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Novel Concepts of Soft Multi Rough Sets with MCGDM for the Selection of Humanoid Robot

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Abstract. This paper comes out with a fascinating fusion of soft sets, multisets and rough sets. We introduce novel concepts of soft multi rough sets (SMR-Sets) and soft multi approximation spaces. We present some fundamental properties of SMR-approximations and their related examples. We also discuss the variation between some properties of Pawlak approximation space, soft approximation spaces and the same properties of soft multi approximation spaces. Furthermore, we present two different algorithms based on soft multi rough set with an application to multi-criteria group decision-making (MCGDM) for the selection of humanoid robot.

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1. Introduction

The foundation of modern mathematics is thought of as having two pillars: mathematical logic and set theory. Mathematical logic and set theory indeed make up the language spanning in almost all fields of mathematics. In fact the rapid development of science has led to an urgent need for the development of modern sets theoretic mathematical modeling. "Keeping in view the uncertainty element Zadeh [57], in 1965, floated the idea of fuzzy sets where a membership degree is assigned to each member of the universe of discourse. In 1983, Atanassov [6, 7] introduced a newfangled sort of sets titled intuitionistic fuzzy sets (IFS)– a set which is written off as by two mappings communicating the degree of association and the degree of non-association of members of the universe to the IFS. In 1999, Molodtsov [27] proposed soft set theory as a novel mathematical model to deal with uncertainty in the real world problems. Pawlak [32] presented rough set theory to overcome the uncertainty and vagueness appears in the input data. Pawlak and Skowron presented some significant results on rough set and its extension [33].

Many researcher including Akram *et al.* [1]-[3], Ali [4]-[5], Çağman *et al.* [13], Feng *et al.* [14]-[17], Garg *et al.* [18]-[19], Hashmi *et al.* [20]-[21], Hayat *et al.* [22], Karaaslan *et al.* [24]-[25], Kryskiewicz [26], Maji *et al.* [28]-[29], Khalid *et al.* [30]-[31], Riaz *et al.* [35]-[37], Riaz and Hashmi [39]-[40], Xueling *et al.* [52], and Yager [53]-[56] have contributed their work in the development of the theories of fuzzy sets, soft sets, and rough sets. These theories are independently generalizations of the crisp set theory.

Riaz and Naeem [38] introduced the idea of measurable soft mapping. Recently, Riaz and Hashmi [41] introduced the notion of linear Diophantine fuzzy Set (LDFS) and its applications towards MCDM problem. Riaz and Hashmi [42] introduced novel concepts of soft rough Pythagorean m-Polar fuzzy sets and Pythagorean m-polar fuzzy soft rough sets with application to decision-making. Riaz and Tehrim [43]-[47] established the idea of bipolar fuzzy soft mappings, bipolar fuzzy soft topology, bipolar neutrosophic soft topology, cubic bipolar fuzzy set and cubic bipolar fuzzy ordered weighted geometric aggregation operators and their application using internal and external cubic bipolar fuzzy data.

Earlier in 1989, Blizard [10, 11] discovered the concept of multiset theory. This theory is also generalization of the crisp set theory. In 2001, Syropoulos [49] defined various operations on multiset. In 2009, Herawan and Mustafa [23] introduced the concept of multi soft set (MS-set) for showing multi valued information system. In 2013, Babitha and John [8] presented the idea of soft multiset. As a broad view of multiset, Yager [53] introduced the notion of fuzzy multiset (FMS) in which a member of a fuzzy multiset can appear a finite number of times which may have same or different membership values. In 2012, Shinoj and John [48] introduced a new concept of intuitionistic fuzzy multiset and used this idea in medical diagnosis. Calude *et al.* [12] worked on multiset processing in 2001. Bakier *et al.* [9] introduced the notion of soft rough topology with application to the medical diagnosis. In 2012, Thivagar *et al.* [50] introduced a modern topology in medical events. In

1998, Kryskiewicz [26] initiate rough set approach to incomplete information systems. In 2016, Wang *et al.* [51] presented properties of multi-granularity soft rough sets. In 2017, Pi-Yu Li *et al.* [34] presented some results On multi-soft rough sets. Zhan *et al.* [58]-[61] introduced certain concepts of soft rough hemirings, Z-soft fuzzy rough set model, soft rough covering, intuitionistic fuzzy rough sets and their multi-criteria group decision making (MCGDM). In 2014, Zhang and Xu *et al.* [62] established an extension of TOPSIS in multiple criteria decision making (MCDM) by means of Pythagorean fuzzy sets. Zhang *et al.* [63] establish covering-based generalized IF rough sets with their applications to multi-attribute decision-making (MADM).

The goal of this analysis is to introduce soft multi rough set (SMR-set). The SMR-set is suitable to find roughness of multi universe with parameters. In most of the real world problems including multi universe we cannot deal with roughness of parameterized multi data by using soft rough set. That is why soft multi rough set (SMR-set) is most suitable model to find the roughness of parameterized multi data." We discuss application of soft multi rough sets in multi-criteria group decision-making (MCGDM) in the artificial intelligence of humanoid robots.

In section 2, we introduce some basic ideas, which helps us to develop SMR-set theory. In section 3, we combine soft multi set and rough set to find roughness of soft multi set, which gives rise to new concepts of soft multi approximation spaces, soft multi rough approximations and soft multi rough sets. We find some fundamental properties of SMR-approximations and introduce several related concepts. SMR-set infers a good solution to a problem with multiple circumstances and outcomes by approximating the parameterized data. Section 4, implies the novel algorithms based on SMR-set and gives a numerical example of decision-marking in the content of artificial intelligence. The validity of the proposed approach is checked by applying two different algorithms yielding the same result. In end we compare both algorithms. We conclude our work in section 5.

2. Preliminaries

This section include some basic definitions like soft set, multiset, power multiset, power whole multiset, soft rough set, operations of multiset, soft multiset (SM-set), operations of SM-set, and rough set.

Definition 2.1. [27] "Let $\mathcal{J}$ be the universal set. Let $\mathcal{I}(\mathcal{J})$ be the collection of all subsets of $\mathcal{J}$. The pair $(\Gamma, \mathcal{L})$ is said to be a soft set over the initial universe $\mathcal{J}$, Here $\mathcal{L} \subseteq \mathcal{E}$ and $\Gamma : \mathcal{L} \to \mathcal{I}(\mathcal{J})$ is a set-valued mapping. We denote soft set as $(\Gamma, \mathcal{L})$ or $\Gamma_{\mathcal{L}}$ and mathematically we write it as

$$\Gamma_{\mathcal{L}} = \{(l, \Gamma(l)) : l \in \mathcal{L}, \Gamma(l) \in \mathcal{I}(\mathcal{J})\}.$$

For any $l \in \mathcal{L}$, $\Gamma(l)$ is l-approximate elements of soft set $\Gamma_{\mathcal{L}}$. The set containing all soft sets over $\mathcal{J}$ is denoted by $S(\mathcal{J})$".

Example 2.2. Consider $\mathcal{J} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the set containing of six different Light Bulbs and set of features is given by $\mathcal{L} = \{l_1, l_2, l_3, l_4, l_5\} = \mathcal{E}$, where

$$\begin{aligned} l_1 =& \text{ reasonable price,} \\ l_2 =& \text{ emit a small amount of UV rays,} \\ l_3 =& \text{ durable,} \\ l_4 =& \text{ best choice for eye health,} \\ l_5 =& \text{ less electricity consuming.} \end{aligned}$$

The soft set $\Gamma_{\mathcal{L}}$ expresses the "quality of Light Bulbs" that Mr. Zain want to buy. Consider a mapping $\Gamma : \mathcal{L} \to \mathcal{I}(\mathcal{J})$ such that

$$\begin{aligned} \Gamma(l_1) &= \{x_3, x_5\}, \\ \Gamma(l_2) &= \{x_1, x_2, x_5\}, \\ \Gamma(l_3) &= \{x_2, x_4, x_5\}, \\ \Gamma(l_4) &= \{x_2, x_3\}, \\ \Gamma(l_5) &= \{x_2, x_3, x_5\}. \end{aligned}$$

Then the soft set $\Gamma_{\mathcal{L}}$ is the set of approximations.

The tabular form of soft set $(\Gamma, \mathcal{L})$ is given in Table 1.

$(\Gamma, \mathcal{L})$	x_1	x_2	x_3	x_4	x_5
l_1	0	0	1	0	1
l_2	1	1	0	0	1
l_3	0	1	0	1	1
l_4	0	1	1	0	0
l_5	0	1	1	0	1

Table 1. Soft set $(\Gamma, \mathcal{L})$

Definition 2.3. [32] "Suppose we have a set of objects under observation $\mathcal{J}$ and an indiscernibility relation $\Re \subseteq \mathcal{J} \times \mathcal{J}$ which indicates our information about elements of $\mathcal{J}$. For sake of our convenience, we take $\Re$ as an equivalence relation and denote it as $\Re(x)$. The pair $(\mathcal{J}, \Re)$ is referred as *approximation space.* A subset $\mathcal{Y}$ of $\mathcal{J}$ is taken to characterize it w.r.t $\Re$.

(1) The union of the particles entirely included in the set $\mathcal{Y}$ forms lower approximation of the set $\mathcal{Y}$ w.r.t $\Re$. mathematically defined as;
$$\underline{\Re}(\mathcal{Y}) = \bigcup_{x \in \mathcal{J}} \{\Re(x) : \Re(x) \subseteq \mathcal{Y}\}.$$
(2) The union of the granules having non-empty intersection with the set $\mathcal{Y}$ forms upper approximation of the set $\mathcal{Y}$ w.r.t $\Re$. mathematically defined as;
$$\overline{\Re}(\mathcal{Y}) = \bigcup_{x \in \mathcal{J}} \{\Re(x) : \Re(x) \cap \mathcal{Y} \neq \emptyset\}.$$
(3) The difference between upper and lower approximation forms boundary region of the set $\mathcal{Y}$ w.r.t $\Re$. mathematically defined as;
$$B_{\Re}(\mathcal{Y}) = \overline{\Re}(\mathcal{Y}) - \underline{\Re}(\mathcal{Y}).$$

The set $\mathcal{Y}$ is said to be defined if $\overline{\Re}(\mathcal{Y}) = \underline{\Re}(\mathcal{Y})$. The set $\mathcal{Y}$ is (imprecise) rough set w.r.t $\Re$, if $\overline{\Re}(\mathcal{Y}) \neq \underline{\Re}(\mathcal{Y})$ i.e $B_R(\mathcal{Y}) \neq \emptyset$."

Example 2.4. Consider we have set of people $\mathcal{J} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ who are using certain mobile network. Suppose set of attributes as set of mobile network features. Consider $\mathcal{Y} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\} \subseteq \mathcal{J}$ and *Ja Network* as indiscernibility relation. We present the information in tabular form, rows indicate (objects) people, columns shows attributes and entries of table give attribute values. Such tables are known as information systems. We can see that $\{x_1, x_3\}$ are using Ja-Network. Table 2 shows that a person x_2 is using Ja-Network while x_7 is not using Ja-Network, and they have same mobile network features, so x_2 and x_7 lies in boundary region. Hence *lower approximation* of the set w.r.t relation 'Ja-Network' is $\underline{\Re}(\mathcal{Y}) = \{x_1, x_3\}$ and the *upper approximation* of this set is the set $\overline{\Re}(\mathcal{Y}) = \{x_1, x_2, x_3, x_7\}$, while *boundary region* is $B_{\Re}(\mathcal{Y}) = \overline{\Re}(\mathcal{Y}) \setminus \underline{\Re}(\mathcal{Y})$ =$\{x_2, x_7\}$.

