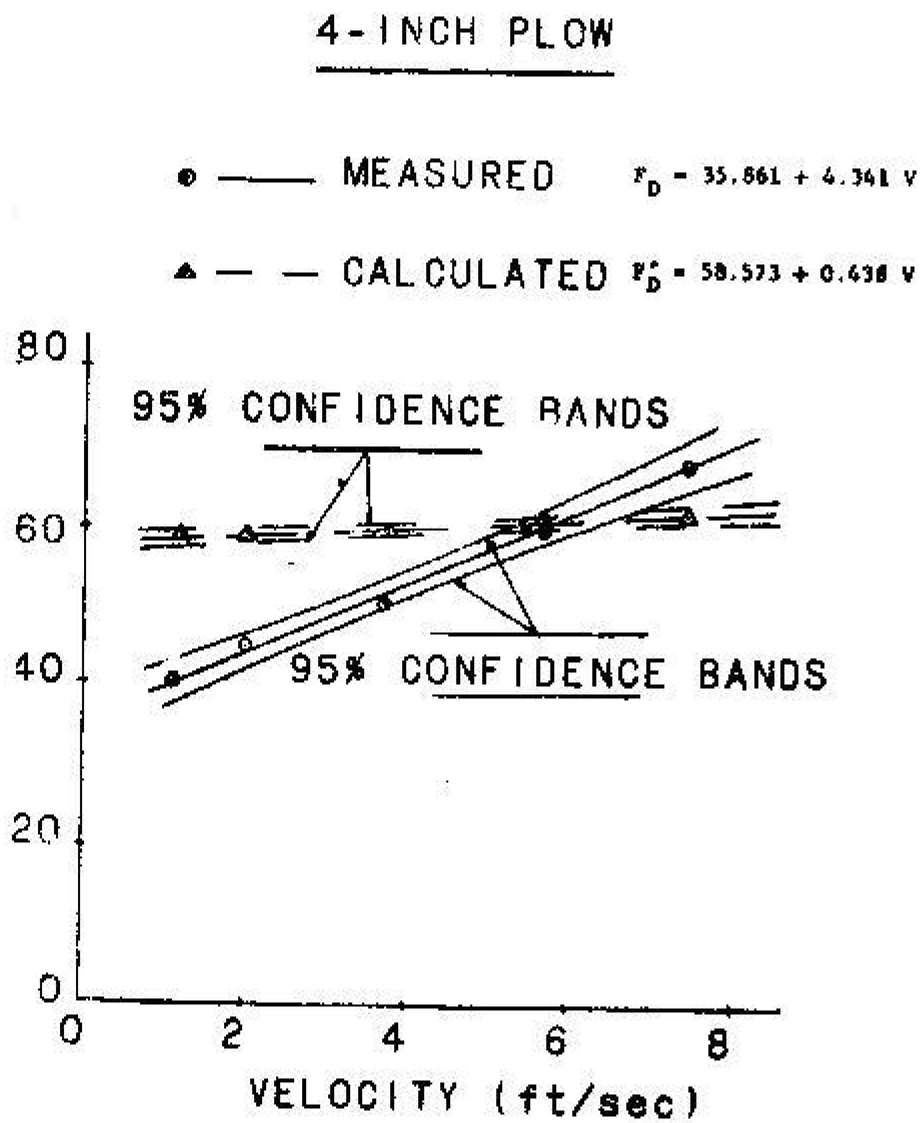


Figure 4 Comparison of measured and calculated drafts for four-inch plow

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