People	Good Signals	Fast Internet	Reasonable Call Charges	*Ja Network*
x_1	✗	✓	✓	✓
x_2	✓	✗	✓	✓
x_3	✓	✓	✓	✓
x_4	✓	✓	✗	✗
x_5	✗	✓	✗	✗
x_6	✓	✗	✗	✗
x_7	✓	✗	✓	✗

TABLE 2. Information system

Definition 2.5. [15] "Consider a soft set $\mathcal{S} = (\Gamma, \mathcal{L})$ over the universe $\mathcal{J}$, where $\mathcal{L} \subseteq \mathcal{E}$ and Γ is a function given as

$$\Gamma : \mathcal{L} \to \mathcal{I}(\mathcal{J}).$$

Then the pair $G = (\mathcal{J}, \mathcal{S})$ is called a *soft approximation space*. Following the soft approximation space G, we get two approximations to every subset $\mathcal{Y} \subseteq \mathcal{J}$ given by

$$\underline{apr}_{G_\star}(\mathcal{Y}) = \{x \in \mathcal{J} : \exists l \in \mathcal{L}, x \in \Gamma(l) \subseteq \mathcal{Y}\},$$

$$\overline{apr}_{G}{}^{\star}(\mathcal{Y}) = \{x \in \mathcal{J} : \exists l \in \mathcal{L}, x \in \Gamma(l) \cap \mathcal{Y} \neq \emptyset\},$$

which we call *soft G-lower approximation* and *soft G-upper approximation* of $\mathcal{Y}$ respectively. Generally, $\underline{apr}_{G_\star}(\mathcal{Y})$ and $\overline{apr}_{G}{}^{\star}(\mathcal{Y})$ are called SR-approximations of $\mathcal{Y}$ w.r.t G. If $\underline{apr}_{G_\star}(\mathcal{Y}) \neq \overline{apr}_{G}{}^{\star}(\mathcal{Y})$ then $\mathcal{Y}$ is said to be soft G-rough set otherwise *soft G-definable*".

Example 2.6. Let $\mathcal{J} = \{x_1, x_2, x_3, x_4, x_5\}$, $\mathcal{E} = \{l_1, l_2, l_3, l_4, l_5, l_6\}$ and $\mathcal{L} = \{l_1, l_2, l_3, l_4\} \subseteq \mathcal{E}$. Let $\mathcal{S} = (\Gamma, \mathcal{L})$ is soft set over $\mathcal{J}$ and

$$\begin{aligned}
&\Gamma(l_1) = \{x_2, x_3\},\\
&\Gamma(l_2) = \{x_2, x_4, x_5\},\\
&\Gamma(l_3) = \{x_1, x_2, x_5\},\\
&\Gamma(l_4) = \{x_3, x_5\}.
\end{aligned}$$

The tabular form of soft set $(\Gamma, \mathcal{L})$ is given in Table 3. Then we obtain soft approximation space $G = (\mathcal{J}, \mathcal{S})$.

$(\Gamma, \mathcal{L})$	x_1	x_2	x_3	x_4	x_5
l_1	0	1	1	0	0
l_2	0	1	0	1	1
l_3	1	1	0	0	1
l_4	0	0	1	0	1

TABLE 3. Soft set $(\Gamma, \mathcal{L})$

For $\mathcal{Y} = \{x_3, x_4, x_5\} \subseteq \mathcal{J}$, we obtain $\underline{apr}_{G\star}(\mathcal{Y}) = \{x_3, x_5\}$ and $\overline{apr}_{G}{}^{\star}(\mathcal{Y}) = \{x_1, x_2, x_3, x_4, x_5\}$.
Since $\underline{apr}_{G\star}(\mathcal{Y}) \neq \overline{apr}_{G}{}^{\star}(\mathcal{Y})$ and hence $\mathcal{Y}$ is said to be a soft G-rough set.

Definition 2.7. "Let $\mathcal{J}$ be a set. A pair $< \mathcal{J}, C_{\mathcal{U}} >$ is said to be multiset, where *Count* $\mathcal{U}$ *or* $C_{\mathcal{U}}$ is a function defined as

$$C_{\mathcal{U}} : \mathcal{J} \to \mathcal{W}$$

and $\mathcal{W}$ is a set of whole numbers. Here $C_{\mathcal{U}(x)}$ is the number of occurrences of the element x in the multiset $\mathcal{U}$
In other words a set consist of unordered collection of objects or repetition of objects is allow in set is called Multiset. A multiset $\mathcal{U}$ is defined as:

$$\mathcal{U} = < \mathcal{J}, C_{\mathcal{U}} > = \left[\frac{s_1}{x_1}, \frac{s_2}{x_2}, \frac{s_3}{x_3}, ..., \frac{s_n}{x_n}\right]$$

Here x_1 occurring s_1 times, x_2 occurring s_2 times and so on" (See [49]).

Example 2.8. Consider $\mathcal{J} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be a crisp set of chairs. Then $\mathcal{M} = \left[\frac{s_1}{x_1}, \frac{s_2}{x_2}, \frac{s_3}{x_3}, \frac{s_4}{x_4}, \frac{s_5}{x_5}, \frac{s_6}{x_6}\right]$ is a multiset of chairs under consideration, here s_i represent multiplicity of $x_i, i = 1, 2, ..., 6$.

Definition 2.9. [49] "The power set of an multiset $\mathcal{U}$ is denoted by $P(\mathcal{U})$ and defined as the set of all sub-multisets of $\mathcal{U}$. The cardinality of the power set $P(\mathcal{U})$ of $\mathcal{U}$ is

$$card(P(\mathcal{U})) = \prod_{x \in \mathcal{U}} (f(x) + 1)$$

Where $f(x)$ is the multiplicity of x.

Definition 2.10. [49] Let $[\mathcal{N}]^m$ denotes the set of all multisets whose elements are in $\mathcal{N}$ such that no element in an multiset occurs more than m times.
Let $\mathcal{U} \in [\mathcal{N}]^m$ be a multiset. The power whole multiset of $\mathcal{U}$ denoted by $PW(\mathcal{U})$ is defined
as the set of all whole sub-multisets of $\mathcal{U}$. The cardinality of $PW(\mathcal{U})$ is

$$card(PW(\mathcal{U})) = 2^n,$$

Where n is the cardinality of the support set (root set) of $\mathcal{U}$".

We use basic operations on multiset as defined in [49].

Definition 2.11. Let $\mathcal{U}$ be an the initial universal multiset, $\mathcal{E}$ be a set of parameters, $PW(\mathcal{U})$ be a power whole multiset of $\mathcal{U}$ and $\mathcal{L} \subseteq \mathcal{E}$. Then an ordered pair $\mathcal{S} = (\Xi, \mathcal{L})$ is called a soft multiset (SM-set), where

$$\Xi : \mathcal{L} \to PW(\mathcal{U}).$$

In other words, a soft multiset over $\mathcal{U}$ is a parameterized family of whole submultisets of $\mathcal{U}$. Also the set of all soft multisets over $\mathcal{U}$ with parameters from $\mathcal{E}$ is denoted by SM($\mathcal{U}$)" (See [8]).

Example 2.12. Let $\mathcal{U} = \left[\frac{s_1}{x_1}, \frac{s_2}{x_2}, \frac{s_3}{x_3}, \frac{s_4}{x_4}, \frac{s_5}{x_5}, \frac{s_6}{x_6}\right]$ be an universal multiset representing microwave oven of different companies, where $x_1 = Dawlance$, $x_2 = Haier$, $x_3 = Homage$, $x_4 = PEL$, $x_5 = LG$, $x_6 = Samsung$, and
$\mathcal{E} = \{$Power consumption, Auto Cook Menu, Timer, Defrost, Reasonable price, Child lock$\}$ be the set of all attribute.
Let $\mathcal{L} = \{$Power consumption, Auto Cook Menu, Timer, Reasonable Price, $\} \subseteq \mathcal{E}$.
Then the soft multiset $\Xi_{\mathcal{E}}$ defined below describe the attractiveness of microwave oven under consideration,

$$\begin{aligned}
\Xi(Power\ comsumption) &= \left[\frac{s_1}{x_1}, \frac{s_3}{x_3}\right], \\
\Xi(Auto\ Cook\ Menu) &= \left[\frac{s_2}{x_2}, \frac{s_3}{x_3}, \frac{s_5}{x_5}\right], \\
\Xi(Timer) &= \left[\frac{s_1}{x_1}, \frac{s_6}{x_6}\right], \\
\Xi(Reasonable\ Price) &= \left[\frac{s_3}{x_3}, \frac{s_5}{x_5}, \frac{s_6}{x_6}\right].
\end{aligned}$$

The tablur form of soft multi set $(\Xi, \mathcal{L})$ is given in Table 4.

$(\Xi, \mathcal{L})$	$\frac{s_1}{x_1}$	$\frac{s_2}{x_2}$	$\frac{s_3}{x_3}$	$\frac{s_4}{x_4}$	$\frac{s_5}{x_5}$	$\frac{s_6}{x_6}$
Power comsumption	1	0	1	0	0	0
Auto Cook Menu	0	1	1	0	1	0
Timer	1	0	0	0	0	1
Reasonable Price	0	0	1	0	1	1

Table 4. Soft multi set $(\Xi, \mathcal{L})$

3. Soft Multi Rough Set

In this section, we introduce the novel concept of soft multi rough set (SMR-set). When we are dealing with approximations of parameterized crisp data we use soft rough set but when we want to deal with approximations of parameterized multi data then this SMR-set model is very suitable for this situation. Soft multi rough set describes the roughness of soft multi set. We also define comparison analysis between some fundamental properties of approximations of Pawlak space, Soft space and Soft multi space.

Definition 3.1. Consider a soft multi set $\mathcal{S} = (\xi, \mathcal{L})$ over the universe of multiset $\mathcal{U}$ and $\mathcal{E}$ be a set of parameters. Where $\mathcal{L} \subseteq \mathcal{E}$ and ξ is a function given as

$$\xi : \mathcal{L} \to PW(\mathcal{U}).$$

Then the pair $P = (\mathcal{U}, \mathcal{S})$ is called a *soft multi approximation space.* Following the soft multi approximation space P, we get two approximations to every whole sub-multiset $\mathcal{Y} \subseteq \mathcal{U}$ given by

$$\underline{apr}_P(\mathcal{Y}) = \{\frac{s}{x} \in \mathcal{U} : \exists\, l \in \mathcal{L}, \left[\frac{s}{x} \in \xi(l) \subseteq \mathcal{Y}\right]\},$$
$$\overline{apr}_P(\mathcal{Y}) = \{\frac{s}{x} \in \mathcal{U} : \exists\, l \in \mathcal{L}, \left[\frac{s}{x} \in \xi(l) \cap \mathcal{Y} \neq \emptyset\right]\},$$

which we call *soft multi P-lower approximation* and *soft multi P-upper approximation* of $\mathcal{Y}$. Generally, $\underline{apr}_P(\mathcal{Y})$ and $\overline{apr}_P(\mathcal{Y})$ are called SMR-approximations of $\mathcal{Y}$ w.r.t P. If $\underline{apr}_P(\mathcal{Y}) \neq \overline{apr}_P(\mathcal{Y})$ then $\mathcal{Y}$ is said to be *soft multi P-rough set* otherwise *soft multi P-definable.* Also, *Soft multi P-positive region set*, *Soft multi P-negative region set* and *Soft multi P-boundary region set* are defined as follows
$PoS_P(\mathcal{Y}) = \underline{apr}_P(\mathcal{Y})$
$Neg_P(\mathcal{Y}) = -\overline{apr}_P(\mathcal{Y})$
$Bnd_P(\mathcal{Y}) = \overline{apr}_P(\mathcal{Y}) - \underline{apr}_P(\mathcal{Y})$.

Example 3.2. Suppose that $\mathcal{U} = \left[\frac{3}{x_1}, \frac{1}{x_2}, \frac{2}{x_3}\right]$ be universal multiset of dresses under consideration, where $3, 1$ *and* 2 is the multiplicity of dresses x_1, x_2 *and* x_3, respectively.
Let $\mathcal{E}$ = {modern style, reasonable price, comfortable, durable, digital printing, expensive} and $\mathcal{L}$ = {modern style, reasonable price, comfortable, durable, digital

priniting}$\subseteq \mathcal{E}$. Let $\mathcal{S} = (\xi, \mathcal{L})$ be soft multi set over $\mathcal{U}$.
Since the cardinality of multiset $\mathcal{U}$ is as follows:

$$\begin{aligned} Card(P(\mathcal{U})) &= (3+1)(1+1)(2+1) \\ &= 4.2.3 \\ &= 24 \end{aligned}$$

All sub-multisets of multiset $\mathcal{U}$ or $P(\mathcal{U})$ are as follows:

$$\begin{aligned}
S_1 &= \left[\tfrac{0}{x_1}, \tfrac{0}{x_2}, \tfrac{0}{x_3}\right], S_2 = \left[\tfrac{3}{x_1}, \tfrac{0}{x_2}, \tfrac{0}{x_3}\right], S_3 = \left[\tfrac{2}{x_1}, \tfrac{0}{x_2}, \tfrac{0}{x_3}\right] \\
S_4 &= \left[\tfrac{1}{x_1}, \tfrac{0}{x_2}, \tfrac{0}{x_3}\right], S_5 = \left[\tfrac{0}{x_1}, \tfrac{1}{x_2}, \tfrac{0}{x_3}\right], S_6 = \left[\tfrac{0}{x_1}, \tfrac{0}{x_2}, \tfrac{2}{x_3}\right] \\
S_7 &= \left[\tfrac{0}{x_1}, \tfrac{0}{x_2}, \tfrac{1}{x_3}\right], S_8 = \left[\tfrac{3}{x_1}, \tfrac{1}{x_2}, \tfrac{0}{x_3}\right], S_9 = \left[\tfrac{2}{x_1}, \tfrac{1}{x_2}, \tfrac{0}{x_3}\right] \\
S_{10} &= \left[\tfrac{1}{x_1}, \tfrac{1}{x_2}, \tfrac{0}{x_3}\right], S_{11} = \left[\tfrac{3}{x_1}, \tfrac{0}{x_2}, \tfrac{2}{x_3}\right], S_{12} = \left[\tfrac{0}{x_1}, \tfrac{1}{x_2}, \tfrac{2}{x_3}\right] \\
S_{13} &= \left[\tfrac{0}{x_1}, \tfrac{1}{x_2}, \tfrac{1}{x_3}\right], S_{14} = \left[\tfrac{2}{x_1}, \tfrac{1}{x_2}, \tfrac{2}{x_3}\right], S_{15} = \left[\tfrac{2}{x_1}, \tfrac{1}{x_2}, \tfrac{1}{x_3}\right] \\
S_{16} &= \left[\tfrac{1}{x_1}, \tfrac{1}{x_2}, \tfrac{2}{x_3}\right], S_{17} = \left[\tfrac{1}{x_1}, \tfrac{1}{x_2}, \tfrac{1}{x_3}\right], S_{18} = \left[\tfrac{3}{x_1}, \tfrac{1}{x_2}, \tfrac{1}{x_3}\right] \\
S_{19} &= \left[\tfrac{2}{x_1}, \tfrac{0}{x_2}, \tfrac{2}{x_3}\right], S_{20} = \left[\tfrac{2}{x_1}, \tfrac{0}{x_2}, \tfrac{1}{x_3}\right], S_{21} = \left[\tfrac{1}{x_1}, \tfrac{0}{x_2}, \tfrac{2}{x_3}\right] \\
S_{22} &= \left[\tfrac{1}{x_1}, \tfrac{0}{x_2}, \tfrac{1}{x_3}\right], S_{23} = \left[\tfrac{3}{x_1}, \tfrac{0}{x_2}, \tfrac{1}{x_3}\right], S_{24} = \left[\tfrac{3}{x_1}, \tfrac{1}{x_2}, \tfrac{2}{x_3}\right]
\end{aligned}$$

The cardinality of power whole multiset $\mathcal{U}$ is as follows:

$$\begin{aligned} card(PW(\mathcal{U})) &= 2^3 \\ &= 8 \end{aligned}$$

Now the $PW(\mathcal{U}) = \{S_1, S_2, S_5, S_6, S_8, S_{11}, S_{12}, S_{24}\}$ and $\xi : \mathcal{L} \to PW(\mathcal{U})$. Then the soft multiset $\xi_{\mathcal{L}}$ defined below describe the attractiveness of dress under consideration,

$$\begin{aligned}
\xi(modern\ style) = \quad S_2 &= \left[\frac{3}{x_1}, \frac{0}{x_2}, \frac{0}{x_3}\right], \\
\xi(reasonable\ price) = \quad S_5 &= \left[\frac{0}{x_1}, \frac{1}{x_2}, \frac{0}{x_3}\right], \\
\xi(comfortable) = \quad S_8 &= \left[\frac{3}{x_1}, \frac{1}{x_2}, \frac{0}{x_3}\right], \\
\xi(durable) = \quad S_{24} &= \left[\frac{3}{x_1}, \frac{1}{x_2}, \frac{2}{x_3}\right], \\
\xi(digital\ priniting) = \quad S_2 &= \left[\frac{3}{x_1}, \frac{0}{x_2}, \frac{0}{x_3}\right].
\end{aligned}$$

The tabular form of soft multi set $\mathcal{S} = (\xi, \mathcal{L})$ is given in Table 4. Then we obtain soft multi approximation space $P = (\mathcal{U}, \mathcal{S})$.

$(\xi,\mathcal{L})$	$\frac{3}{x_1}$	$\frac{1}{x_2}$	$\frac{2}{x_3}$
modern style	1	0	0
reasonable price	0	1	0
comfortable	1	1	0
durable	1	1	1
digital priniting	1	0	0

Table 5. Soft Multi Set $(\xi,\mathcal{L})$

For whole sub-multiset $\mathcal{Y}=\left[\frac{3}{x_1},\frac{2}{x_3}\right]\subseteq\mathcal{U}$, we obtain $\underline{apr}_P(\mathcal{Y})=\left[\frac{3}{x_1}\right]$ and $\overline{apr}_P(\mathcal{Y})=\left[\frac{3}{x_1},\frac{1}{x_2},\frac{2}{x_3}\right]$. Thus $\underline{apr}_P(\mathcal{Y})\neq\overline{apr}_P(\mathcal{Y})$ and $\mathcal{Y}$ is a *soft multi P-rough set.* Here $Pos_P(\mathcal{Y})=\{\left[\frac{3}{x_1}\right]\}$, $Neg_P(\mathcal{Y})=\emptyset$ and $Bnd_P(\mathcal{Y})=\left[\frac{1}{x_2},\frac{2}{x_3}\right]$.
If $\underline{apr}_P(\mathcal{Y})=\overline{apr}_P(\mathcal{Y})$ then $\mathcal{Y}$ is said to be a *soft multi P-definable set.*

Remark:
Its clear from above example the approximations of SMR-set are multi sets. So the operations use in SMR-set are multi operations.

3.1. **Proposition.** Let $\mathcal{S}=(\xi,\mathcal{L})$ be a soft multi set over $\mathcal{U}$ and $P=(\mathcal{U},\mathcal{S})$ a soft multi approximation space. Then we have

$$\underline{apr}_P(\mathcal{Y})=\bigcup_{l\in\mathcal{L}}\{\xi(l):\xi(l)\subseteq\mathcal{Y}\}$$

$$\overline{apr}_P(\mathcal{Y})=\bigcup_{l\in\mathcal{L}}\{\xi(l):\xi(l)\cap\mathcal{Y}\neq\emptyset\}$$

for all whole sub-multiset $\mathcal{Y}\subseteq\mathcal{U}$.
Suppose that $\mathcal{S}=(\xi,\mathcal{L})$ is a soft multi set over multiset $\mathcal{U}$ and $P=(\mathcal{U},\mathcal{S})$ is a corresponding soft multi approximation space. One can verify that soft multi rough approximations satisfy the following properties:

(i) $\underline{apr}_P(\emptyset)=\overline{apr}_P(\emptyset)=\emptyset$
(ii) $\underline{apr}_P(\mathcal{U})=\overline{apr}_P(\mathcal{U})=\bigcup_{l\in\mathcal{L}}\xi(l)$
(iii) $\underline{apr}_P(\mathcal{X}\cap\mathcal{Y})\subseteq\underline{apr}_P(\mathcal{X})\cap\underline{apr}_P(\mathcal{Y})$
(iv) $\underline{apr}_P(\mathcal{X}\cup\mathcal{Y})\supseteq\underline{apr}_P(\mathcal{X})\cup\underline{apr}_P(\mathcal{Y})$
(v) $\overline{apr}_P(\mathcal{X}\cup\mathcal{Y})=\overline{apr}_P(\mathcal{X})\cup\overline{apr}_P(\mathcal{Y})$
(vi) $\overline{apr}_P(\mathcal{X}\cap\mathcal{Y})\subseteq\overline{apr}_P(\mathcal{X})\cap\overline{apr}_P(\mathcal{Y})$
(vii) $\mathcal{X}\subseteq\mathcal{Y}\Rightarrow\underline{apr}_P(\mathcal{X})\subseteq\underline{apr}_P(\mathcal{Y})$
(viii) $\mathcal{X}\subseteq\mathcal{Y}\Rightarrow\overline{apr}_P(\mathcal{X})\subseteq\overline{apr}_P(\mathcal{Y})$

Example 3.3. We verify above soft multi rough approximations properties by considering following example. Let $\mathcal{U}=\left[\frac{2}{x_1},\frac{1}{x_2},\frac{2}{x_3},\frac{1}{x_4}\right]$ be a multi universe and $\mathcal{S}=(\xi,\mathcal{L})$ a soft multi set over multi set $\mathcal{U}$, where $\mathcal{L}=\{l_1,l_2,l_3,l_4,l_5\}$, is set of parameter

$$\begin{aligned}
\xi(l_1) &= \left[\frac{2}{x_1}\right], \\
\xi(l_2) &= \left[\frac{2}{x_1}, \frac{2}{x_3}\right], \\
\xi(l_3) &= \left[\frac{1}{x_2}, \frac{2}{x_3}, \frac{1}{x_4}\right], \\
\xi(l_4) &= \left[\frac{1}{x_2}, \frac{1}{x_4}\right], \\
\xi(l_5) &= \left[\frac{2}{x_1}, \frac{1}{x_2}, \frac{2}{x_3}\right].
\end{aligned}$$

Then $P = (\mathcal{U}, \mathcal{S})$ is soft multi approximation space. The tabular form of soft multi set $\mathcal{S} = (\xi, \mathcal{L})$ is given in Table 6.

$(\xi, \mathcal{L})$	$\frac{2}{x_1}$	$\frac{1}{x_2}$	$\frac{2}{x_3}$	$\frac{1}{x_4}$
l_1	1	0	0	0
l_2	1	0	1	0
l_3	0	1	1	1
l_4	0	1	0	1
l_5	1	1	1	0

TABLE 6. Soft Multi Set $(\xi, \mathcal{L})$

Its obvious property (i) and (ii) hold.
(iii) Suppose $\mathcal{X} = \left[\frac{2}{x_1}, \frac{1}{x_2}, \frac{1}{x_4}\right] \subseteq \mathcal{U}$ and $\mathcal{Y} = \left[\frac{2}{x_1}, \frac{2}{x_2}\right] \subseteq \mathcal{U}$. So $\mathcal{X} \cap \mathcal{Y} = \left[\frac{2}{x_1}\right]$ and $\underline{apr}_P(\mathcal{X} \cap \mathcal{Y}) = \left[\frac{2}{x_1}\right]$. Also $\underline{apr}_P(\mathcal{X}) = \left[\frac{2}{x_1}, \frac{1}{x_2}\right]$, $\underline{apr}_P(\mathcal{Y}) = \left[\frac{2}{x_1}, \frac{2}{x_3}\right]$ and $\underline{apr}_P(\mathcal{X}) \cap \underline{apr}_P(\mathcal{Y}) = \left[\frac{2}{x_1}\right]$. Its clear property (iii) hold as $\underline{apr}_P(\mathcal{X} \cap \mathcal{Y}) \subseteq \underline{apr}_P(\mathcal{X}) \cap \underline{apr}_P(\mathcal{Y})$.
(iv) We have $\mathcal{X} \cup \mathcal{Y} = \left[\frac{2}{x_1}, \frac{1}{x_2}, \frac{2}{x_3}, \frac{1}{x_4}\right]$ and $\underline{apr}_P(\mathcal{X} \cup \mathcal{Y}) = \left[\frac{2}{x_1}, \frac{1}{x_2}, \frac{2}{x_3}, \frac{1}{x_4}\right]$. Also $\underline{apr}_P(\mathcal{X}) = \left[\frac{2}{x_1}, \frac{1}{x_2}\right]$, $\underline{apr}_P(\mathcal{Y}) = \left[\frac{2}{x_1}, \frac{2}{x_3}\right]$ and $\underline{apr}_P(\mathcal{X}) \cup \underline{apr}_P(\mathcal{Y}) = \left[\frac{2}{x_1}, \frac{1}{x_2}, \frac{2}{x_3}\right]$. So property (iv) satisfy.
(v) As $\overline{apr}_P(\mathcal{X} \cup \mathcal{Y}) = \mathcal{U}$, $\overline{apr}_P(\mathcal{X}) = \mathcal{U}$, $\overline{apr}_P(\mathcal{Y}) = \mathcal{U}$ and $\overline{apr}_P(\mathcal{X}) \cup \overline{apr}_P(\mathcal{Y})$. Hence this property also satisfy.
(vi) While $\overline{apr}_P(\mathcal{X}) \cap \overline{apr}_P(\mathcal{Y}) = \mathcal{U}$ and $\overline{apr}_P(\mathcal{X} \cap \mathcal{Y}) = \left[\frac{2}{x_1}, \frac{1}{x_2}, \frac{2}{x_3}\right]$. Therefore property (vi) proved here.
For property (vii) and (viii), we take $\mathcal{Y} = \left[\frac{2}{x_1}, \frac{1}{x_2}, \frac{1}{x_4}\right] \subseteq \mathcal{U}$ and $\mathcal{X} = \left[\frac{2}{x_1}, \frac{1}{x_2}\right] \subseteq \mathcal{U}$. As $\mathcal{X} \subseteq \mathcal{Y}$, $\underline{apr}_P(\mathcal{X}) = \left[\frac{2}{x_1}\right]$ and $\underline{apr}_P(\mathcal{Y}) = \left[\frac{2}{x_1}, \frac{1}{x_2}\right]$. Consequently it is clear $\mathcal{X} \subseteq \mathcal{Y} \Rightarrow \underline{apr}_P(\mathcal{X}) = \left[\frac{2}{x_1}\right] \subseteq \underline{apr}_P(\mathcal{Y}) = \left[\frac{2}{x_1}, \frac{1}{x_2}\right]$.

Again if $\mathcal{X} \subseteq \mathcal{Y}$, $\overline{apr}_P(\mathcal{X}) = \left[\frac{2}{x_1}\right]$ and $\overline{apr}_P(\mathcal{Y}) = \left[\frac{2}{x_1}, \frac{1}{x_2}\right]$. It is clear $\mathcal{X} \subseteq \mathcal{Y} \Rightarrow \overline{apr}_P(\mathcal{X}) = \mathcal{U} \subseteq \overline{apr}_P(\mathcal{Y}) = \mathcal{U}$. All properties proved here successfully by the help of example.

3.2. **Comparison Analysis.** Consider the soft multi approximation space $P = (\mathcal{U}, \mathcal{S})$, where $\mathcal{S} = (\xi, \mathcal{L})$ is soft multi set over multiset $\mathcal{U}$. The following Table 7 indicates some deviations between Pawlak space, Soft space and Soft multi space on some properties of lower and upper approximations.

Pawlak Space	$\underline{\Re}(\mathcal{U}) = \overline{\Re}(\mathcal{U}) = \mathcal{U}$ $\underline{\Re}(\mathcal{X} \cap \mathcal{Y}) = \underline{\Re}(\mathcal{X}) \cap \underline{\Re}(\mathcal{Y})$ - $\overline{\Re}(\mathcal{X} \cup \mathcal{Y}) = \overline{\Re}(\mathcal{X}) \cup \overline{\Re}(\mathcal{Y})$ -
Soft Space	$\underline{apr}_{P_\star}(\mathcal{Y}) = \overline{apr}_{P^\star}(\mathcal{Y}) = \bigcup_{l \in \mathcal{L}} \Gamma(l)$ $\underline{apr}_{P_\star}(\mathcal{X} \cap \mathcal{Y}) \subseteq \underline{apr}_{P_\star}(\mathcal{X}) \cap \underline{apr}_{P_\star}(\mathcal{Y})$ $\underline{apr}_{P_\star}(\mathcal{X} \cup \mathcal{Y}) \supseteq \underline{apr}_{P_\star}(\mathcal{X}) \cup \underline{apr}_{P_\star}(\mathcal{Y})$ $\overline{apr}_{P^\star}(\mathcal{X} \cup \mathcal{Y}) = \overline{apr}_{P^\star}(\mathcal{X}) \cup \overline{apr}_{P^\star}(\mathcal{Y})$ $\overline{apr}_{P^\star}(\mathcal{X} \cap \mathcal{Y}) \subseteq \overline{apr}_{P^\star}(\mathcal{X}) \cap \overline{apr}_{P^\star}(\mathcal{Y})$
Soft Multi Space	$\underline{apr}_P(\mathcal{U}) = \overline{apr}_P(\mathcal{U}) = \bigcup_{l \in \mathcal{L}} \xi(l)$ $\underline{apr}_P(\mathcal{X} \cap \mathcal{Y}) \subseteq \underline{apr}_P(\mathcal{X}) \cap \underline{apr}_P(\mathcal{Y})$ $\underline{apr}_P(\mathcal{X} \cup \mathcal{Y}) \supseteq \underline{apr}_P(\mathcal{X}) \cup \underline{apr}_P(\mathcal{Y})$ $\overline{apr}_P(\mathcal{X} \cup \mathcal{Y}) = \overline{apr}_P(\mathcal{X}) \cup \overline{apr}_P(\mathcal{Y})$ $\overline{apr}_P(\mathcal{X} \cap \mathcal{Y}) \subseteq \overline{apr}_P(\mathcal{X}) \cap \overline{apr}_P(\mathcal{Y})$

TABLE 7. Comparison Analysis

3.3. **Proposition.** Let $\mathcal{S} = (\xi, \mathcal{L})$ be a soft multi set over multiset $\mathcal{U}$ and $P = (\mathcal{U}, \mathcal{S})$ a soft multi approximation space. Then for any whole sub-multiset $\mathcal{Y} \subseteq \mathcal{U}$, $\mathcal{Y}$ *is soft multi P-definable* if and only if $\overline{apr}_P(\mathcal{Y}) \subseteq \mathcal{Y}$.

Proof

Firstly if $\mathcal{Y}$ is *soft multi P-definable*, then $\underline{apr}_P(\mathcal{Y}) = \overline{apr}_P(\mathcal{Y})$, and so $\underline{apr}_P(\mathcal{Y}) = \overline{apr}_P(\mathcal{Y}) \subseteq \mathcal{Y}$.

Conversely, suppose that $\overline{apr}_P(\mathcal{Y}) \subseteq \mathcal{Y}$ for $\mathcal{Y} \subseteq \mathcal{U}$. To show that Y is *soft P-definable*, we only need to prove that $\overline{apr}_P(\mathcal{Y}) \subseteq \underline{apr}_P(\mathcal{Y})$ since the reverse inequality is trivial. Let $\frac{s}{x} \in \overline{apr}_P(\mathcal{Y})$. Then $\frac{s}{x} \in \xi(l)$ and $\xi(l) \cap \mathcal{Y} \neq \emptyset$ for some $l \in \mathcal{L}$. It follows that $\frac{s}{x} \in \xi(l) \subseteq \overline{apr}_P(\mathcal{Y}) \subseteq \mathcal{Y}$. Hence $\frac{s}{x} \in \underline{apr}_P(\mathcal{Y})$, and so $\overline{apr}_P(\mathcal{Y}) \subseteq \underline{apr}_P(\mathcal{Y})$ as required.

3.4. **Theorem.** If $\mathcal{S} = (\xi, \mathcal{L})$ be a soft multi set over multiset $\mathcal{U}$ and $P = (\mathcal{U}, \mathcal{S})$ be soft multi-approximation space. Then following hold:

(1) $\underline{apr}_P(\overline{apr}_P(\mathcal{Y})) = \overline{apr}_P(\mathcal{Y})$

(2) $\overline{apr}_P(\underline{apr}_P(\mathcal{Y})) \supseteq \underline{apr}_P(\mathcal{Y})$
(3) $\underline{apr}_P(\underline{apr}_P(\mathcal{Y})) = \underline{apr}_P(\mathcal{Y})$
(4) $\overline{apr}_P(\overline{apr}_P(\mathcal{Y})) \supseteq \underline{apr}_P(\mathcal{Y})$

for all $\mathcal{Y} \subseteq \mathcal{U}$

Proof:

(1) If $\mathcal{X} = \overline{apr}_P(\mathcal{Y})$ and $\frac{s}{x} \in \mathcal{X}$. Then $\frac{s}{x} \in \xi(l) \neq \emptyset$ for some $l \in \mathcal{L}$. By using Proposition 3.1 $\mathcal{X} = \overline{apr}_P(\mathcal{Y}) = \bigcup_{l\in\mathcal{L}}\{\xi(l) : \xi(l) \cap \mathcal{Y} \neq \emptyset\}$. So, $\exists l \in \mathcal{L}$ such that $\frac{s}{x} \in \xi(l) \subseteq \mathcal{X}$. Hence $\frac{s}{x} \in \underline{apr}_P(\mathcal{X})$, and so $\mathcal{X} \subseteq \underline{apr}_P(\mathcal{X})$. Also, we know that for any $\mathcal{X} \subseteq \mathcal{U}$, $\underline{apr}_P(\mathcal{X}) \subseteq \mathcal{X}$ holds. From this we obtain our required result which is $\mathcal{X} = \underline{apr}_P(\mathcal{X})$.

(2) If $\mathcal{X} = \underline{apr}_P(\mathcal{Y})$ and $\frac{s}{x} \in \mathcal{X}$. Then $\frac{s}{x} \in \xi(\mathcal{X})$ for some $l \in \mathcal{L}$. Since by Proposition 3.1 $\mathcal{Y} = \overline{apr}_P(\mathcal{X}) = \bigcup_{l\in\mathcal{L}}\{\xi(l) : \xi(l) \cap \mathcal{Y}\}$. We obtain that $\frac{s}{x} \in \xi(l)$ and $\xi(l)\cap\mathcal{X} = \xi(l) \neq \emptyset$. Hence $\frac{s}{x} \in \overline{apr}_P(\mathcal{X})$, and so $\mathcal{X} \subseteq \overline{apr}_P(\mathcal{X})$.

(3) Consider $\mathcal{X} = \underline{apr}_P(\mathcal{Y})$ and $\frac{s}{x} \in \mathcal{X}$. Then $\frac{s}{x} \in \xi(\mathcal{X})$ for some $l \in \mathcal{L}$. But $\mathcal{X} = \underline{apr}_P(\mathcal{Y}) = \bigcup_{l\in\mathcal{L}}\{\xi(l) : \xi(l) \cap \mathcal{Y}\}$. We deduce that $\frac{s}{x} \in \xi(l) \subseteq \mathcal{X} for l \in \mathcal{L}$. Thus $\frac{s}{x} \in \underline{apr}_P(\mathcal{X})$, and so $\mathcal{X} \subseteq \underline{apr}_P(\mathcal{X})$.Also $\underline{apr}_P(\mathcal{X}) \subseteq \mathcal{X}$ for any $\mathcal{Y} \subseteq \mathcal{U}$. Hence $\mathcal{X} = \underline{apr}_P(\mathcal{X})$.

(4) Consider $\mathcal{X} = \overline{apr}_P(\mathcal{Y})$ and $\frac{s}{x} \in \xi(l)$. Then $\frac{s}{x} \in \xi(l)$ and $\xi(l) \cap \mathcal{Y} \neq \emptyset$ for some $l \in \mathcal{L}$. But $\mathcal{X} = \overline{apr}_P(\mathcal{Y}) = \bigcup_{l\in\mathcal{L}}\{\xi(l) : \xi(l) \cap \mathcal{Y} \neq \emptyset\}$, this implies $\frac{s}{x} \in \xi(l)$ and $\xi(l) \cap \mathcal{X} \neq \emptyset$. Thus, $\frac{s}{x} \in \overline{apr}_P(\mathcal{X})$ and then $\mathcal{X} \subseteq \overline{apr}_P(\mathcal{X})$.

4. SMR-set in multi-criteria group decision-making problem for the selection of humanoid robots

In this section we present the technique of SMR-set in object estimation and multi-criteria group decision-making. Consider $\mathcal{U} = \left[\frac{s_1}{x_1}, \frac{s_2}{x_2}, \frac{s_3}{x_3}, ..., \frac{s_n}{x_n}\right]$ be the multi-set of objects under observation, $\mathcal{E}$ be the set of criterions to find the objects in $\mathcal{U}$. Here s_1 is the multiplicity of x_1, s_2 is the multiplicity of x_2 and so on. Suppose $\mathcal{L} \subseteq \mathcal{E}$. We take a soft multi set $\mathcal{S} = (\xi, \mathcal{L})$ for real worlds problems. For the sake of betterment we take full soft multi set over $\mathcal{U}$. Consider $\mathcal{Z} = \{\mathfrak{D}_1, \mathfrak{D}_2, \mathfrak{D}_3, ..., \mathfrak{D}_\mathfrak{n}\}$ be the set consisting to decision makers who examine the objects to identify the possible solution and $\mathfrak{X}_\mathfrak{i}$ be the initial estimation derived by members of experts $\mathfrak{D}_\mathfrak{i}$ which is express by the soft multi set $\Omega = (\omega, \mathcal{S})$. To get better results we find out SMR-approximation of initial estimated results $\mathfrak{X}_\mathfrak{i}$ according to soft multi-approximation space $\mathcal{P} = (\mathcal{U}, \mathcal{S})$, consequently we obtain two soft multi sets $\Omega_\star = (\omega_\star, \mathcal{S})$ and $\Omega^\star = (\omega^\star, \mathcal{S})$. Following these soft multi sets define fuzzy multi sets $u_{\Omega_\star}(\left[\frac{s_k}{x_k}\right])$, $u_\Omega(\left[\frac{s_k}{x_k}\right])$ and $u_{\Omega^\star}(\left[\frac{s_k}{x_k}\right])$ defined as:

$$u_{\Omega_\star}(\left[\frac{s_k}{x_k}\right]) = \frac{1}{n}\sum_{i=1}^{n} C_{\omega_\star\mathfrak{D}_\mathfrak{i}}(\left[\frac{s_k}{x_k}\right]),$$

$$u_{\Omega}(\left[\frac{s_k}{x_k}\right]) = \frac{1}{n}\sum_{i=1}^{n} C_{\omega\mathfrak{D}_\mathfrak{i}}(\left[\frac{s_k}{x_k}\right]),$$

$$u_{\Omega^\star}(\left[\frac{s_k}{x_k}\right]) = \frac{1}{n}\sum_{i=1}^{n} C_{\omega^\star\mathfrak{D}_\mathfrak{i}}(\left[\frac{s_k}{x_k}\right]).$$

Soft multi-set and fuzzy results are then combined.

Suppose $C = \{\text{not recommended}, \text{recommended}, \text{highly recommended}\}$ is the set of parameters. Define fuzzy soft multi set $\mathcal{R}(\gamma, C)$ over $\mathcal{U}$ as
$\gamma(\text{not recommended}) = u_{\Omega}{}^{\star}$
$\gamma(\text{recommended}) = u_{\Omega}$
$\gamma(\text{highly recommended}) = u_{\Omega_{\star}}$
Calculate the choice value c_i corresponding to each object $\left[\frac{s_k}{x_k}\right]$ as: $c_i = \sum_j \left[\frac{s_{ij}}{x_{ij}}\right]$
Here s_{ij} is the multiplicity of x_{ij}. At the end we are in position to choose the favorable substitute having maximum choice value c_i.

Algorithm 1:

The strategy of the algorithm is given as:
Input
Step-1: Write the soft multi-set $\mathcal{S} = (\xi, \mathcal{L})$ which describes the given data.
Step-2: Based on initial estimated results of the group of advisers $\mathcal{Z}$, define a soft multi-set.
Step-3: Obtain SMR-approximations in the form of soft multi sets $\Omega_{\star} = (\omega_{\star}, \mathcal{Z})$ and $\Omega^{\star} = (\omega^{\star}, \mathcal{Z})$.
Step-4: Define fuzzy multi sets $u_{\Omega_{\star}}$, u_{Ω} and $u_{\Omega}{}^{\star}$ corresponding to the soft multi sets $\Omega_{\star} = (\omega_{\star}, \mathcal{Z})$, $\Omega = (\omega, \mathcal{Z})$ and $\Omega^{\star} = (\omega^{\star}, \mathcal{Z})$.
Step-5: Characterize the recommended level of experts in the form of parameter set
C={not recommended,recommended,highly recommended}.
Step-6: Define fuzzy soft multi set $\mathcal{R}(\gamma, C)$ over $\mathcal{U}$ using fuzzy multi sets $u_{\Omega_{\star}}$, u_{Ω} and $u_{\Omega}{}^{\star}$.
Output
Step-7: Calculate choice value c_i for each object. Select the object having maximum choice value.

Case Study:
In this problem we consider case of Artificial Intelligence. It is clear from the name Artificial means Fake and Intelligence means understanding, sense or brainpower. Artificial Intelligence(AI) is basically a ability of a computer program or a machine to think, learn or act like human. Best advantage of AI is that machines don,t require sleep or break.
In this modern era, we are using AI in different fields of life like Agriculture, Cars, Education, Healthcare and of course in security system. Common examples of AI software are Siri Siri in iphone, Tesla in smart phones and automobiles, Delta Cars, Flying Drones, Robots and Humanoid Robots.
In 1920 Czech writer Karel Capek published a science fiction play named *"Rossum,s Uuniversal Robots"* also named as *(RUR)*.
In this problem our area of interest is humanoid robots, so we talk about AI of humanoid robots only. Humanoid means resembles with human. So simply humanoid robot may be defined as a robot that resembles or looks like a human and having characteristic like ability to walk, talk, facial expression and eye contact just like

human. First recorded designs of humanoid robot was made by *Leonardo da Vinci* (1452-1519) around in 1495.
Some known humanoid robots names are *Sophia robot, Han robot, Actroid robot, Albert Einstein Hubo robot, Erica robot, Alice robot, Jia Jia robot* and many others. The detail infomation about *Sophia robot, Actroid robot, Erica robot and Jia Jia robot* are given below:

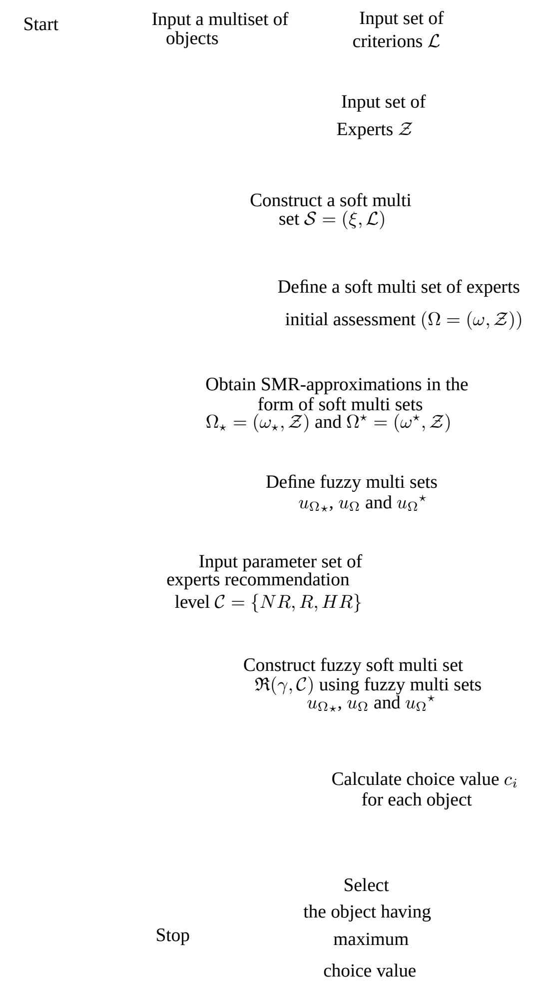

FIGURE 1. Graphical representation of Algorithm 1

Sophia: *Sophia* was first activated on April 19, 2015 in Hong Kong Japan. She is also a world first humanoid robot who receive the citizenship from Saudi Arabia.

She has seven siblings. *Sophia* humanoid robot can walk also.

FIGURE 2. Sophia Humanoid Robot

Erica: Erica is humanoid robot made in Japan. Erica, is developed by Hiroshi, the director of the Intelligent Robotics Laboratory at Osaka University Japan. This robot use first time as News Anchor in Japan.

FIGURE 3. Erica Humanoid Robot

Jia Jia: Jia jia humanoid robot is made by China. Jia jia is first chinese humanoid robot. She looks fairly realistic, with a flexible plastic face.

FIGURE 4. Jia Jia Humanoid Robot

Actroid: In 2003 Actroid first version come in market. But with the passage of time Japan make improvement in Actroid robot. Best quality of this humanoid robot is that it seems like breathing which look realistic.

FIGURE 5. Actroid Humanoid Robot

Example 4.1. Now we apply the concept of SMR-set for the selection of humanoid robots for hotel staff members. Suppose a multinational hotels company name *JAL Hotels Company* decided to have AI humanoid robot staff members in their hotels. For this purpose the CEO of the company contact with three Artificial Intelligence experts to decide which humanoid robot is better as hotel staff.
Consider a multi set of Humanoid Robots

$$\mathcal{U} = \left[\frac{20}{x_1}, \frac{30}{x_2}, \frac{25}{x_3}, \frac{35}{x_4}\right]$$

for the selection as a hotels staff member. Here the multiplicity of humanoid robots denotes the number of robots that are required as a staff in hotels.
Let $\mathcal{L} = \{e_1, e_2, e_3, e_4\}$ be the set of features considered for humanoid robots where,

$$\begin{aligned} e_1 &= Facial\ Recognition, \\ e_2 &= Conversations\ Skills, \\ e_3 &= Movable, \\ e_4 &= Affordable\ Price. \end{aligned}$$

Construct a soft multi set $\mathcal{S} = (\xi, \mathcal{L})$ which specify the Artificial Intelligence of humanoid robots. Consider a team of AI experts $\mathcal{Z} = \{\mathfrak{D}_1, \mathfrak{D}_2, \mathfrak{D}_3\}$ to evaluate the Artificial Intelligence of robots in $\mathcal{U}$. The tabular form of soft multi set $\mathcal{S} = (\xi, \mathcal{L})$ is given in Table 8.

	$\left[\frac{20}{x_1}\right]$	$\left[\frac{30}{x_2}\right]$	$\left[\frac{25}{x_3}\right]$	$\left[\frac{35}{x_4}\right]$
e_1	0	1	0	1
e_2	1	0	1	0
e_3	0	1	0	1
e_4	0	1	1	0

TABLE 8. Soft multi set $\mathcal{S} = (\xi, \mathcal{L})$

Let $\mathfrak{X}_{\mathfrak{i}}$ be the initial estimated results of the experts team. We represent this evaluation by means of soft multi-set $\Omega = (\omega, \mathcal{Z})$ whose tabular representation is given in Table 9.

	$\left[\frac{20}{x_1}\right]$	$\left[\frac{30}{x_2}\right]$	$\left[\frac{25}{x_3}\right]$	$\left[\frac{35}{x_4}\right]$
$\mathfrak{D}_1$	1	0	0	1
$\mathfrak{D}_2$	0	1	0	1
$\mathfrak{D}_3$	1	1	1	0

TABLE 9. Soft multi set $\Omega = (\omega, \mathcal{Z})$

From this soft multi set $\Omega = (\omega, \mathcal{Z})$ initial evaluated result of experts are

$$\begin{aligned}
\mathfrak{X}_1 &= \omega(\mathfrak{D}_1) = \left[\frac{20}{x_1}, \frac{35}{x_4}\right], \\
\mathfrak{X}_2 &= \omega(\mathfrak{D}_2) = \left[\frac{30}{x_2}, \frac{35}{x_4}\right], \\
\mathfrak{X}_3 &= \omega(\mathfrak{D}_3) = \left[\frac{20}{x_1}, \frac{30}{x_2}, \frac{25}{x_3}\right].
\end{aligned}$$

Now we find out the SMR-approximation as

$$\begin{aligned}
\omega_\star(\mathfrak{D}_1) &= \underline{apr}_P(\mathfrak{X}_1) = [\emptyset], \\
\omega_\star(\mathfrak{D}_2) &= \underline{apr}_P(\mathfrak{X}_2) = \left[\frac{30}{x_2}, \frac{35}{x_4}\right], \\
\omega_\star(\mathfrak{D}_3) &= \underline{apr}_P(\mathfrak{X}_3) = \left[\frac{20}{x_1}, \frac{30}{x_2}, \frac{25}{x_3}\right]
\end{aligned}$$

and

$$\begin{aligned}
\omega^\star(\mathfrak{D}_1) &= \overline{apr}_P(\mathfrak{X}_3) = \left[\frac{20}{x_1}, \frac{30}{x_2}, \frac{25}{x_3}, \frac{35}{x_4}\right], \\
\omega^\star(\mathfrak{D}_2) &= \overline{apr}_P(\mathfrak{X}_3) = \left[\frac{30}{x_2}, \frac{25}{x_3}, \frac{35}{x_4}\right], \\
\omega^\star(\mathfrak{D}_3) &= \overline{apr}_P(\mathfrak{X}_3) = \left[\frac{20}{x_1}, \frac{30}{x_2}, \frac{25}{x_3}, \frac{35}{x_4}\right].
\end{aligned}$$

Following these SMR-approximations, we get two soft multi sets $\Omega_\star = (\omega_\star, \mathcal{Z})$ and $\Omega^\star = (\omega^\star, \mathcal{Z})$ where, $\omega_\star(\mathfrak{D}_{\mathfrak{i}}) = \underline{apr}_P(\mathfrak{X}_{\mathfrak{i}})$ and $\omega_\star(\mathfrak{D}_{\mathfrak{i}}) = \overline{apr}_P(\mathfrak{X}_{\mathfrak{i}})$.
Tabular representation of soft multi sets $\Omega_\star = (\omega_\star, \mathcal{Z})$ and $\Omega^\star = (\omega^\star, \mathcal{Z})$ are given in Table 10 and Table 11.

	$\left[\frac{20}{x_1}\right]$	$\left[\frac{30}{x_2}\right]$	$\left[\frac{25}{x_3}\right]$	$\left[\frac{35}{x_4}\right]$
$\mathfrak{D}_1$	0	0	0	0
$\mathfrak{D}_2$	0	1	0	1
$\mathfrak{D}_3$	1	1	1	0

TABLE 10. Soft multi set $\Omega_\star$

	$\left[\frac{20}{x_1}\right]$	$\left[\frac{30}{x_2}\right]$	$\left[\frac{25}{x_3}\right]$	$\left[\frac{35}{x_4}\right]$
$\mathfrak{D}_1$	1	1	1	1
$\mathfrak{D}_2$	0	1	1	1
$\mathfrak{D}_3$	1	1	1	1

TABLE 11. Soft multi set $\Omega^\star$

Now we define fuzzy multi sets $u_{\Omega\star}(\left[\frac{s_k}{x_k}\right])$, $u_{\Omega}(\left[\frac{s_k}{x_k}\right])$ and $u_{\Omega}{}^\star(\left[\frac{s_k}{x_k}\right])$ as follows

$$u_{\Omega\star}(\left[\frac{s_k}{x_k}\right]) = \frac{1}{3}\sum_{i=1}^{3} C_{\omega_\star\mathfrak{D}_i}(P_k),$$

$$u_{\Omega}(\left[\frac{s_k}{x_k}\right]) = \frac{1}{3}\sum_{i=1}^{3} C_{\omega\mathfrak{D}_i}(P_k),$$

$$u_{\Omega}{}^\star(\left[\frac{s_k}{x_k}\right]) = \frac{1}{3}\sum_{i=1}^{3} C_{\omega^\star\mathfrak{D}_i}(P_k),$$

$$u_{\Omega\star}(\left[\frac{s_k}{x_k}\right]) = \{(\left[\frac{20}{x_1}\right], \frac{1}{3}), (\left[\frac{30}{x_2}\right], \frac{2}{3}), (\left[\frac{25}{x_3}\right], \frac{1}{3}), (\left[\frac{35}{x_4}\right], \frac{1}{3})\},$$

$$u_{\Omega}(\left[\frac{s_k}{x_k}\right]) = \{(\left[\frac{20}{x_1}\right], \frac{2}{3}), (\left[\frac{30}{x_2}\right], \frac{2}{3}), (\left[\frac{25}{x_3}\right], \frac{1}{3}), (\left[\frac{35}{x_4}\right], \frac{2}{3})\},$$

$$u_{\Omega}{}^\star(\left[\frac{s_k}{x_k}\right]) = \{(\left[\frac{20}{x_1}\right], \frac{2}{3}), (\left[\frac{30}{x_2}\right], 1), (\left[\frac{25}{x_3}\right], 1), (\left[\frac{35}{x_4}\right], 1)\}.$$

Suppose $\mathcal{C} = \{\mathcal{NR}, \mathcal{R}, \mathcal{HR}\}$ be the set of parameters by AI experts which represents "*Not Recommended*", "*Recommended*" and "*Highly Recommended*". Then we get the fuzzy soft multi set $\mathfrak{R}(\gamma, \mathcal{C})$ over $\mathcal{U}$ by setting
$\gamma(\mathcal{NR}) = u_{\Omega^\star}$,
$\gamma(\mathcal{R}) = u_\Omega$
and
$\gamma(\mathcal{HR}) = u_{\Omega_\star}$.

Calculating choice value corresponding to each humanoid robot. Fuzzy soft multi set $\mathfrak{R}(\gamma, \mathcal{C})$ with estimated values is given in Table 12.

	$\left[\frac{20}{x_1}\right]$	$\left[\frac{30}{x_2}\right]$	$\left[\frac{25}{x_3}\right]$	$\left[\frac{35}{x_4}\right]$
Not Recommended	2/3	1	1	1
Recommended	2/3	2/3	1/3	2/3
Highly Recommended	1/3	2/3	1/3	1/3
choice value c_i	1.6	2.3	1.6	2

TABLE 12. $\mathfrak{R}(\gamma, \mathcal{C})$

From table we can arrange all the alternatives according to their choice evaluation values:

$$\left[\frac{30}{x_2}\right] \succ \left[\frac{35}{x_4}\right] \succ \left[\frac{25}{x_3}\right] \succeq \left[\frac{20}{x_1}\right].$$

Thus, $\left[\frac{30}{x_2}\right]$ is the robot to be selected for the hotels staff member.

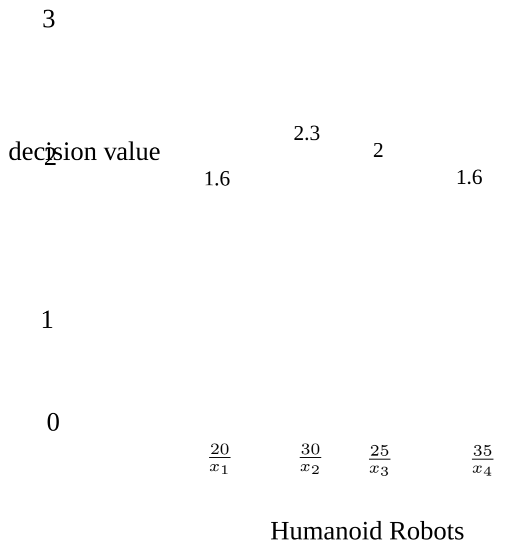

FIGURE 6. Bar chart of Algorithm 1

Algorithm 2

The scheme of the algorithm is given as:
Step-1: Write the soft multi set $\mathcal{S} = (\xi, \mathcal{L})$ which describes the given data.

Step-2: Based on initial assessment results of the group of analyst $\mathcal{Z}$, define a soft multi set.
Step-3: Obtain SMR-approximations in the form of soft multi sets $\Omega_\star = (\omega_\star, \mathcal{Z})$ and $\Omega^\star = (\omega^\star, \mathcal{Z})$.
Step-4: Find choice value for all selected soft multi sets $\Omega_\star = (\omega_\star, \mathcal{Z})$, $\Omega = (\omega, \mathcal{Z})$ and $\Omega^\star = (\omega^\star, \mathcal{Z})$.
Step-5: Find the decision set by adding all the choice values of obtained soft multi sets.
Step-6: Characterize the recommendation level of experts in the form of parameter set $\mathcal{C} = \{not\ recommended, recommended, highly\ recommended\}$.
Input the weighting vector $W = (w_{NR}, w_R, w_{HR})$ and compute the weighted evaluation value for each object.
Step-7: Find the decision set by adding all the weighted values $\sum_i w_i$.Choose the object having maximum value.

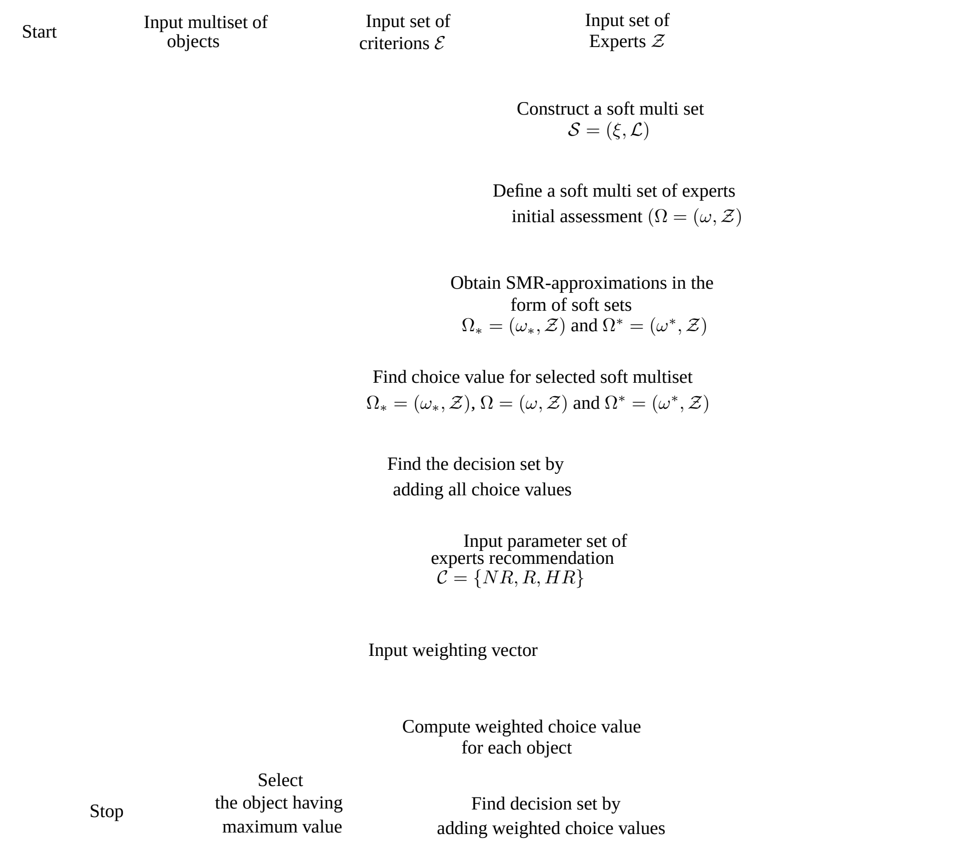

FIGURE 7. Graphical representation of Algorithm 2

Example 4.2. Consider Example 4.1. First three steps same as done by algorithm 1. Find choice value for all selected soft multi sets $\Omega_\star = (\omega_\star, \mathcal{Z})$, $\Omega = (\omega, \mathcal{Z})$ and $\Omega^\star = (\omega^\star, \mathcal{Z})$. Choice value for soft multi sets $\Omega_\star = (\omega_\star, \mathcal{Z})$, $\Omega = (\omega, \mathcal{Z})$ and $\Omega^\star = (\omega^\star, \mathcal{Z})$ are given in Table 13, Table 14 and Table 15.

	$\boxed{\frac{20}{x_1}}$	$\boxed{\frac{30}{x_2}}$	$\boxed{\frac{25}{x_3}}$	$\boxed{\frac{35}{x_4}}$
$\mathfrak{D}_1$	0	0	0	0
$\mathfrak{D}_2$	0	1	0	1
$\mathfrak{D}_3$	1	1	1	0
choice value c_1	1	2	1	1

TABLE 13. Choice value for soft multi set $\Omega_\star$

	$\boxed{\frac{20}{x_1}}$	$\boxed{\frac{30}{x_2}}$	$\boxed{\frac{25}{x_3}}$	$\boxed{\frac{35}{x_4}}$
$\mathfrak{D}_1$	1	0	0	1
$\mathfrak{D}_2$	0	1	0	1
$\mathfrak{D}_3$	1	1	1	0
choice value c_2	2	2	1	2

TABLE 14. Choice value for soft multi set Ω

	$\boxed{\frac{20}{x_1}}$	$\boxed{\frac{30}{x_2}}$	$\boxed{\frac{25}{x_3}}$	$\boxed{\frac{35}{x_4}}$
$\mathfrak{D}_1$	1	1	1	1
$\mathfrak{D}_2$	0	1	1	1
$\mathfrak{D}_3$	1	1	1	1
choice value c_3	2	3	3	3

TABLE 15. Choice value for soft multi set $\Omega^\star$

Now, we find the decision table by adding choice values for each humanoid robot. Choice values for each humanoid robot is given in Table 16.

	$\boxed{\frac{20}{x_1}}$	$\boxed{\frac{30}{x_2}}$	$\boxed{\frac{25}{x_3}}$	$\boxed{\frac{35}{x_4}}$
c_1	1	2	1	1
c_2	2	2	1	2
c_3	2	3	3	3
final choice value	5	7	5	6

TABLE 16. Choice value

Let $\mathcal{C} = \{not\ recommended, recommended, highly\ recommended\}$ be the set of parameters. Suppose that weighting vector $W = (w_{NR}, w_R, w_{HR}) = (.2, .3, .5)$. Calculate weighted choice value for each robot. Find the decision set by adding all the weighted values $\sum_i w_i$. Final weighted choice value is given in Table 17.

	$\left[\frac{20}{x_1}\right]$	$\left[\frac{30}{x_2}\right]$	$\left[\frac{25}{x_3}\right]$	$\left[\frac{35}{x_4}\right]$
$notrecommended = \{NR\}(.2)$	1	1.4	1	1.2
$recommended = \{R\}(.3)$	1.5	2.1	1.5	1.8
$highlyrecommended = \{HR\}(.5)$	2.5	3.5	2.5	3
final weighted choice value=W_c	5	6.7	5	5.5

TABLE 17. Final weighted choice value

Select the robot having maximum final weighted choice value. we can arrange all the humanoid robots according to their choice evaluation values:

$$\left[\frac{30}{x_2}\right] \succ \left[\frac{35}{x_4}\right] \succ \left[\frac{25}{x_3}\right] \succeq \left[\frac{20}{x_1}\right].$$

Thus, $\left[\frac{30}{x_2}\right]$ is the robot to be selected for the hotel stuff member.

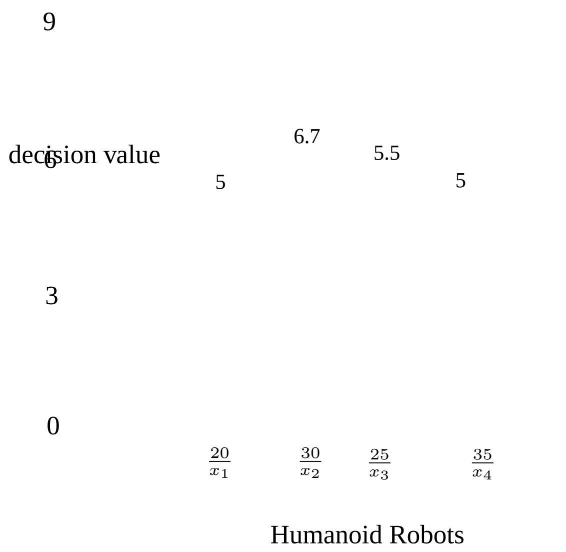

FIGURE 8. Bar chart of Algorithm 2

Comparison Analysis:
Since both algorithms yields the same result. So both algorithms are valid and strong we can use any algorithm according to our decision marking problem. Also both algorithms have different formulation strategies we can produced lightly different results but the final optimal choices would same in any decision marking problem. Table 18 gives the comparison analysis between Algorithm 1 and Algorithm 2.

Algorithms	Alternative selected	Analysis
Algorithm 1	$\left[\frac{30}{x_2}\right]$	Final result remains same
Algorithm 2	$\left[\frac{30}{x_2}\right]$	Final result remains same

TABLE 18. Comparison Analysis

Remark:

(i) It is great significance mention here that both algorithms as given above give the same result.

(ii) In this models we can notice that the use of SMR- scientific procedure refines the primary evaluation results and permit the experts to choose the optimal alternative in a suitable manner. Incredibly, SMR-upper approximation can be used to add the optimal objects possibly neglected by the experts in primary evaluation while SMR- lower approximation can be used to remove the objects that are asymmetrically selected as optimal. Hence SMR-reduce the error to some extent caused by personal nature of analyst during group decision-making.

5. Conclusion

We introduced novel concept of soft multi rough set (SMR-set) with a fascinating fusion of soft set, multiset and rough set. We proposed soft multi rough approximation spaces (SMR-approximations spaces). We presented some fundamental properties of SMR-approximations along with their examples and results. We also discussed the variation between some properties of Pawlak approximation space, soft approximation spaces and the same properties of soft multi approximation spaces. Furthermore, we presented Algorithm 1 and Algorithm 2 based on soft multi rough sets for multi-criteria group decision-making (MCGDM) for the selection of humanoid robot and to deal with vagueness and uncertainties in the field of artificial intelligence.

References

[1] M. Akram, A. Adeel, J. C. R. Alcantud, *Group decision-making methods based on hesitant N-soft sets*, Expert System With Applications, **115**(2019), 95-105.

[2] M. Akram, A. Adeel, J. C. R. Alcantud, *Fuzzy N-soft sets: A novel model with applications*, Journal of Intelligent & Fuzzy Systems, **35**(4)(2018), 4757-4771.

[3] M. Akram, F. Feng, A. B. Saeid and V. Leoreanu-Fotea, *A new multiple criteria decision-making method based on bipolar fuzzy soft graphs*, Iranian Journal of Fuzzy Systems, **15**(4)(2018), 73-92.

[4] M. I. Ali, F. Feng, X. Y. Liu, W. K. Min and M. Shabir, *On some new operations in soft set theory*, Computers and Mathematics with Applications, **57**(2009), 1547-1553.

[5] M. I. Ali, *A note on soft sets, rough soft sets and fuzzy soft sets*, Applied Soft Computing **11**(2011), 3329-3332.

[6] K. T. Atanassov, *Intuitionistic Fuzzy sets*, VII ITKRs Session, Sofia (deposed in Central Science-Technical Library of Bulgarian Academy of Science 1697(84)(1983).

[7] K. T. Atanassov, *Intuitionistic Fuzzy sets*, Fuzzy sets and Systems, **20**(1986), 87-96.

[8] K. V. Babitha and S. J. John, *On soft multi-set*, Annals of Fuzzy Mathematics and Informatics, **5**(1)(2013), 35-44.

[9] M. Y. Bakier, A. A. Allam and SH. S. Abd-Allah, *Soft rough topology*, Annals of Fuzzy Mathematics and Informatics, **11**(2)(2016), 4-11.

[10] W. D. Blizard, *Multiset theory*, Notre Dame J. Formal. Logic, **30**(1989), 36-65.

[11] W. D. Blizard, *Dedekind multisets and function shells*, Theoretical Computer Science, **110**(1993), 79-98.

[12] C. S. Calude, G. Paun, G. Rozenberg and A. Salomaa *Multiset Processing*, Springer-Verlag, Germany,(2001).

[13] N. Çağman, S. Karataş and S. Enginoglu, *Soft topology*, Computers and Mathematics with Applications, **62**(2011), 351-358.

[14] F. Feng, Changxing Li, B. Davvaz and M. I. Ali, *Soft sets combined with fuzzy sets and rough sets: a tentative approach*, Soft Computing, **14**(2010), 899-911.

[15] F. Feng, X. Liu, V. Leoreanu-Fotea and Y. B. Jun, *Soft sets and Soft rough sets*, Information Sciences, **181** (2011), 1125-137.

[16] F. Feng, X. Liu, V. Leoreanu-Fotea and Y. B. Jun, *Soft sets and Soft rough sets*, Information Sciences, **181** (2011), 1125-137.

[17] F. Feng, H. Fujita, M.I. Ali, R.R. Yager, X. Liu, *Another view on generalized intuitionistic fuzzy soft sets and related multiattribute decision making methods*, IEEE Transactions on Fuzzy Systems, **27**(3)(2019), 474-488.

[18] H. Garg and R. Arora, *Distance and similarity measures for dual hesitant fuzzy soft sets and their applications in multicriteria decision-making problem*, International Journal for Uncertainty Quantification, **7**(3)(2017), 229-248.

[19] H. Garg and R. Arora, *Dual hesitant fuzzy soft aggregation operators and their applicatio in decision-making*, Cognitive Computation **10**(5)(2018), 769-789.

[20] M. R. Hashmi, M. Riaz and F. Smarandache, *m-polar Neutrosophic Topology with Applications to Multi-Criteria Decision-Making in Medical Diagnosis and Clustering Analysis*, International Journal of Fuzzy Systems, (2019), https://doi.org/10.1007/s40815-019-00763-2.

[21] M. R. Hashmi and M. Riaz, *A Novel Approach to Censuses Process by using Pythagorean m-polar Fuzzy Dombi's Aggregation Operators*, Journal of Intelligent & Fuzzy Systems, (2019), DOI: 10.3233/JIFS-190613.

[22] K. Hayat, M. I. Ali, B.Y.Cao, and F.Karaaslan, *New results on type-2 soft sets*, Hacettepe Journal of Mathematices and Statistic, DOI:10.15672/HJMS.2017.484

[23] T. Herawan and M. M. Deris, *On Multi-Soft Set Construction in information Systems*, Emerging Intelligent Computing Technology and Applications. With Aspects of Artificial Intelligence, 5th International Conference on Intelligent Computing, ICIC 2009, Ulsan, South Korea, September 16-19, (2009), DOI: 10.1007/978-3-642-04020-7-12.

[24] F. Karaaslan, *Soft Classes and Soft Rough Classes with Applications in Decision Making*, Mathematical Problems in Engineering, 2016, 1-11. DOI:10.1155/2016/1584528.

[25] F. Karaaslan, N. Çağman and S. Enginoglu, *Soft Lattices*, Journal of New Results in Science(1), 5-17-2012.

[26] M. Kryskiewicz, *Rough set approach to incomplete information systems*, Information Sciences **112**(1998), 39-49.

[27] D. Molodtsov, *Soft set theoty-first results*, Computers and Mathematics with Applications, **37**(4-5)(1999),19-31.

[28] P. K. Maji, A.R. Roy and R.Biswas, *An application of soft sets in a decision making problem*, Computers and Mathematics with Applications, **44**(8-9)(2002),1077-1083.

[29] P. K. Maji, A.R. Roy and R.Biswas, *Soft set theory*, Computers and Mathematics with Applications, **45**(4-5)(2003), 555-562.

[30] K. Naeem, M. Riaz, X.D. Peng and D. Afzal, *Pythagorean Fuzzy Soft MCGDM Methods Based on TOPSIS, VIKOR and Aggregation Operators*, Journal of Intelligent & Fuzzy Systems, **37**(5)(2019), 6937-6957. DOI:10.3233/JIFS- 190905.

[31] K. Naeem, M. Riaz, and Deeba Afzal, *Pythagorean m-polar Fuzzy Sets and TOPSIS method for the Selection of Advertisement Mode*, Journal of Intelligent & Fuzzy Systems, **37**(6)(2019), 8441-8458. DOI: 10.3233/JIFS-191087.

[32] Z. Pawlak, *Rough sets*, Int. J. Inf. Comput. Sci. **11**(1982),341-356.

[33] Z. Pawlak and A. Skowron, *Rough sets: Some extensions*, Information Sciences **177**(2007),28-40.

[34] Pi-Yu Li, Z. Kong, Wen-Li Liu, Chang-Tao Xue, *On multi-soft rough sets*, 2017 29th Chinese Control And Decision Conference (CCDC), IEEE xplore (2017),3051-3055. doi.10.1109/CCDC.2017.7979032.

[35] M. Riaz, F. Samrandache, A. Firdous and F. Fakhar, *On Soft Rough Topology with Multi-Attribute Group Decision Making*, Mathematics **7**(67)(2019), doi:10.3390/math7010067.

[36] M. Riaz, B. Davvaz, A. Firdous and F. Fakhar, *Novel Concepts of Soft Rough Set Topology with Applications*, Journal of Intelligent & Fuzzy Systems **36**(4)(2019), 3579-3590.

[37] M. Riaz, N. Çağman, I. Zareef and M. Aslam, *N-Soft Topology and its Applications to Multi-Criteria Group Decision Making*, Journal of Intelligent & Fuzzy Systems 36(6)(2019), 6521-6536. DOI:10.3233/JIFS-182919.

[38] M. Riaz and K. Naeem, *Measurable Soft Mappings*, Punjab University Journal of mathematics, **48**(2)(2016), 19-34.

[39] M. Riaz, M. R. Hashmi, *Fuzzy parameterized fuzzy soft compact spaces with decision-making*, Punjab University Journal of Mathematics 50(2)(2018),131-145.

[40] M. Riaz and M. R. Hashmi, *MAGDM for agribusiness in the environment of various cubic m-polar fuzzy averaging aggregation operators*, Journal of Intelligent & Fuzzy Systems, **37**(3)(2019), 3671-3691. DOI:10.3233/JIFS-182809.

[41] M. Riaz and M. R. Hashmi, *Linear Diophantine fuzzy set and its applications towards multi-attribute decision making problems*, Journal of Intelligent & Fuzzy Systems, **37**(4)(2019), 5417-5439. DOI:10.3233/JIFS-190550.

[42] M. Riaz and M. R. Hashmi, Soft Rough Pythagorean m-Polar Fuzzy Sets and Pythagorean m-Polar Fuzzy Soft Rough Sets with Application to Decision-Making, Computational and Applied Mathematics (2019), DOI: 10.1007/s40314-019-0989-z.

[43] M. Riaz and S. T. Tehrim, *Certain properties of bipolar fuzzy soft topology via Q-neighborhood*, Punjab University Journal of Mathematics **51**(3)(2019), 113-131.

[44] M. Riaz and S. T. Tehrim, *Cubic bipolar fuzzy ordered weighted geometric aggregation operators and their application using internal and external cubic bipolar fuzzy data*, Computational & Applied Mathematics, **38**(2)(2019), 1-25.

[45] M. Riaz and S. T. Tehrim, *Multi-attribute group decision making based cubic bipolar fuzzy information using averaging aggregation operators*, Journal of Intelligent & Fuzzy Systems, **37**(2)(2019), 2473-2494.

[46] M. Riaz and S. T. Tehrim, *Bipolar fuzzy soft mappings with application to bipolar disorders*, International Journal of Biomathematics, (2019). Doi.org/10.1142/S1793524519500803.

[47] S. T. Tehrim and M. Riaz, *A novel extension of TOPSIS to MCGDM with bipolar neutrosophic soft topology*, Journal of Intelligent and Fuzzy Systems (2019). DOI:10.3233/JIFS-190668.

[48] T. K. Shinoj and S. J. John, *Intuitionistic Fuzzy Multiset and its application in Medical Diagnosis*, International Journal of Mathematical and Computational Sciences, WASET **6**,(2012),34-38.

[49] A. Syropoulos, *Mathematics of Multisets*, GR-671 00 Xanith, GREECE.
[50] M.L. Thivagar, C. Richard and N.R. Paul, *Mathematical Innovations of a Modern Topology in Medical Events*, International Journal of Information Science **2(4)**(2012),33-36.
[51] X. Wang, Y. Liu, P. Li1, J. Liu, *On multi-granularity soft rough sets*, 2016 28th Chinese Control and Decision Conference (CCDC), IEEE xplore (2016), 6657-6662. doi.10.1109/CCDC.2016.7532195.
[52] Xueling Ma, Q. Liu and J. Zhan, *A survey of decision making methods based on certain hybrid soft set models*, Artificial Intelligence Review, **47** (2017),507-530.
[53] R. R. Yager, *On the theory of bags*, International Journal of General Systems **13**(1986),23-37.
[54] R. R. Yager, *Pythagorean fuzzy subsets*, IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), 2013 Joint, Edmonton, Canada, IEEE (2013),57-61.
[55] R. R. Yager and A. M. Abbasov, *Pythagorean membership grades, complex numbers, and decision making*. International Journal of Intelligent Systems **28**(5)(2013),436-452.
[56] R. R. Yager, *Pythagorean membership grades in multi-criteria decision making*, IEEE Transactions on Fuzzy Systems, **22**(4)(2014),958-965.
[57] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (1965),338-353.
[58] J. Zhan, Q. Liu and T. Herawan, *A novel soft rough set: soft rough hemirings and its multicriteria group decision making*, Applied Soft Computing, **54**(2017), 393-402.
[59] J. Zhan, M.I. Ali and N. Mehmood, *On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods*, Applied Soft Computing, **56**(2017), 446-457.
[60] J. Zhan and J.C.R. Alcantud, *A novel type of soft rough covering and its application to multicriteria group decision making*, Artificial Intelligence Review, **52**(4)(2019), 2381-2410.
[61] J. Zhan, H. M. Malik and M. Akram, *Novel decision-making algorithms based on intuitionistic fuzzy rough environment*, International Journal of Machine Learning and Cybernetics, **10**(6)(2019), 1459-1485. doi.org/10.1007/s13042-018-0827-4.
[62] X. L. Zhang, Z. S. Xu, *Extension of TOPSIS to multiple criteria decision making with pythagorean fuzzy sets*, International Journal of Intelligent Systems, **29**(2014), 1061-1078.
[63] L. Zhang, J. Zhan, Z.X. Xu, *Covering-based generalized IF rough sets with applications to multi-attribute decision-making*, Information Sciences, **478**(2019), 275-302.